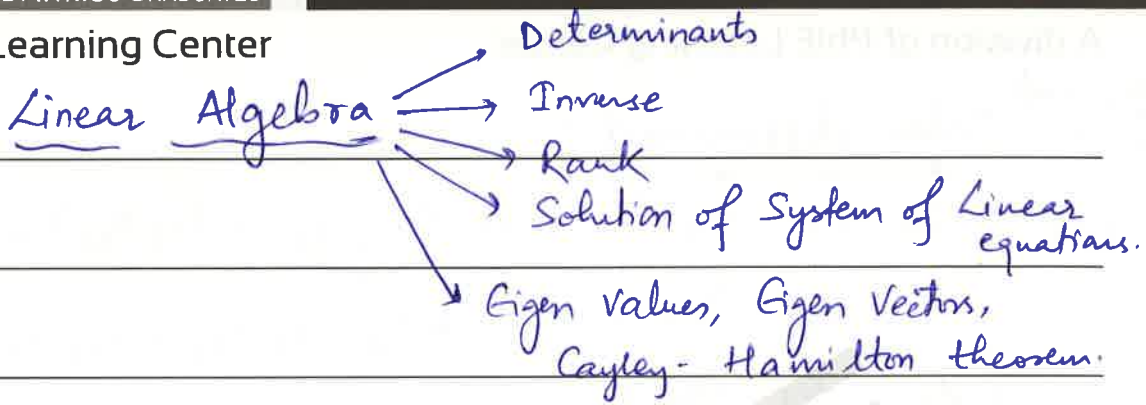


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① Determinants:-

→ $\|\vec{x}\|$ → norm of \vec{x}

→ Determinants is for Square matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \rightarrow \text{arrangement is called Matrix.}$$

② 2nd Order determinant

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then the expression $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ is called

2nd order determinant of $A_{2 \times 2}$ & it is denoted by $|A|$ or $\det(A)$.

The expansion of $|A| = (a_{11}a_{22} - a_{12}a_{21})$

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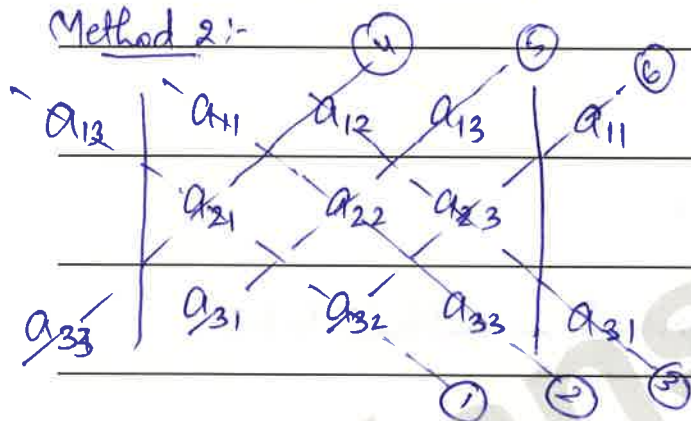
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③ 3rd Order determinant :-

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Method 2:-



$$|A| = \{ \textcircled{1} + \textcircled{2} + \textcircled{3} \} - \{ \textcircled{4} + \textcircled{5} + \textcircled{6} \}$$

④ 4th Order Determinant :-

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} - a_{14} \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}$$

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⑤ Elementary operations:-

- (i) $R_i \leftrightarrow R_j$
- (ii) $R_i \rightarrow KR_i (K \neq 0)$
- (iii) $R_j \rightarrow R_j + KR_i$

Note: ① Always select that Row or Column which has more No. of zero to evaluate the value.

② Total no. of terms in the expansion of a determinant of order

n is $n!$

$$\begin{cases} 2 \times 2 \rightarrow 2 = 2! \\ 3 \times 3 \rightarrow 6 = 3! \\ 4 \times 4 \rightarrow 24 = 4! \end{cases}$$

③ If A is $(m \times n)$ and B is $(n \times p)$, then how many no. of multiplication

& addition are involved in computing matrix product $A \times B$?

$$[A]_{m \times n} \times [B]_{n \times p} = [AB]_{m \times p}$$

equal always for product of Matrix.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & \dots & a_{mn} \end{bmatrix}_{m \times n} \begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{1p} \\ b_{21} & b_{22} & b_{23} & \dots & b_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & \dots & b_{np} \end{bmatrix}_{n \times p}$$

$$= (a_{11}b_{11} + a_{12}b_{21} + \dots + a_{1n}b_{n1}) + (a_{11}b_{12} + a_{22}b_{22} + \dots + a_{1n}b_{n2}) + \dots$$

(n-1) additions for 1 term.
 $n \rightarrow$ multiplication for 1 terms

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Addition:- $mp(n-1)$ Multiplication (mpn)

Properties of determinants :-

① $|A_{n \times n} B_{n \times n}| = |A_{n \times n}| |B_{n \times n}|$

② $|A^k| = |A|^k$

③ $|A^T| = |A|$ (Transpose).

④ If two rows are same, $|A| = 0$

⑤ If two rows are interchanged, $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ b_{21} & b_{22} & b_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$

$|B| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ c_{31} & c_{32} & c_{33} \\ b_{21} & b_{22} & b_{23} \end{vmatrix} = -|A|$

or $(-1)^k |A|$

$|C| = \begin{vmatrix} c_{31} & c_{32} & c_{33} \\ b_{21} & b_{22} & b_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix} = |A|$ $K = \text{no. of times of interchange of two rows.}$

⑥ $\begin{vmatrix} 1 & 2 & 3 \\ 4K & 5K & 6K \\ 7 & 8 & 9 \end{vmatrix} = K \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \rightarrow$ for determinants.

for Matrix,

$K \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} K & 2K & 3K \\ 4K & 5K & 6K \\ 7K & 8K & 9K \end{bmatrix} \rightarrow$ for Matrix.
 $|K A_{n \times n}| = K^n |A_{n \times n}|$

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- ⑦ $A = \begin{bmatrix} 2 & 0 & 8 \\ 0 & 9 & 0 \\ 0 & 0 & 10 \end{bmatrix}$
- Upper Triangular Matrix
 - Lower Triangular Matrix
 - Diagonal Matrix.
 - Scalar matrix. = k (Identity Matrix).
 - Identity matrix.
 - Null matrix.

⊛ Singular Matrix $\rightarrow |A| = 0$

Inverse of Square Matrix:-

$$A^{-1} = \frac{\text{Adj}(A)}{|A|}$$

⊛ Idempotent; $(A^2 = A)$
Matrix

⊛ Involutionary Matrix;
 $(A^2 = I)$

$$\text{Adj}(A) = \frac{(\text{Cofactor matrix of } A)^T}{|A|} \quad (\text{Explained Afterwards}).$$

⊛ Equality of Matrices:- (Add)

$$\begin{bmatrix} x-y & p+q \\ p-q & x+y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 10 \end{bmatrix}$$

$$\left. \begin{array}{l} \text{then, } x-y=2 \\ p+q=5 \\ p-q=10 \\ x+y=10 \end{array} \right\}$$

⊛ Addition / Subtraction of Matrices:-

$$A \pm B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \pm \begin{bmatrix} 4 & 6 \\ 7 & 8 \end{bmatrix} = \begin{cases} \begin{bmatrix} 5 & 8 \\ 10 & 13 \end{bmatrix} & (\text{Addition}) \\ \begin{bmatrix} -3 & -4 \\ -4 & -3 \end{bmatrix} & (\text{Subtraction}) \end{cases}$$

Note:- ① Properties of Matrix Multiplication:-

(a) $(AB)C = A(BC)$

(b) $AB = 0 \Rightarrow A \neq 0 \text{ or } B \neq 0$ {not necessary}

(c) $AB \neq BA$.

(d) $AB = AC \Rightarrow B = C$ (iff A is non-singular Matrix)