GATE Aerospace Coaching by Team IGC

Engineering Mathematics

LINEAR ALGEBRA

- Determinant
- Inverse
- Rank
- Solution of system of Linear equation
- Eigen values, Eigen Vectors, Cayley-Hamittom theorem

(1). Determinants

$\|\vec{x}\|$ norm of $\vec{x}$

Determinants is for square matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & \cdots & a_{nm} \end{bmatrix}$$ ------Arrangement is called Matrix.

(2). 2^{nd} order determinant

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then the expression $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ is called 2^{nd} order determinant of A_{2x2} and it is denoted by $|A|$ or det(A) the expansion of

$$|A| = (a_{11}a_{22} - a_{12}a_{21})$$

(3). 3^{rd} order determinant

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

(4). 4^{th} order determinant

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} - a_{14}\begin{vmatrix} a_{12} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}$$

(5). Elementary operations

(i). $R_i \leftrightarrow R_j$

(ii). $R_i \rightarrow KR_i$ $(K \neq 0)$

(iii). $R_j \rightarrow R_j + KR_i$

Note:-

(1). Always select that Row or Column which has more No of zero to evaluate the value.

(2). Total No of terms in the expansion of a determinant of order n is n!

$$\begin{cases} 2X2 \rightarrow 2 = 2! \\ 3X3 \rightarrow 6 = 3! \\ 4X4 \rightarrow 24 = 4! \end{cases}$$

(3). If A is (m x n) and B is (n x p), then how many no of multiplication and addition are involved in computing matrix product A x B

$[A_{m \times n}] \times [B]_{n \times p} = [AB]_{m \times p}$

*Equal always for product of Matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots\cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots\cdots & a_{mn} \end{bmatrix}_{m \times n} \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots\cdots & b_{1n} \\ b_{21} & b_{22} & b_{23} & \cdots\cdots & b_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & b_{n3} & \cdots\cdots & b_{np} \end{bmatrix}_{n \times p}$$

$= (a_{11}b_{11}+a_{12}b_{21}+\ldots+a_{1n}b_{n2})+(a_{11}b_{12}+a_{22}b_{22}+\ldots+a_{1n}b_{n2})+\ldots$

Note:-

(n-1) additions for 1term

N multiplication for 1 term

Addition:-

mp(n-1)

Multiplication

mpn

Properties of determinants:-

(1). $|A_{n \times n} B_{n \times n}| = |A_{n \times n}|\,|B_{n \times n}|$

(2). $|A^K| = |A|^K$

(3). $|A^T| = |A|$ (Transpose)

(4). If two rows are same, |A| = 0

(5). If two rows are interchanged,

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ b_{21} & b_{22} & b_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

$$|B| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ c_{31} & c_{32} & c_{33} \\ b_{21} & b_{22} & b_{23} \end{vmatrix}$$

$|B| = -|A|$ or $(-1)^K|A|$

K= no of times of interchange of two rows.

$$|C| = \begin{vmatrix} c_{31} & c_{32} & c_{33} \\ b_{21} & b_{22} & b_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix} = |A|$$

(6). $\begin{vmatrix} 1 & 2 & 3 \\ 4k & 5k & 6k \\ 7 & 8 & 9 \end{vmatrix} = k \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ for determinants

For matrix,

$$k\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix} = \begin{bmatrix}k & 2k & 3k\\ 4k & 5k & 6k\\ 7k & 8k & 9k\end{bmatrix}$$ for matrix $|K A_{n \times n}| = K^n |A_{n \times n}|$

(7). $A = \begin{bmatrix}2 & 0 & 8\\ 0 & 9 & 0\\ 0 & 0 & 10\end{bmatrix}$

- Upper triangle matrix
- Lower triangle matrix
- Diagonal matrix
- Scalar matrix = k (identity matrix)
- Identity matrix
- Dull matrix

Singular matrix |A| = 0

Idempotent matrix ($A^2 = A$)

Involutory matrix ($A^2 = I$)

Inverse of square matrix

$$A^{-1} = \frac{Adj(A)}{|A|}$$

Adj (A) = $\frac{(A)^T}{|A|}$, here, $(A)^T$ is cofactor matrix

Equality of matrices

$$\begin{bmatrix}x-y & p+q\\ p-q & x+y\end{bmatrix} = \begin{bmatrix}2 & 5\\ 1 & 10\end{bmatrix}$$

Then,

x-y = 2

p+q = 5

p-q = 1

x+y = 10

Addition/Subtraction of matrix

$$A \pm B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \pm \begin{bmatrix} 4 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\text{In addition} = \begin{bmatrix} 5 & 8 \\ 10 & 13 \end{bmatrix}$$

$$\text{In subtraction} = \begin{bmatrix} -3 & -4 \\ -4 & -3 \end{bmatrix}$$

Note:-

(1). Properties of matrix multiplication

(a). $(AB)C = A(BC)$

(b). $AB = 0 \Rightarrow A \neq 0$ or $B \neq 0$ { not necessary}

(c). $AB \neq BA$

(d). $AB = AC \Rightarrow B = C$ (if A is non-singular matrix)