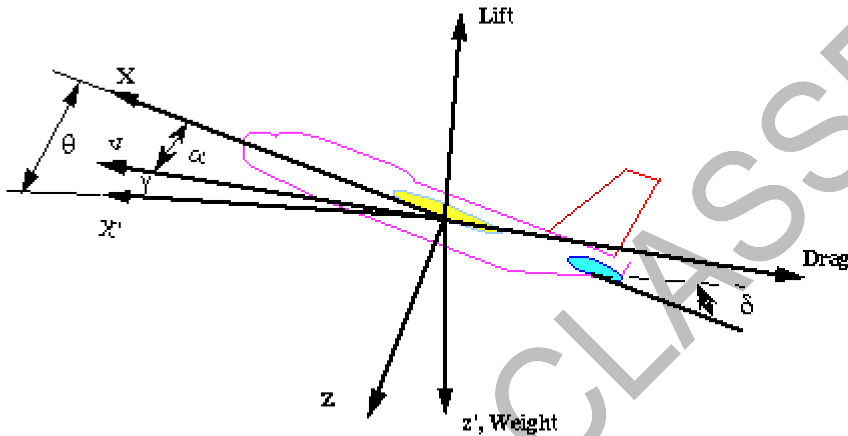
GATE Aerospace Coaching by Team IGC

Aircraft Performance

Steady Level Flight



Where,

α – is angle of attack

α_T – is angle of inclination b/w flight path direction and thrust vector.

γ - is climb angle (angle between flight path and horizontal)

β – is pitch angle (angle between chord line and horizontal)

$\beta = \alpha + \gamma$

(i.e).Pitch angle = AOA + Climb angle

Four physical forces:-

L – Lift perpendicular to flight path direction (relative wind)

D – Drag parallel to relative wind.

T – Thrust (at an angle α_T w.r.t to flight path)

W – Weight perpendicular to horizontal.

According to Newton's second law (for curvilinear motion)

$$\sum F_{\|el} = m\frac{dv}{dt} \qquad \sum F_{\perp r} = \frac{mv_2}{r_c}$$

Where, r_c – is radius of curvature.

Resolving forces $\|^{el}$ and $\perp^{r}$ to flight path we get,

$$\sum F_{\|el} = T\cos\alpha_T - W\sin\gamma - D = m\frac{dv}{dt}$$

$$\sum F_{\perp r} = L - W\cos\gamma + T\sin\alpha_T = \frac{mv_2}{r_c}$$

Un-accelerated flight performance:-

For un-accelerated flight,

$$\frac{dv}{dt} = 0, \frac{v^2}{r_c} = 0$$

Then,

$$T\cos\alpha_T - W\sin\gamma - D = 0$$

$$L - W\cos\gamma + T\sin\alpha_T = 0$$

In case of straight steady and level flight,

$$\alpha_T = 0, \gamma = 0$$

$$T - D = 0 \quad ; \quad L - W = 0$$

Consider for straight and level flight

T = D ; L = W

I). Condition of minimum drag:-

$$L = W\ ,\ T = D$$

$$\frac{L}{D} = 1 = \frac{D}{T}$$

$$T = \frac{W}{\left(\frac{L}{D}\right)} \qquad (1)$$

$$(T_R)_{\min} = D_{\min} = \frac{1}{\left(\frac{L}{D}\right)_{\max}} W$$

Where,

$$C_D = a + b\,C_L^2$$

$$a = C_{D,O}$$

$$b = \frac{1}{\pi \rho AR}$$

For minimum drag, $\left(\frac{L}{D}\right)$ should be maximum

$$T_R = D = C_D \frac{1}{2}\rho_\infty v_\infty^{\ 2} s \qquad L = W = C_L \frac{1}{2}\rho_\infty v_\infty^{\ 2} s$$

$$= (a + bC_L^{\ 2})\frac{1}{2}\rho_\infty v_\infty^{\ 2} s \qquad C_L = \frac{W}{\frac{1}{2}\rho_\infty v_\infty^{\ 2} s}$$

$$= \left(a\frac{1}{2}\rho_\infty s\right) v_\infty^{\ 2} + b\left[\frac{W^2}{\frac{1}{2}\rho_\infty s}\right]\frac{1}{v_\infty^{\ 2}}$$

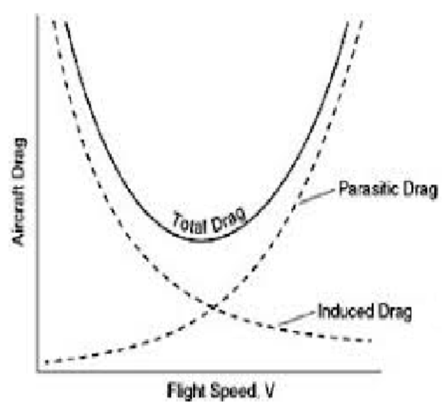
$$T_R = k_1 v_\infty^{\,2} + \frac{k_2}{v_\infty^{\,2}} \qquad (1A)$$

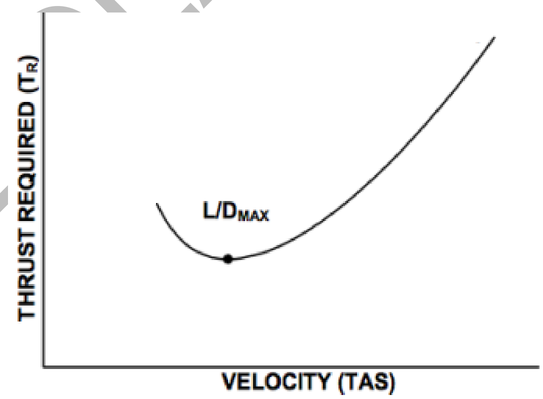
T_R = profile drag + induced drag

Where,

$$k_1 = a\frac{1}{2}\rho_\infty s$$

$$k_2 = \frac{bW^2}{\frac{1}{2}\rho_\infty s}$$





$$\frac{dT_R}{dv_\infty} = 0$$

$$k_1(2v_\infty) - \frac{2k_2}{v_\infty^{\,3}} = 0$$

$$v_\infty = \sqrt[4]{k_2/k_1}$$

$$v_{md} = \sqrt[4]{b/a}\sqrt{\frac{2W}{\rho s}} \qquad (1B)$$

Sub (1B) in (1A) we get

$$D_{min} = 2W\sqrt{ab} \qquad (1C)$$

Minimum drag is independent of attitude from equ (1)

$$T_R = \frac{W}{\left(\frac{L}{D}\right)} = \frac{W}{\left(\frac{C_L}{C_D}\right)}$$

For T_R minimum drag is minimum.

(i.e) $\left(\frac{C_L}{C_D}\right)$ is maximum.(i.e) $\left(\frac{C_D}{C_L}\right)$ should be minimum.

(i.e) $\frac{d\frac{C_D}{C_L}}{dC_L} = 0 \qquad \frac{C_D}{C_L} = \frac{a + bC_L^2}{C_L}$

$$\frac{d\frac{C_D}{C_L}}{dC_L} = \frac{(a + bC_L^2).1 - C_L(2bC_L)}{C_L^2} = 0$$

$$a - \frac{bC_L^2}{C_L^2} = 0$$

$$a - bC_L^2 = 0$$

$a = bC_L^2$ (i.e). $C_{Do} = C_{Di}$

$$C_{Lmd} = \sqrt{a/b}$$

$$C_{Dmd} = 2a$$

$$\left(\frac{C_L}{C_D}\right)_{md} = \frac{1}{2\sqrt{ab}}$$

It is the condition for max endurance of jet engine aircraft and condition for ma range of piston engine aircraft.

(2). Minimum power required:-

$$p_R = T_R v_\infty \qquad \text{fromequ (1A)}$$

$$= \left[k_1 v_\infty{}^2 + \frac{k_2}{v_\infty{}^2}\right] v_\infty$$

$$p_R = k_1 v_\infty{}^3 + \frac{k_2}{v_\infty} \qquad (2A)$$

For p_R to be minimum,

$$\frac{dp_R}{dv_\infty} = 0$$

$$3k_1 v_\infty{}^2 - \frac{k_2}{v_\infty{}^2} = 0$$

$$3k_1 v_\infty{}^2 = \frac{k_2}{v_\infty{}^2}$$

$$v_\infty = \sqrt[4]{\frac{k_2}{3k_1}}$$

$$v_{mp} = \sqrt[4]{\frac{b}{3a}}\sqrt{\frac{2w}{\rho_\infty s}} \qquad (2B)$$

$$v_{mp} = \frac{1}{\sqrt[4]{3}} v_{md}$$

$$v_{mp} = 0.7598 v_{md} \quad (2C)$$

$$p_R = T_R v_\infty$$

$$= \frac{w}{\left(\frac{C_L}{C_D}\right)} \sqrt{\frac{2w}{\rho_\infty s C_L}}$$

$$p_R = \frac{w^{3/2}}{\left(\frac{C_L{}^{3/2}}{C_D}\right)} \sqrt{\frac{2}{\rho_\infty s}}$$

$$p_R \alpha \frac{1}{\left(\frac{C_L{}^{3/2}}{C_D}\right)}$$

$$(p_R)_{\min} \alpha \frac{1}{\left(\frac{C_L{}^{3/2}}{C_D}\right)_{\max}}$$

For minimum power required $\left(\frac{C_L{}^{3/2}}{C_D}\right)$ should be maximum (i.e) $\left(\frac{C_D}{C_L{}^{3/2}}\right)$ should be minimum

for min p_R

$$\frac{d\left(\frac{C_D}{C_L{}^{3/2}}\right)}{dC_L} = 0 \qquad \frac{C_D}{C_L} = \frac{a + bC_L{}^2}{C_L{}^{3/2}}$$

$$\frac{\left(a + bC_L{}^2\right)\left(3/2 C_L{}^{1/2}\right) - C_L{}^{3/2}\left(2bC_L\right)}{C_L{}^3} = 0$$

On simplification we get

$$3a = bC_L^{\,2} \qquad \text{(i.e)}\ C_{Do} = \frac{1}{3}C_{Di}$$

$$C_{Lmp} = \sqrt{\frac{3a}{b}} \qquad C_{Lmp} = \sqrt{3}C_{Lmd}$$

$$C_{Lmp} = .732C_{Lmd}$$

$$C_{Dmp} = 4a \qquad C_{Dmp} = 2(2a)$$

$$= 2(C_{Lmd})$$

$$C_{Dmp} = 2C_{Lmd}$$

$$\left(\frac{L}{D}\right)_{mp} = \frac{\sqrt{3}}{2}\frac{1}{2\sqrt{ab}} = 0.866\left(\frac{L}{D}\right)_{md}$$

It is the condition for max endurance for piston engine aircraft.

(3). Minimum drag to velocity ratio

w.k.t $$T_R = D = k_1 v_\infty^{\,2} + \frac{k_2}{v_\infty^{\,2}}$$

$$\frac{D}{v_\infty} = k_1 v_\infty + \frac{k_2}{v_\infty^{\,3}} \qquad \text{(3A)}$$

For min $\left(\dfrac{D}{v_\infty}\right)$

$$\frac{d\left(\frac{D}{v_\infty}\right)}{dv_\infty} = 0$$

$$k_1 - \frac{3k_2}{v_\infty{}^4} = 0$$

$$v_\infty = \sqrt[4]{\frac{3k_2}{k_1}}$$

$$V_{m\left(\frac{D}{v_\infty}\right)} = \sqrt[4]{\frac{3b}{a}}\sqrt{\frac{2w}{\rho_\infty s}} \qquad \text{(3B)}$$

$$V_{m\left(\frac{D}{v_\infty}\right)} = \sqrt[4]{3}V_{md}$$

$$Vm\left(\frac{D}{v_\infty}\right) = 1.316V_{md} \qquad \text{(3C)}$$

$$\frac{D}{v_\infty} = \frac{w}{\left(\frac{C_L}{C_D}\right)}\frac{1}{v_\infty}$$

$$= \frac{1}{\left(\frac{C_L}{C_D}\right)}\frac{w}{\sqrt{\frac{2w}{\rho_\infty s C_L}}}$$

$$\frac{D}{v_\infty} = \frac{1}{\left(\frac{C_L{}^{1/2}}{C_D}\right)}\sqrt{\frac{w\rho_\infty s}{2}}$$

$$\left(\frac{D}{v_\infty}\right)_{\min} \alpha \frac{1}{\left(\frac{C_L{}^{1/2}}{C_D}\right)_{\max}}$$

For $\left(\frac{D}{v}\right)_{\min}$ then $\left(\frac{C_L{}^{1/2}}{C_D}\right)$ should be maximum (i.e) $\left(\frac{C_D}{C_L{}^{1/2}}\right)$ is minimum.

$$\frac{d\left(\frac{C_D}{C_L^{1/2}}\right)}{dC_L} = 0$$

$$\frac{d\left(\frac{a + bC_L^{1/2}}{C_L^{1/2}}\right)}{dC_L} = \frac{\left(a + bC_L^{2}\right)\left(\frac{1}{2}C_L^{-1/2}\right) - C_L^{1/2}\left(2bC_L\right)}{C_L} = 0$$

On simplification we get

$$2bC_L^{2} = a \qquad \text{(i.e) } C_{Do} = 3C_{Di}$$

$$C_{Lm\left(\frac{D}{v_\infty}\right)} = \sqrt{\frac{a}{3b}}$$

$$C_{Lm\left(\frac{D}{v_\infty}\right)} = \frac{1}{\sqrt{3}}C_{L\,md} = 0.577C_{Lmd}$$

$$C_{Dm\left(\frac{D}{v_\infty}\right)} = a + bC_L^{2} = a + b\left(\frac{a}{3b}\right)$$

$$C_{Dm\left(\frac{D}{v_\infty}\right)} = \frac{4a}{3} = \frac{2}{3}(2a)$$

$$= \frac{2}{3}C_{Dmd}$$

$$CDm\left(\frac{D}{v_\infty}\right) = 0.667C_{Dmd}$$