GATE Aerospace Coaching by Team IGC

Aircraft Structures Basics

Topics to Study

- **Basic Elasticity**
  - **Stresses**
  - **Strain**
  - **Stress-Strain Relationship**
  - **Volumetric Strain**
  - **Strain Energy**
  - **Thermal Stress**
  - **Compatibility Eqn and Airy Stress Function**
- **Axially loaded Member**
- **Torsion of Shaft**
- **Beam**
  - **SFBM Diagram**
  - **Bending and Shear Stresses**
  - **Deflection of Beam**
  - **Indeterminate Beam**
- **Thin Shell (Cylindrical and Spherical Shell)**
- **Buckling of Column (Euler Buckling Theory)**
- **Theory of Failure**

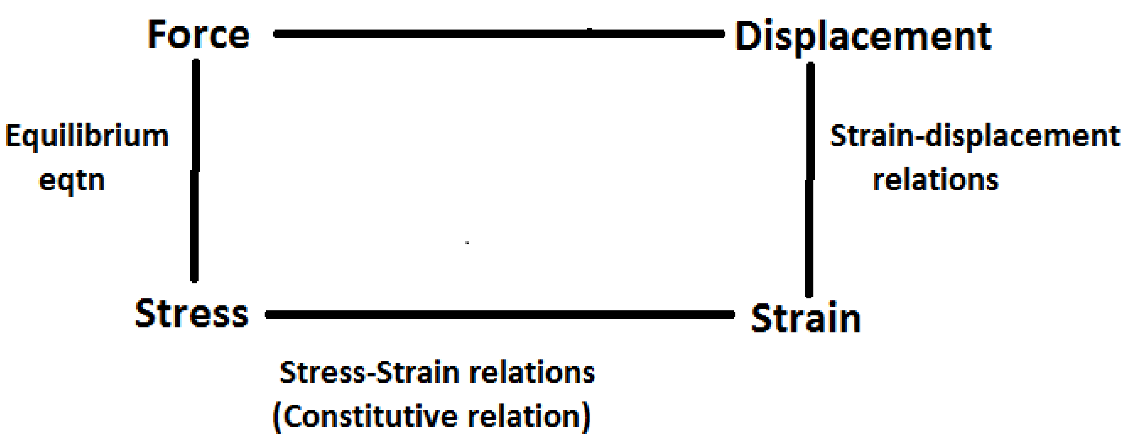
Ref: Mechanics of Material by GERE

Aircraft Structures by Megson (1st Two Chapter)

BASIC ELASTICITY

Topics:-

1. Stresses
2. Strains
3. Stress-strain relationships
4. Volumetric Strain
5. Strain Energy
6. Thermal Stress
7. Compatibility equations
8. Airy stress function

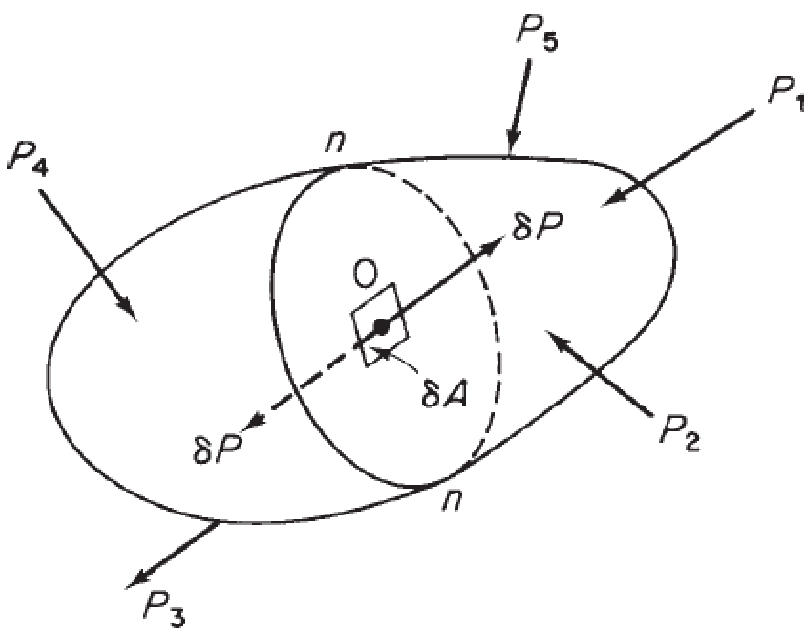


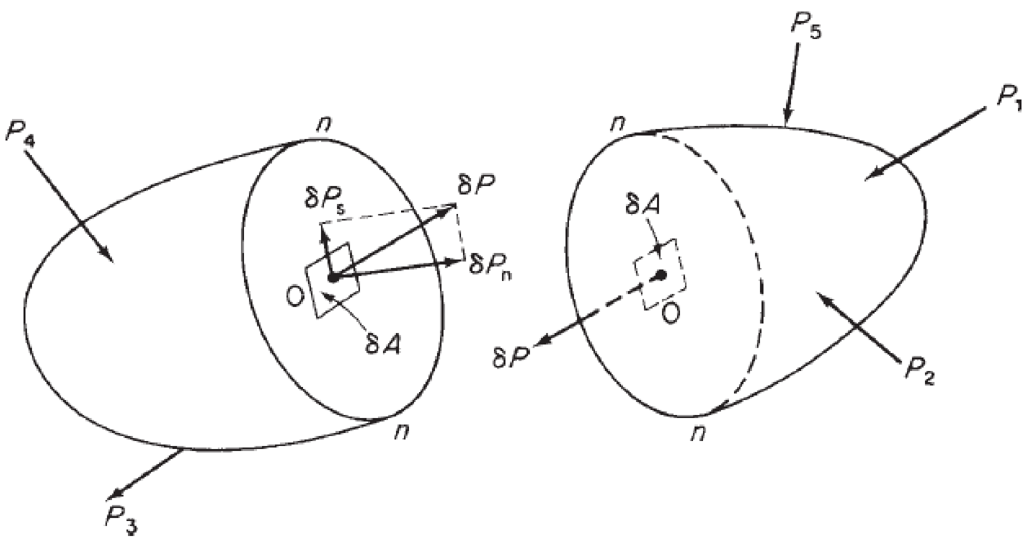
- **Stress**

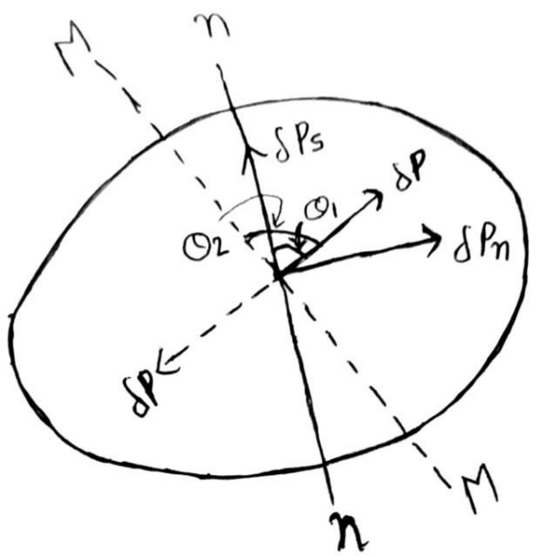
When a body undergoes deformation under the application of external forces, a restoring/resistance force is induced within the body. The intensity of force, i.e. restoring force per unit area is termed as stress.

$$stress\ (\sigma) = \lim_{\delta A \to 0} \frac{\delta P}{\delta A}$$

Consider an arbitrary shaped body subjected to several loads as shown in figure







Here,

$$\delta P_n = \delta P \sin\theta$$

$$\delta P_s = \delta P \cos\theta$$

Reference plane for stresses

Under complex loading, for any given small area δA the resultant force can be at any inclination. This resultant force (here stress) is resolved in two components.

Normal Stress: - (Normal to plane)

$$\sigma = \lim_{\delta A \to 0} \frac{\delta Pn}{\delta A}$$

Shear Stress: - (Parallel to plane)

$$\tau = \lim_{\delta A \to 0} \frac{\delta Ps}{\delta A}$$

Stress is a tensor quantity (2nd order tensor) (i.e.), it depends on magnitude, direction and the plane on which it acts.

Normal stress (σ) can be tensile or compressive in nature depends on loading.

Generally, normal stress pointing away from plane or section is considered as tensile stress (+ve) while normal stress pointing towards the plane or section is considered as compressive stress (-ve).

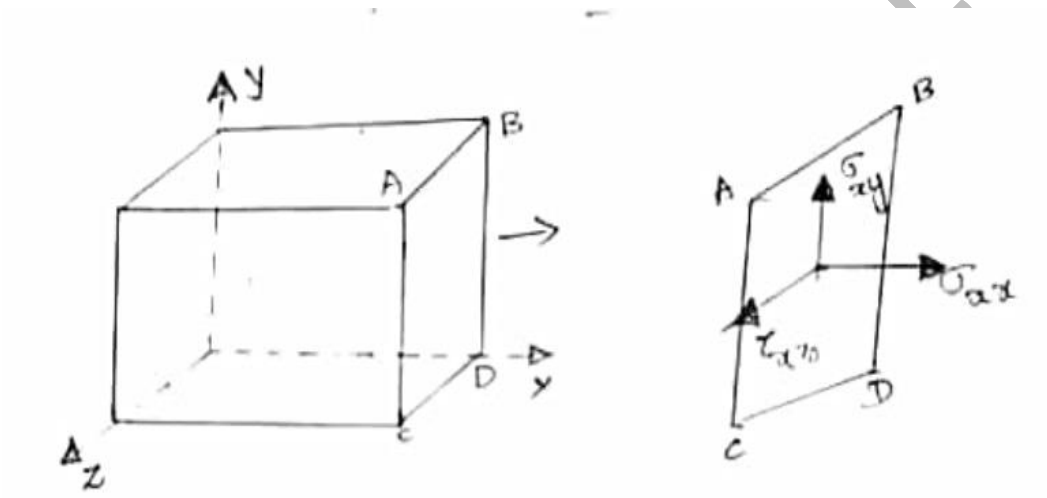
→Normal stress is also termed as direct stress.

- **Notation and Direction of stresses**

In tonsorial notation the stress is generally termed as σ_{ij} or τ_{ij} two suffix for 2nd order tensor, where

i – Indicates plane in which it acts

j – Indicates direction of action



In above figure plane ABCD is 'X-plane'. There is a normal force and two shear stress in a plane of '3-D structure'. The normal stress in plane ABCD is noted as σ_{xx}. Similarly, shear stress τ_{xy} in y-direction and τ_{xz} in z-direction.

Some time, σ_{xx} is also termed with σ_x (single σ_x).

Direction:-

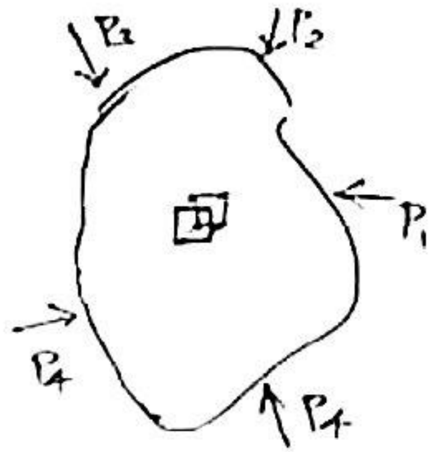
The normal stresses are defined as positive when they are directed away from their related surface.

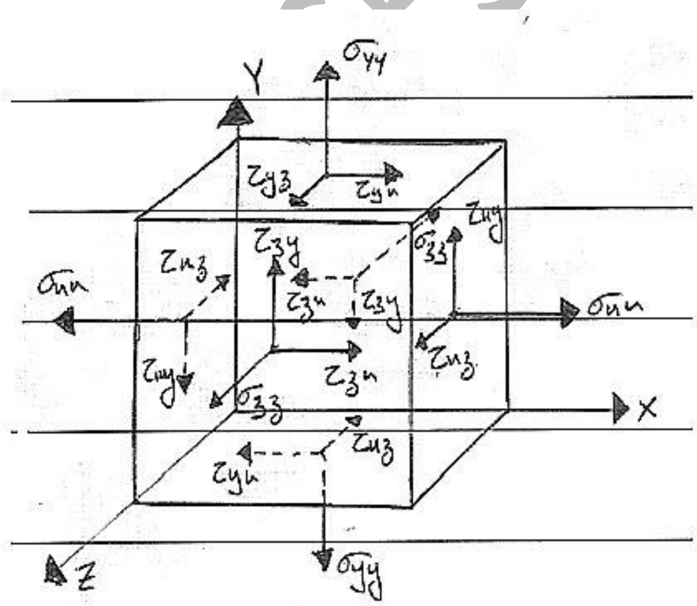
Direction of shear stress depends on the corresponding direction of Normal stress

If the tensile stress is in positive direction of the axis (say x), then corresponding shear stress are positive in other two positive direction of axis (y and z), while if tensile stress in opposite direction of axis (say -x), then corresponding positive shear stress are in direction opposite to positive direction of axis (-y and –z).

- **Stresses in 3-D Body**

If any 3D body is subjected to external load it undergoes deformations which include strains and stresses. Consider a small (infinitesimal) particle of a 3-D body.





There are 3-planes in a cubical partial (X, Y, Z). In each plane (one normal stress and two shear stress).

There are total 9-stresses in a 3-D body. In terms of matrix,

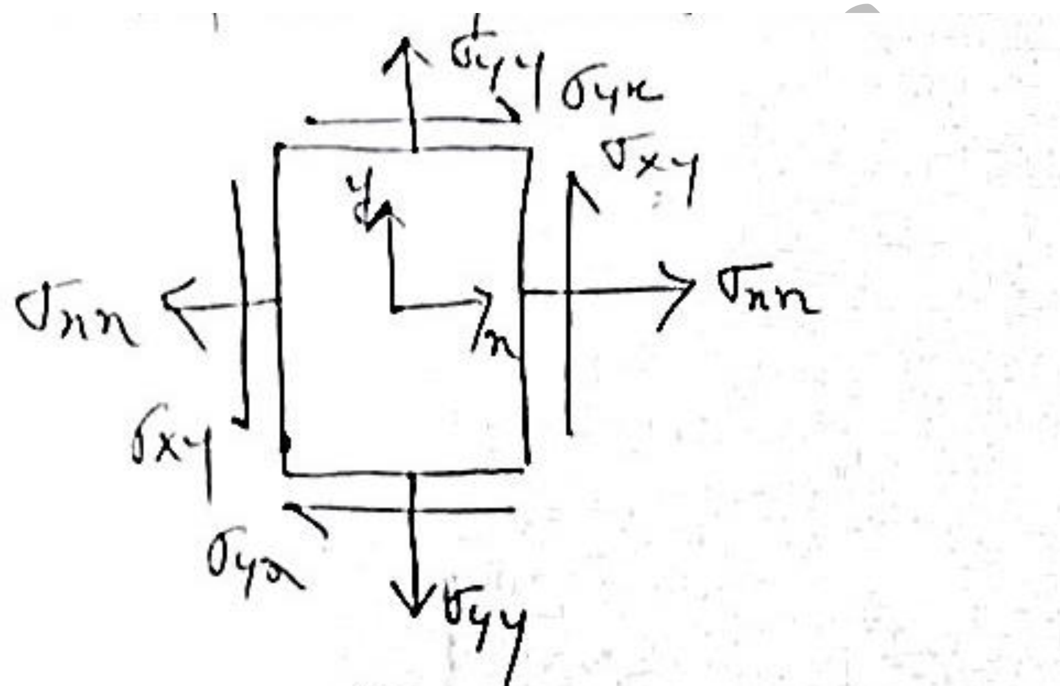
$$[\sigma]=\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

→Row indicate plane

→Column indicate direction

- **Stresses in 2-D Body**

Consider a plate of unit thickness as shown

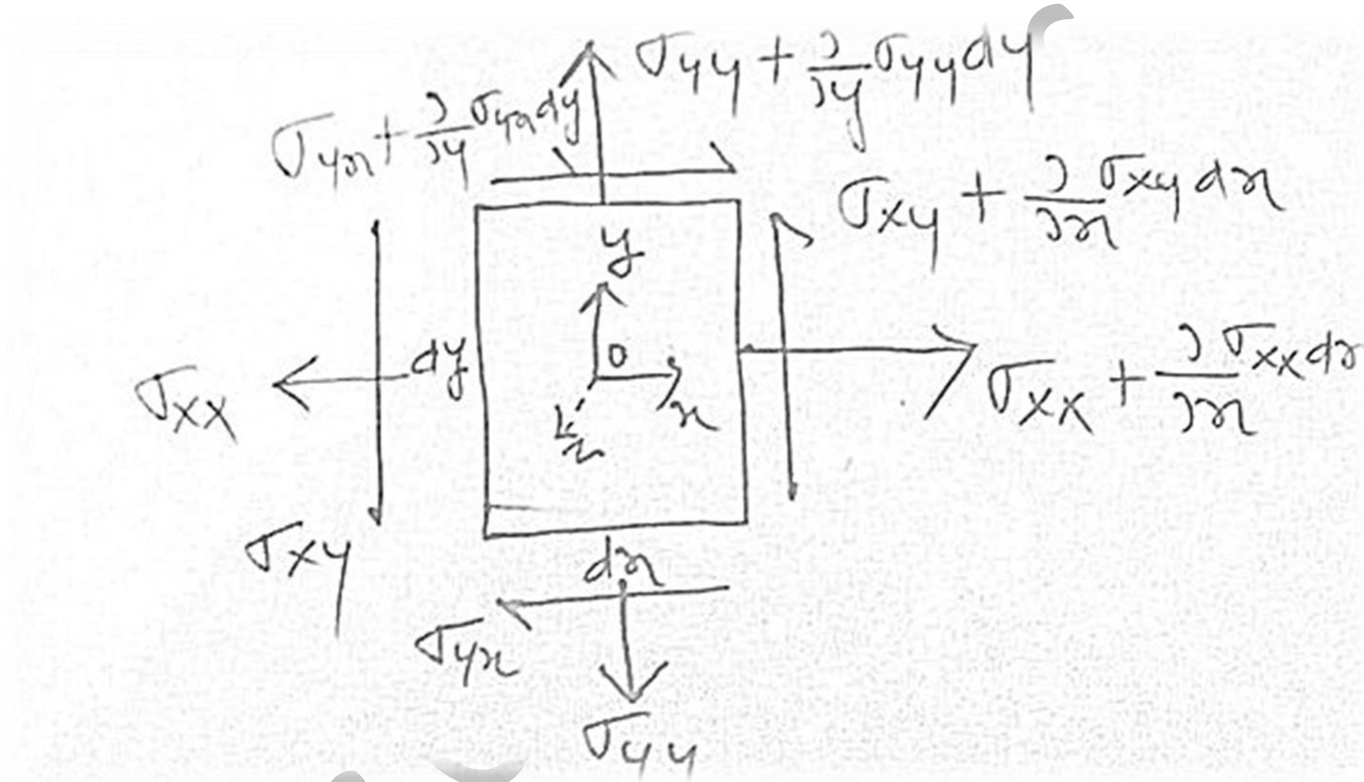


$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

- **Equations of Equilibrium**

For 2-D stress system

Consider the variation of stress along the element as well. Let the body forces in x and y directions are X and Y per unit volume.



⇒ Taking moments about an axis through the centre of the element parallel to the z-axis,

$\sum M_{oz} = 0$

$$(\sigma_{xy} dy)\frac{dx}{2} + \left(\sigma_{xy} + \frac{\partial \sigma_{xy}}{\partial x} dx\right) dy \frac{dx}{2} - \sigma_{yx} dx\ \frac{dy}{2} - \left(\sigma_{yx} + \frac{\partial \sigma_{yx}}{\partial y} dy\right) dx \frac{dy}{2} = 0$$

Ignoring higher order term and dividing by dx.dy;

$$\sigma_{xy} = \sigma_{yx}$$

Above expression shows that shear stresses are of complementary nature and on two perpendicular plane, shear stresses has same magnitude and either they approach each other or will go away from each other.

Now Force Equilibrium

⇒ Considering the equilibrium of the element in x-direction, $\sum F_x = 0$

$$\left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right) dy - \sigma_x dy + \left(\sigma_{yx} + \frac{\partial \sigma_{yx}}{\partial y} dy\right) dx - \sigma_{yx} dx + X\, dxdy = 0$$

$$\rightarrow \frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + X = 0 \qquad \text{.................. (1)}$$

Similarly,

$$\rightarrow \frac{\partial \sigma_y}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} + Y = 0 \qquad \text{.................. (2)}$$

→These above two equations are equilibrium equations for stresses in 2-D system.

- **For 3-D stress system**

$$\rightarrow \frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + X = 0 \qquad \text{.................. (1)}$$

$$\rightarrow \frac{\partial \sigma_y}{\partial y} + \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} + Y = 0 \qquad \text{.................. (2)}$$

$$\rightarrow \frac{\partial \sigma_z}{\partial z} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial x} + Z = 0 \qquad \text{.................. (3)}$$

$$\sigma_{xy} = \sigma_{yx}$$

$$\sigma_{yz} = \sigma_{zz}$$

$$\sigma_{xz} = \sigma_{zx}$$

Problems

1.

The components of stress in a body under plane stress condition, in the absence of body forces, is given by:
$\sigma_{xx} = Ax^2$; $\sigma_{yy} = 12x^2 - 6y^2$ and $\sigma_{xy} = 12xy$.
The coefficient, A, such that the body is under equilibrium is _____ (accurate to one decimal place).

Sol. -

Applying Equilibrium Equation for 2D stress system in the absence of body forces

$$\rightarrow \frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} = 0 \qquad \text{..................} \ (1)$$

$$\rightarrow \frac{\partial \sigma_y}{\partial y} + \frac{\partial \sigma_{yx}}{\partial x} = 0 \qquad \text{..................} \ (2)$$

From equation-1

2Ax+ 12x=0

A= -6