GATE Aerospace Coaching by Team IGC

Aircraft Structures Basics

Bredth Betho Theory

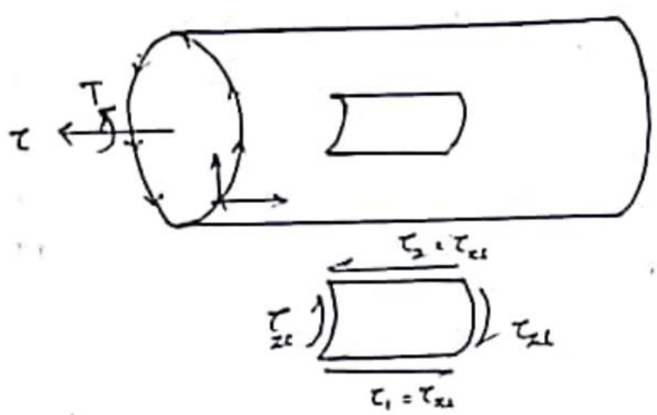
Torsion of thin walled section

Assumption:-

Wapping displacement are freely permitted

$\tau_{xs}$is presents and every other stress component is zero

τ_{xs}doesn't vary along thickness direction



Force equilibrium along Z-axis

$$-\tau_1 t_1 dz + \tau_2 t_2 dz = 0$$

$$\tau_1 t_1 = \tau_2 t_2 = q$$

Shear flow per unit length

q in terms of torque

Diagram

$$dT = qds.s$$

$$T = \int dT = \int qsds = 2\int q\frac{1}{2}sds$$

$$T = 2\int q\,dA$$

$$T = 2q\int dA$$

$$T = 2Aq$$

$$q = \frac{T}{2A}$$

Angle of Twist

Shear strain energy stored in the structure

$$= \frac{1}{2}T\theta' \rightarrow angle\ of\ twist$$

$$= \int \frac{1}{2}.\tau_v.volume$$

$$= \int \frac{1}{2}\frac{\tau^2}{G}(tds\ X\ 1)$$

$$\frac{1}{2}T\theta' = \int \frac{\tau^2}{2G}tds$$

$$\theta' = \int \frac{\tau^2}{TG}ds$$

$$= \int \frac{q^2}{tTG}ds$$

$$\theta' = \int \frac{q}{2\pi TG}ds$$

Torsional Rigidity

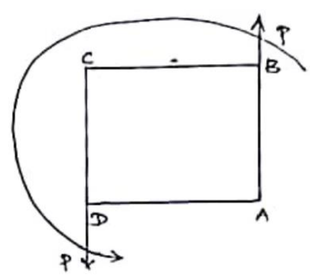
$$GT = \frac{T}{\theta'} \qquad ; \qquad J = \frac{4A^2}{\int \frac{ds}{t}}$$

$$GT = \frac{4A^2}{\int \frac{ds}{Gt}} \rightarrow torstonal\ Rigidity$$

$$J = I_P \rightarrow for\ circular\ crossection$$

Problems:-

(1). In a thin walled rectangular subjected to equal and opposite forces as shown in fig, the shar stress along lay is.



(a). Zero (b). constant non-zero (c). yasles linearly (d). yaslesparabolitally

Sol:-

From breathbetha theory

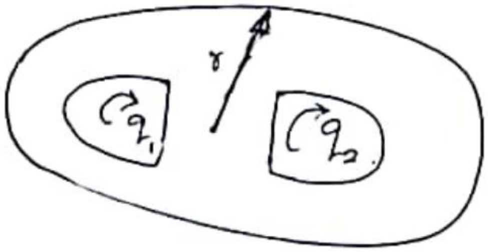
$$T = 2Aq$$

$$T = \frac{P_1}{2} + \frac{P_2}{2}$$

$$q = \frac{T}{2A}$$

$$T = P_2 \qquad \tau_t = \frac{T}{2A}$$

(2). A thin walled tube of circles cross section with mean radius R has a central way which divide it into two symmetrical cell a torque on is acting on the section. The shear flow q in central web is



(a). $q = \frac{M}{2\pi r^2}$ (b). $q = 0$ (c). $q = \frac{M}{4\pi r^2}$ (d). $q = \frac{K}{\pi r^2}$

Sol:-

$$q = q_1 - q_2$$

$$T = 2A_1q_1 + 2A_2q_2$$

$$\theta_1' = \theta_2' \ (from\ compatability\ equation)$$

$$\theta_1' = \int \frac{q_1 ds}{2A_1Gt_1}$$

$$\theta_2' = \int \frac{q_2 ds}{2A_2Gt_2}$$

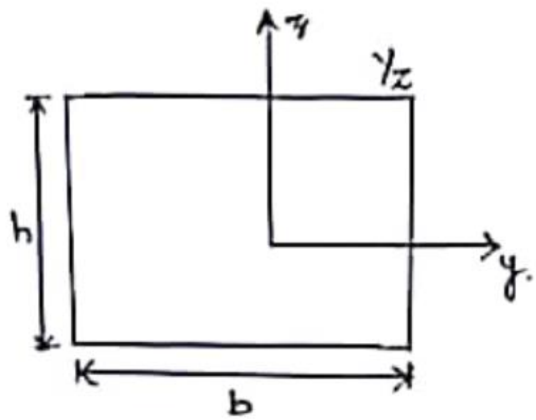
$$\theta_1' = \theta_2'$$

$$\int \frac{q_1 ds}{2A_1Gt_1} = \int \frac{q_2 ds}{2A_2Gt_2}$$

$$q_1 = q_2$$

$$q = 0$$

(3). An Euler Bernoulli's Beam has rectangular cross section Beam shear in fig and subjected to non-uniform BM along its length $v_2 = \frac{dM}{v_2 y}$, the shear stress distribution τ_{xx} across the cross section



(a). $\tau_{xx} = \frac{V_\tau}{2I_y\left(\frac{h}{2}\right)}$ (b). $\tau_{xx} = \frac{V_\tau\left(\frac{h}{2}\right)^2}{2I_y}\left(1 - \frac{\tau^2}{\frac{h}{2}}\right)$ (c). $\tau_{xx} = \frac{V_\tau}{2I_y}\left(\frac{2}{\frac{h}{2}}\right)^2$ (d). $\tau_{xx} = \frac{V_2\left(\frac{h}{2}\right)^2}{2I_y}$

(4). Find the torsional constant for a ring of radius is

$$J = \frac{4A^2}{\int \frac{ds}{t}}$$

$$= \frac{4(\pi r)^2}{\frac{2\pi r}{t}}$$

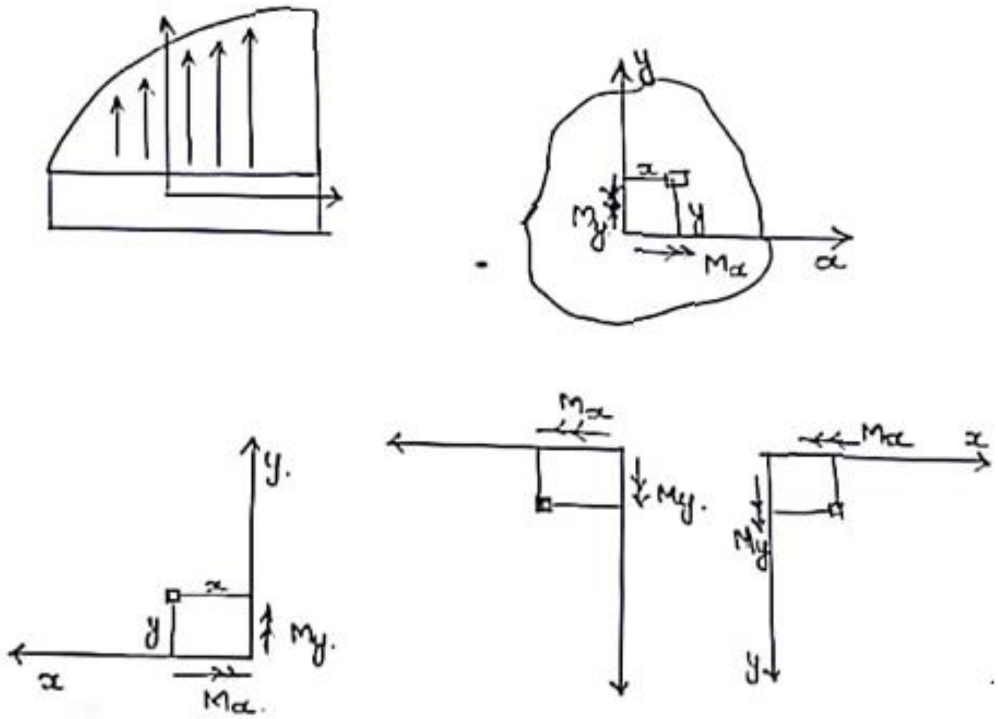
$$= \frac{4\pi^2 rt}{2\pi r}$$

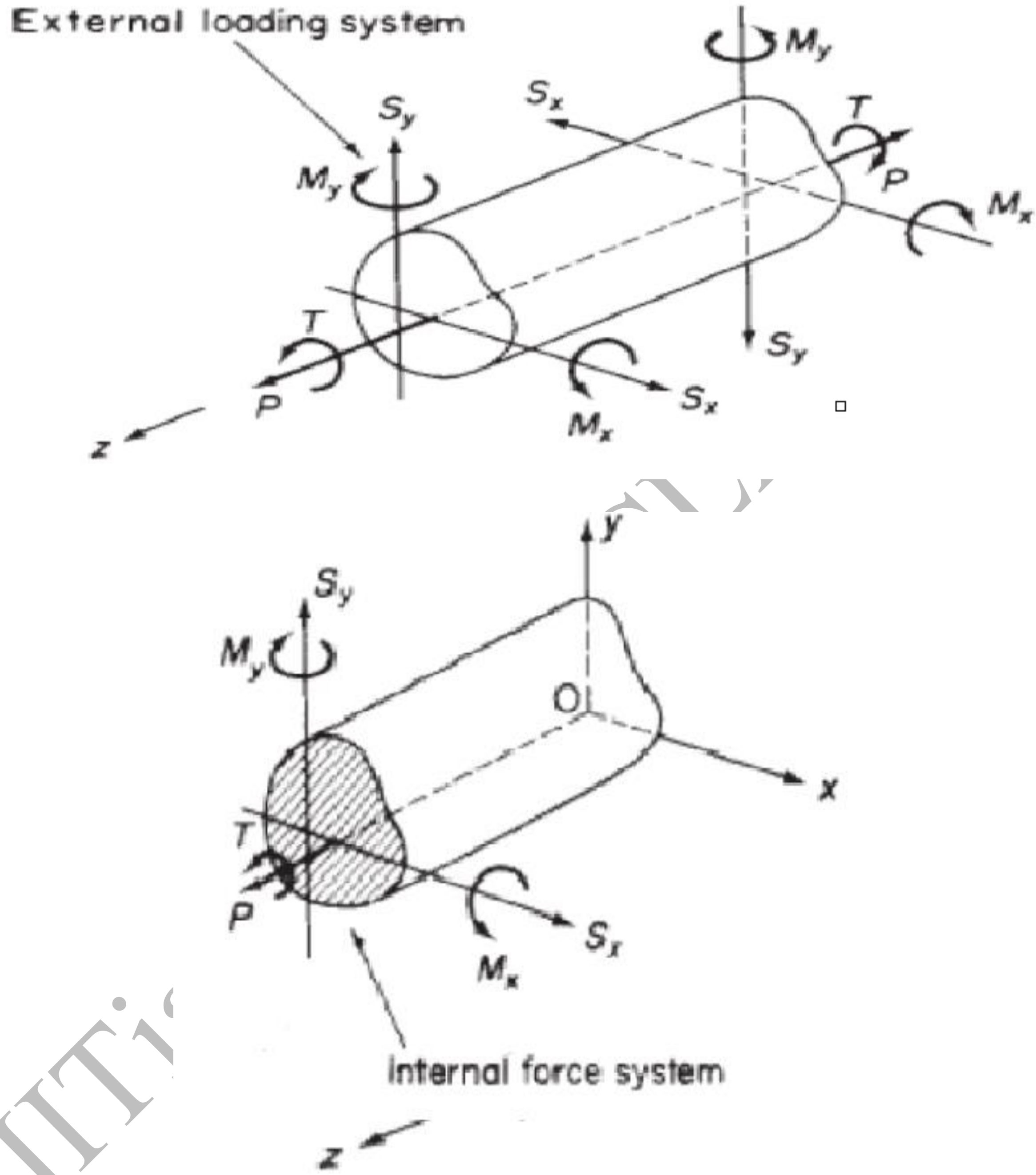
$$J = 2\pi r^3 t$$

Unsymmetrical Bending:-

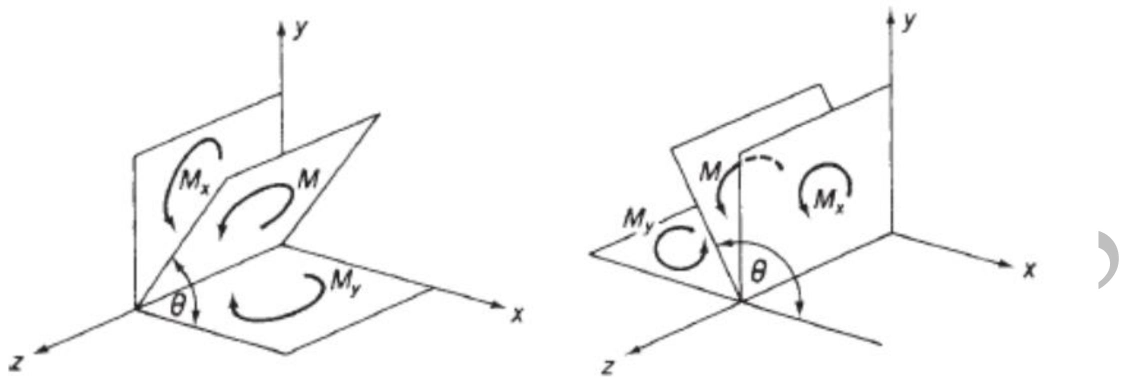
If the cross section of the beam is not symmetrical about any axis or applied load is not acting through plane of symmetry than bending will be unsymmetrical

Consider a beam obituary cross section as shown in fig,





Internal Force System



(a) $\theta < 90°$ (b) $\theta > 90°$

Resolution of bending moments

Sign depending on the size of θ.In both cases, for the sense of M shown

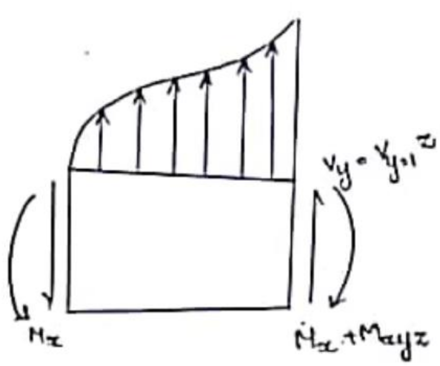
$$M_x = M sin\theta$$

$$M_y = M cos\theta$$

Which give,

For $\theta < \frac{\pi}{2}, M_x$ and M_y positive (fig (a)) and for $\theta > \frac{\pi}{2}, M_x$ positive and M_y negative (fig (b)).

If the neutral axis made angle α with x-axis



$$y' = xsin\alpha + ycos\alpha$$

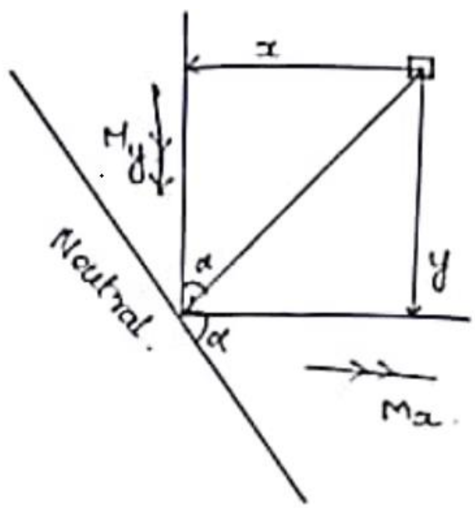
Strain

$$\varepsilon = \frac{y\prime}{R}$$

$$\varepsilon = \frac{xsin\alpha + ycos\alpha}{R}$$

$$\sigma_z = E\varepsilon$$

$$\sigma_x = \frac{E}{R}(xsin\alpha + ycos\alpha)$$



Strain relations

$$\int \sigma_z dA = 0$$

(zero about Neutral axis)

Moment resultant

$$M_x = \int \sigma_z\, ydA$$

$$M_y = \int \sigma_z xdA$$

$$M_x = \frac{E}{R}[sin\alpha \int xydA + cos\alpha \int y^2 dA]$$

$$= \frac{E}{R}[sin\alpha I_{xy} + cos\alpha I_{xx}]$$

Similarly

$$M_y = \frac{E}{R}[cos\alpha I_{xy} + sin\alpha I_{yy}]$$

This is matrix form

$$\begin{bmatrix} M_x \\ M_y \end{bmatrix} = \frac{E}{R}\begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}\begin{Bmatrix} cos\alpha \\ sin\alpha \end{Bmatrix}$$

$$\begin{Bmatrix} cos\alpha \\ sin\alpha \end{Bmatrix} = \frac{E}{R}\begin{bmatrix} M_x \\ M_y \end{bmatrix}\begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}^{-1}$$

$$\begin{Bmatrix} cos\alpha \\ sin\alpha \end{Bmatrix} = \frac{R}{E(I_{xx}I_{yy} - I_{xy}{}^2)}\begin{bmatrix} I_{yy} & -I_{xy} \\ -I_{xy} & I_{yy} \end{bmatrix}\begin{Bmatrix} M_x \\ M_y \end{Bmatrix}$$

$$cos\alpha = \frac{R}{E(I_{xx}I_{yy} - I_{xy}{}^2)}\left(I_{yy}M_x - I_{xy}M_y\right)$$

$$sin\alpha = \frac{R}{E(I_{xx}I_{yy} - I_{xy}{}^2)}\left(-I_{xy}M_x + I_{xx}M\right)$$

$$\sigma_z = \frac{E}{R}(xsin\alpha + ycos\alpha)$$

$$\sigma_z = \frac{(I_{xx}M_y - I_{xx}M_x)}{(I_{xx}I_{yy} - I_{xy}{}^2)}x + \frac{(I_{yy}M_x - I_{xy}M_y)}{(I_{xx}I_{yy} - I_{xy}{}^2)}y$$

$$\sigma_z = \frac{M_y}{I_{yy}}x + \frac{M_x}{I_{xx}}y$$

At $M_y = 0$

$$\sigma_z = \frac{M_x y}{I_{xx}}$$

Problems:-

1). An idealized thin walled cross-section of the beam and perspective areas of boom are as shown in a bending moment M_y is acting on the cross-section the ratio of magnitude of normal stress in the top boom that of bottom boom.

(a). $\frac{5}{11}$

(b). $\frac{2}{5}$

(c). 1

(d). $\frac{5}{2}$

Sol:-

Since it is symmetric

$$\sigma_x = \frac{M_x}{I_{xx}} y + \frac{M_y}{I_{yy}} x$$

Since, $M_x = 0$(No bending stress at x-axis)

$$\sigma_z = \frac{M_y}{I_{yy}} x$$

$$\sigma_{z\,top} = \frac{M_y}{I_{yy}} x_{top}; \qquad \sigma_{z\,bottom} = \frac{M_y}{I_{yy}} x_{bottom}$$

$$\frac{\sigma_{z\,top}}{\sigma_{z\,bottom}} = \frac{\frac{M_y}{I_{yy}} x_{top}}{\frac{M_y}{I_{yy}} x_{bottom}}$$

$$\frac{\sigma_{z\,top}}{\sigma_{z\,bottom}} = \frac{x_{top}}{x_{bottom}}$$

$$x_{top} = 2 - \bar{x}$$

$$x_{bottom} = 2 + \bar{x}$$

$$\bar{x} = \frac{\sum A\bar{x}}{\sum A} = \frac{2\ X\ 2\ X\ 0.2 + 0.1\ X\ 2 + 0 - 0.2\ X\ 2}{3\ X\ 0.2 + 2\ X 0.1} = \frac{0.8 + 0.2 - 0.4}{0.6 + 0.2}$$

$$\bar{x} = 0.75 = \frac{3}{4}$$

$$x_{top} = 2 - \frac{3}{4} = \frac{16}{4}$$

$$\frac{\sigma_{z\,top}}{\sigma_{z\,bottom}} = \frac{x_{top}}{x_{bottom}} = \frac{\frac{5}{4}}{\frac{11}{4}} = 5/11$$