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QUICK REVISION *FORMULA SHEET*

for
GATE -ME THEORY OF MACHINES



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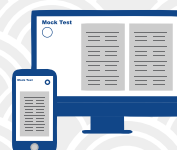
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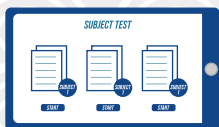
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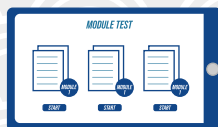
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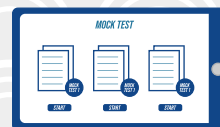
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Theory of Machines

Chapter 1: Simple Mechanism

Link: Smallest unit of machine, which should transfer relative motion.

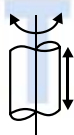
Spring: used as restoring forces, we cannot consider it as kinematic link.

Classification of Kinematic Pairs

A. On the Basic of DOF

Example:

1)(cylindrical pair) C – pair [DOF = 2]



(Spatial joint)

2)(Prismatic pair) P – pair [DOF = 1]

(Planar joint)

3)(Revolute pair) R – pair [DOF = 1] (Planar joint)

4)H – pair [DOF = 1] (Screw pair)

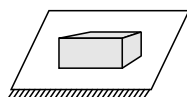
(Spatial joint)

5)G – pair [DOF = 3] (Spherical pair)

(Spatial joint)

6)E – pair [DOF = 3] (Planar or Evena pair)

Eg: Cube on surface



B. According to type of contact:

- Lower pair (surface contact)
 - Linear motion pairs
DOF = 1 **Eg:** R, P, H pair
 - Surface motion pairs

DOF > 1 **Eg:** C, E pair

- Higher pair (Point/Line contact, DOF = 2)
- Wrapping pair when one link wrapped over the other link

All the surface motion pairs will violate Kutzbach equation.

C. On the basic of type of closure

- Self-closed pair (Permanent contact)
Eg: Turning, sliding pair
- Forced closed pair (open pair)
Eg: H.P in cam & follower

Door closure

Automatic clutch

D. On the basis of motion between links

- Completely constraint motion (Self-desired motion)
Eg: P pair
- Successfully constraint motion (Forcefully desired motion)
Eg: Foot step bearing
- Incompletely constraint motion (Unconstraint motion)
Eg: C- pair

Degree of Freedom (DOF):

$$3(n - 1) - 2J - h - F_r$$

Where n = no. of link

J = No. of binary joint

h = no. of higher pair (Kutzbach equation) valid for R-pair; P pair.

- $DOF < 0$ (Super structure)
- $DOF = 0$ Structure/Frame
- $DOF > 1$
 - $DOF = 1$ constraint/Kinematic mechanism
 - $DOF > 1$ unconstraint mechanism

F_r = redundant dof

Grubler's equation:

$$3n - 2j = 4$$

Here $DOF = 1$; $h = 0$

Minimum number of link to form mechanism = 4.

Modified Kutzbach Equation:

$$DOF = 3[n - n_r - 1] - 2[J - J_r] - h - F_r$$

Where:

n = No. of links

n_r = No. of redundant links

J_r = No. of redundant joints

J = No. of Binary joint

F_r = redundant dof

If there are n no. of links, then possible inversion will be $\leq n$

Grashof's Law:

A. $S + l \leq p + q$

S = shortest link

l = longest link

If S (Shortest link) fixed = Double crank mechanism.

If S is adjacent to fixed = Crank rocker

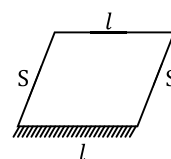
If S is opposite to fixed = Double Rocker or Lever mechanism.

B. $S + l > p + q$

Double rocker only

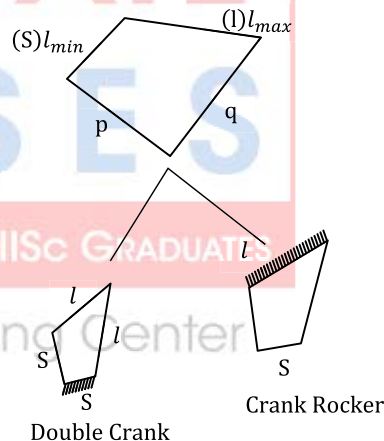
C. If $S + l = p + q$

- All links are equal length (Rhombus linkage)

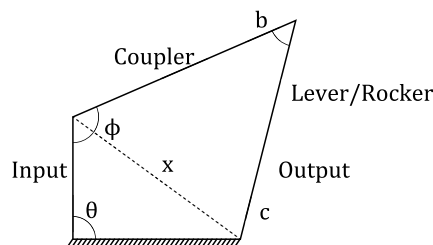


Double crank only.

- Two links of equal length



Transmission Angle (γ): angle between coupler and lever/Rocker



Y_{\min} at $\theta = 0^\circ$
 Y_{\max} at $\theta = 180^\circ$ } (but before this) check $x < b + c$

Toggle Positions $\phi = 180^\circ$ and $\phi = 0^\circ$

Mechanical Advantage (MA):

$$\begin{aligned} \frac{F_{\text{output}}}{F_{\text{input}}} &= \frac{(\text{Torque})_{\text{output}}}{(\text{Torque})_{\text{input}}} \\ &= \frac{\omega_{\text{input}}}{\omega_{\text{output}}} \times \eta_{\text{mech}} \end{aligned}$$

MA = ∞ at Toggle positions.

ω = Angular velocity

Simple Mechanism:

- A. Four bar mechanism
- B. Single slider crank mechanism
- C. Double slider crank mechanism

A. Four Bar Mechanism:

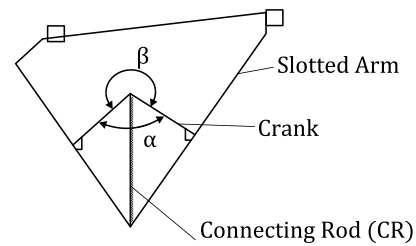
Inversions:

- Double crank mechanism
Eg: Coupling rod of locomotives
- Crank rocker mechanism
Eg: Beam engine
- Double rocker mechanism
Eg: Watt's indicator

B. Single Slider Crank Mechanism

- Cylinder fixed
Eg: Compressor, engine
- Crank fixed:
Eg: Whitworth QRM, Rotary IC engine
- Connecting Rod (CR) Fixed:
Eg: Crank slotted lever QRM
Oscillatory cylinder engine
- Piston/Slider Fixed:
Eg: Hand pump, Pendulum pump

C. Crank Slotted Lever Mechanism (CR is fixed):

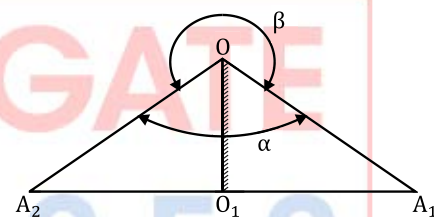


$$\text{QRR} = \frac{\beta}{\alpha} > 1 \text{ (Always } > 1 \text{)}$$

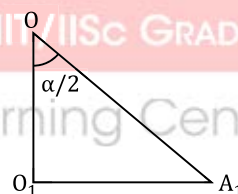
$$\text{Quick Return ratio (QRR)} = \frac{\text{Cutting time}}{\text{Return Time}}$$

$$\text{Stroke length} = \frac{2 \times \left(\text{Length of slotted bar} \right) \left(\text{Crank length} \right)}{\text{Length of CR}}$$

Whitworth QRM: (Crank fixed)



$$\text{QRR} = \frac{\beta}{\alpha}$$



$$\cos \left(\frac{\alpha}{2} \right) = \frac{\text{crank length}}{\text{CR length}}$$

$$\text{Stroke length} = A_1A_2 = 2 (O_1A_1)$$

D. Double Slider Crank Mechanism:

- Slotted bar fixed: Elliptical trammels
- One slider fixed: Scotch yoke mechanism.
- CR fixed: Oldham's coupling.

Chapter 2: Motion Analysis

Number of Instantaneous Centre

$$= n_{c_2} = \frac{n(n-1)}{2}$$

Where n = No. of Links

Kennedy's Theorem:

For relative motion between No. of links in a mechanism any three links, their I-centre Must lie on a straight line.

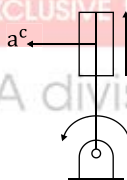
Theorem of Angular Velocity:

- I_{mn}
 - Can be treated on link m
 - Can be treated on link n

$$V_{I_{nn}} = \omega_n(I_{mn}I_{1m}) = \omega_n(I_{mn}I_{1n})$$

If I_{1n} and I_{1m} both are on same side of I_{mn} then sense of rotation is same for ω_m and ω_n

Coriolis Acceleration (a^c):



$$a^c = 2(\vec{\omega} \times \vec{V})$$

Where, V = Velocity of slider

ω = angular velocity of link on which slider is sliding.

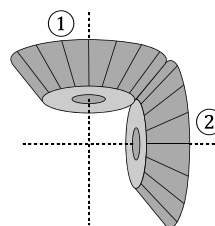
Direction: Rotate the \vec{V} in the same sense as of $\vec{\omega}$.

Chapter 3: GEARS

Positive Drive: Slip is not possible. Eg: (gears)

Negative Drive: Slip is possible, Eg: Belt, rope, Chain.

Mitre Gear:



Similar Gear at 90°

$$\frac{\omega_1}{\omega_2} = 1$$

Circular Pitch:

$$(P_c) = \frac{\pi D}{T}$$

Where D = Diameter of gear / pinion

T = No. of teeth

Module (m):

$$= \frac{D}{T} \text{ (Module is same for mating gears)}$$

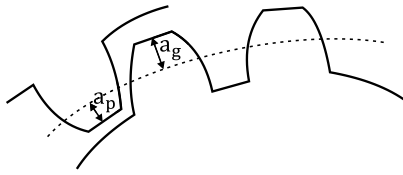
Diametral Pitch (P_d): $\frac{T}{D}$

$$P_d \cdot P_c = \frac{T}{D} \times \frac{\pi D}{T} = \pi$$

Working depth = Addendum + Dedendum-clearance.

= sum of addendum of both mating gears.

Tooth space - Tooth thickness of mating gear = Backlash



Where a_g = Addendum of gear

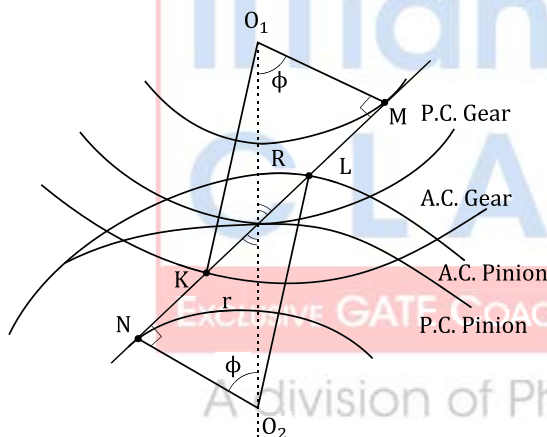
a_p = Addendum of Pinion

Law of Gearing:

Line of action must always pass through the fixed point (pitch point) on the line joining the centres of rotation of gear.

$$V_{\text{sliding}} = V_s = (\omega_1 + \omega_2)QP$$

= Sum of angular Velocity \times distance between pitch point and point of contact.



A.C = Addendum circle

Path of contact = Path of approach (KP) + path of recess (PL)

$$KP = \sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$PL = \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$\rightarrow \text{Arc of contact} = \frac{\text{Path of contact}}{\cos \phi} = \frac{KL}{\cos \phi}$$

– Angle turned by pinion

$$= \frac{\text{Arc of contact}}{r} \quad (\text{in radian})$$

\rightarrow Angle turned by Gear

$$= \frac{\text{Arc of contact}}{R} \quad (\text{in radian})$$

$$\rightarrow \text{Contact ratio} = \frac{\text{Arc of contact}}{\text{circular pitch } (P_c)} \geq 1$$

$$\rightarrow \text{Gear ratio} = G = \frac{T}{t}$$

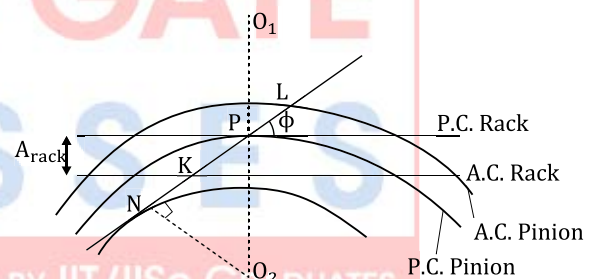
Where T = No. of teeth on gear

t = No. of teeth on pinion.

$$\rightarrow \text{Velocity ratio} = \frac{\omega_p}{\omega_g} = \frac{T}{t} > 1$$

$$\Rightarrow \frac{\omega_g}{\omega_p} = \frac{t}{T} < 1$$

\rightarrow Path of contact in rack and pinion



A_{rack} = Addendum of rack

A.C pinion \Rightarrow Addendum circle of pinion

P.C. pinion \Rightarrow pitch circle of pinion

$$K.L = KP + PL$$

$$= \frac{A_{\text{rack}}}{\sin \phi} + \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi$$

Methods to Prevent Interference:

- Under cut gears
- Increase ϕ (Pressure angle)
- By stubbing the teeth
- Increasing No. of teeth

Minimum No. of teeth required for pinion?

Teeth on Gear:

$$\Rightarrow T_{\min} = \frac{2A_G}{\sqrt{1 + \frac{1}{G} \left(\frac{1}{G} + 2 \right) \sin^2 \phi} - 1}$$

Teeth on Pinion:

$$t_{\min} = \frac{2A_P}{\sqrt{1 + G(G + 2) \sin^2 \phi} - 1}$$

A_G = Fractional addendum of gear

A_P = Fractional Addendum of pinion

Always make gear safe first if $A_G = A_P$

For Rack and Pinion

$$t_{\min} = \frac{2A_R}{\sin^2 \phi}$$

In all, $\left. \begin{matrix} A_G \\ A_P \\ A_R \end{matrix} \right\}$ These are fractional addendum

$m \cdot A_P$ = A of pinion

$m \cdot A_G$ = A of Gear

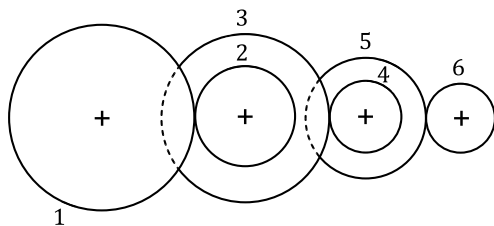
$m \cdot A_R$ = A of Rack

Chapter 4: GEAR TRAINS

$$\text{Speed ratio} = (SR) = \frac{\omega_1}{\omega_2} = \frac{\omega_{\text{driver}}}{\omega_{\text{driven}}}$$

$$\text{Train value} = \frac{1}{SR}$$

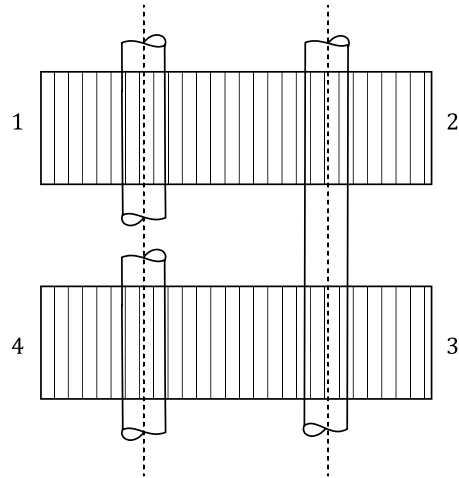
Compound Gear Train:



$$SR = \frac{\omega_1}{\omega_6} = \frac{T_2 T_4 T_6}{T_1 T_3 T_5}$$

T = No. of teeth on gears.

Reverted Gear Train:



m = module of 1, 2

m' = module of 3, 4

$$SR = \frac{\omega_1}{\omega_4} = \frac{T_2 T_4}{T_1 T_3}$$

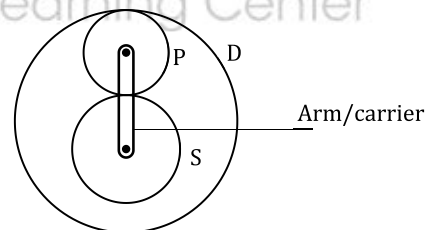
$$r_1 + r_3 = r_2 + r_4$$

$$m \left(\frac{T_1}{2} + \frac{T_2}{2} \right) = m' \left(\frac{T_3}{2} + \frac{T_4}{2} \right)$$

If speed reduction is same

$$\frac{T_2}{T_1} = \frac{T_4}{T_3}$$

Planetary Gear Train (Epicyclic):



$$r_s + 2r_p = r_D$$

$$T_s + 2T_p = T_D$$

$$\Rightarrow T_{\text{input}} + T_{\text{output}} + T_{\text{fixing}} = 0$$

$$\rightarrow \eta_{GT} [T_{\text{input}} \cdot \omega_{\text{input}}] + T_{\text{output}} \times \omega_{\text{output}} = 0$$

η_{GT} = Efficiency of gear train

Chapter 5: GYROSCOPE

$$\text{Gyroscope Couple } (C_g) = L \times \frac{d\theta}{dt}$$

$$c_g = I \cdot \omega \cdot \omega_p$$

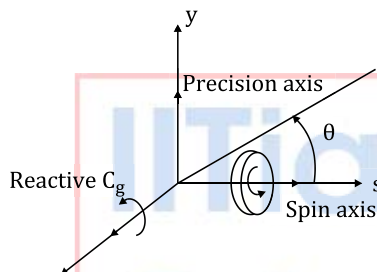
$$L = \text{Angular momentum of rotor} = I \cdot \omega$$

$$\omega = \text{Angular Velocity of rotor}$$

$$I = \text{Mass moment of Inertia of rotor}$$

$$\omega_p = \text{Angular Velocity of axis on which rotor is rotating.}$$

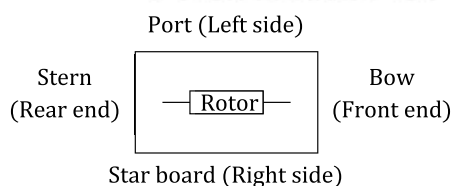
Calculation of Reactive Gyroscopic Couple:



Reactive c_g direction

$$= \text{Spin axis } (\omega) \times \text{Precision axis } (\omega_p)$$

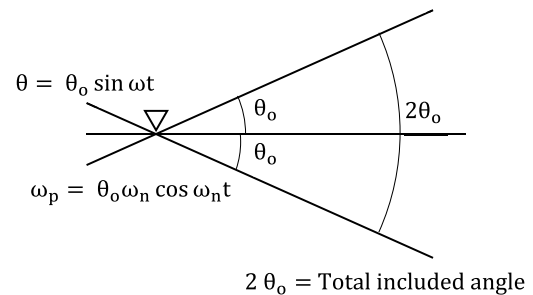
Gyroscopic Effects in Water Boat:



Motion

1. Pitching:

Up and down movement of bow and stern



$2\theta_o = \text{Total included angle}$

$$\omega_n = \frac{2\pi}{T} \text{ (Natural frequency)}$$

$$(\omega_p)_{\max} = \theta_o \times \omega_n = \theta_o \times \frac{2\pi}{T}$$

$T = \text{Time period of pitching}$

$$C_g = I\omega(\omega_p)_{\max}$$

2. Steering:

Left or right turn of ship

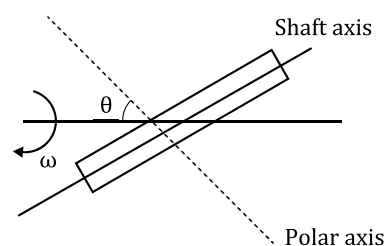
$$C_g = I\omega\omega_p$$

$$\omega_p = \frac{V}{R} = \frac{\text{Velocity of boat}}{\text{Radius of curvature / circle}}$$

3. Rolling:

No couple formed as precision axis and spin axis coincide.

Gyroscopic couple by bearing due to misalignment of disc:



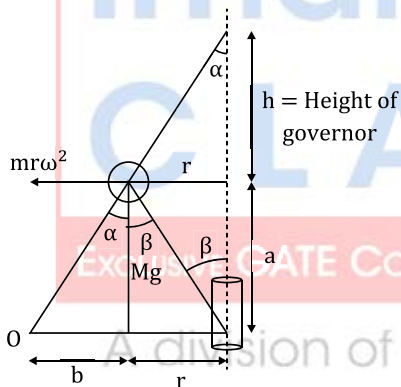
$$G = \frac{m}{8} \omega^2 r^2 \sin(2\theta)$$

$\theta = \text{misalignment in degree}$

Chapter 6: GOVERNORS

- Governor
 - Inertia Governor
 - Centrifugal Governor
 - Pendulum Type
Eg: Watt Governor
 - Gravity Controlled
 - ♦ Porter
 - ♦ Proell
 - Spring Controlled
 - ♦ Hartnell
 - ♦ Hartung
 - ♦ Pickering
 - ♦ Wilson Hartnell

1. Watt Governor:



$$\sum M_o = 0$$

$$mg b = mr\omega^2(a)$$

$$mr\omega^2 = mg \left(\frac{b}{a}\right)$$

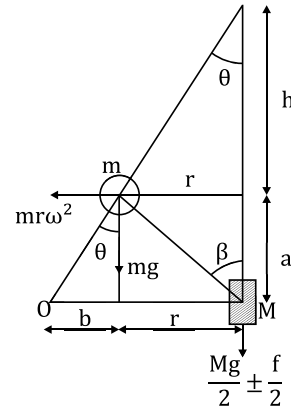
$$r\omega^2 = g \tan \alpha$$

$$r\omega^2 = g \times \frac{r}{\omega}$$

$$\omega^2 = \frac{g}{h}$$

$$N^2 = \frac{895}{h}$$

2. Porter Governor:



$$\sum M_o = 0$$

$$\omega^2 = \frac{2mg + (Mg \pm f)(1 + k)}{2mh}$$

$$k = \frac{\tan \beta}{\tan \theta}$$

f = friction force on sleeve.

Case 1:

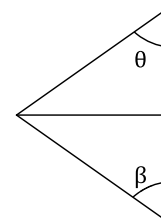
If f (friction) = 0

$$\omega^2 = \frac{2mg + [Mg][1 + k]}{2mh}$$

Case 2:

If $f = 0$

$$\omega^2 = \left(\frac{m + M}{m}\right) \frac{g}{h}$$

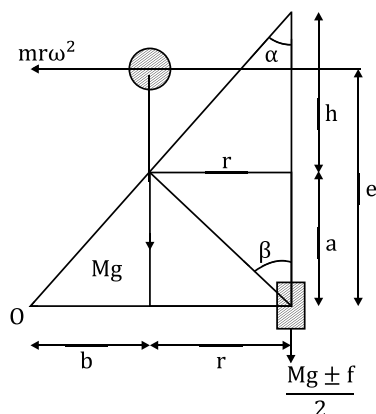


$$\theta = \beta$$

arms are of equal length

M = Mass of Sleeve

3. Proell Governor:



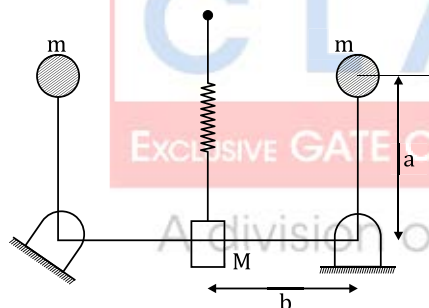
$$\sum M_o = 0$$

$$\omega^2 = \frac{a}{e} \left[\frac{2mg + (Mg \pm f)(1+k)}{2mh} \right]$$

If $f = 0$, and $k = 1$

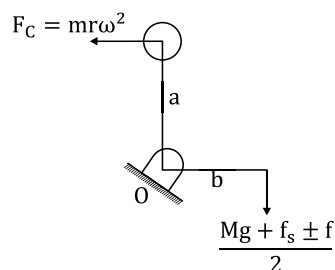
$$\omega^2 = \frac{a}{e} \left[\frac{m+M}{m} \right] \times \frac{g}{h}$$

4. Hartnell Governor:



$$mr_1\omega_1^2(a) = \frac{Mg + fs_1 \pm f}{2} \quad (b)$$

$$mr_1\omega_1^2(a) = \frac{Mg + fs_2 \pm f}{2} \quad (b)$$

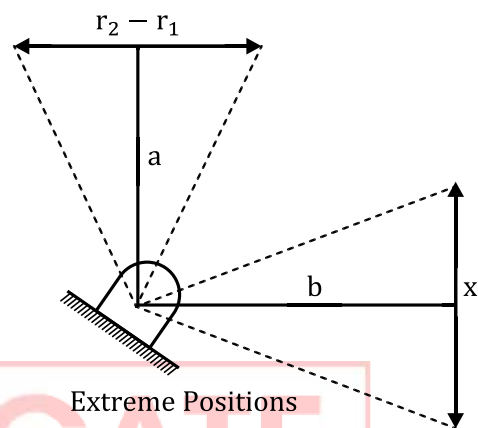


f_s = spring force, f = frictional force

If $f = 0$

$$\frac{r_1\omega_1^2}{r_2\omega_2^2} = \frac{Mg + fs_1}{Mg + fs_2}$$

$$\text{Total sleeve movement (x)} = \frac{b}{a} (r_2 - r_1)$$



Extreme Positions

$$(f_{c_2} - f_{c_1}) \frac{2a}{b} = f_{s_2} - f_{s_1}$$

Additional compression of spring

$$(x_2 - x_1) = \frac{b(r_2 - r_1)}{a}$$

$$\text{Stiffness of spring (k)} = \frac{2(f_{c_2} - f_{c_1})}{(r_2 - r_1)} \left(\frac{a}{b} \right)^2$$

$$\text{Sensitiveness} = \frac{\text{Range of speed}}{\text{Mean speed}}$$

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{(N_2 + N_1)}$$

$$\text{Sensitivity} = \frac{1}{\text{Sensitiveness}} = \frac{N_1 + N_2}{2(N_2 - N_1)}$$

Sensitivity = ∞ for isochronous governor

Hunting: An excessively fast to & fro motion of sleeve between stopper.

Isochronism: A governor is said to be isochronous if its equilibrium speed is constant for all radii of rotations.

$$\text{Range of speed} = N_{\max} - N_{\min} = 0$$

This means ω is same for all sleeve position which is not possible in case of Watt and porter.

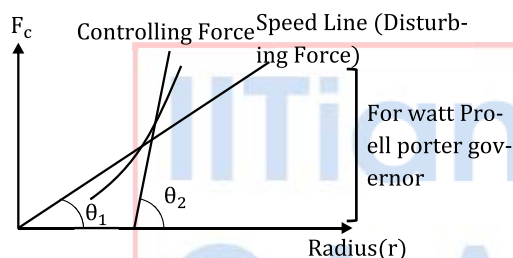
For Hartnell governor: it can be made isochronous governor

$$\omega_1 = \omega_2 = \omega$$

$$\frac{r_2}{r_1} = \frac{Mg + F_{s_2}}{Mg + F_{s_1}}$$

Stability of Governor:

Stable governor: Slope of restoring force > Slope of disturbing force.

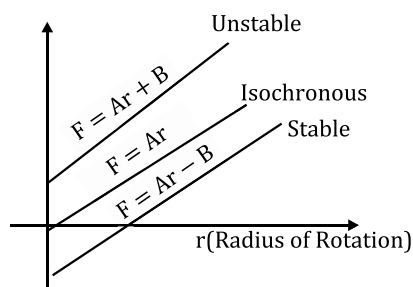


$\theta_2 > \theta_1$ for stable governor

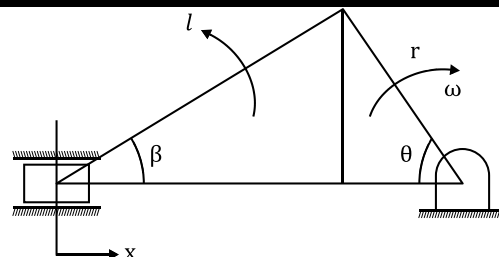
$$\frac{dF(r)}{dr} > m\omega^2$$

If $r \uparrow \Rightarrow N \uparrow$ (Stable governor)

For Spring controlled governor



Chapter 7: DYNAMICS & FLYWHEEL



n = obliquity ratio

r = radius of crank

l = length of connecting rod

$$n = \frac{l}{r}$$

$$l \sin \beta = r \sin \theta$$

$$x = r \left[1 - \cos \theta + n - \sqrt{n^2 - \sin^2 \theta} \right]$$

$$V = r\omega \left[\sin \theta + \frac{\sin 2\theta}{2n} \right] \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Approx equation}$$

$$a = r\omega^2 \left[\cos \theta + \frac{\cos 2\theta}{n} \right] \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \downarrow$$

$$\left. \begin{array}{l} \omega_{cR} = \frac{\omega \cos \theta}{n} = \frac{d\beta}{dt} \\ \alpha_{cR} = \frac{-\omega^2 \sin \theta}{n} \end{array} \right\} \text{Valid when } n \text{ is large}$$

V = Velocity of piston

a = acceleration of piston

ω_{cR} = Angular Velocity of connecting rod

α_{cR} = Angular acceleration of connecting rod

Inertia force on piston = $m a = F_i$

$$= m \left[r\omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n} \right) \right]$$

$$F_{\text{primary}} = mr\omega^2 \cos \theta$$

$$F_{\text{secondary}} = mr\omega^2 \left(\frac{\cos 2\theta}{n} \right)$$

m = mass of piston

F = net force on piston

$$= F_p - F_i + F_g - F_f$$

Where; $F_p = P_1 A_1 - P_2 A_2$

$$F_g = Mg$$

F_f = Kinetic friction

P_1 = pressure of gas at cover end

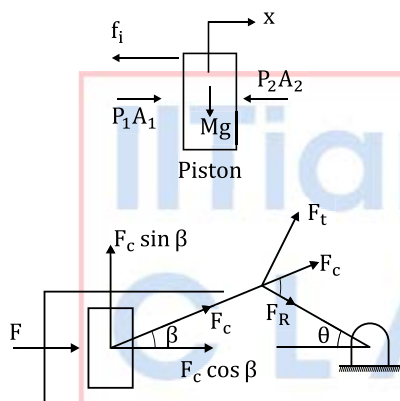
$$A_1 = \left(\frac{\pi}{4}\right) D^2, \text{ where } D$$

= diameter of piston

P_2 = pressure of gas at crank end
side of piston

$$A_2 = \frac{\pi}{4} (D^2 - d^2)$$

d = diameter of piston rod



$$1. F_{CR} = \frac{F}{\cos \beta}$$

2. $F_n = F_c \sin \beta$ = Normal thrust to cylinder walls

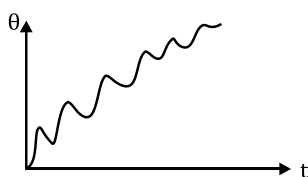
$$3. F_r = F_c \cos(\theta + \beta)$$

F_r = Radial thrust to crank shaft bearing

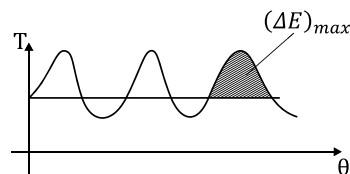
$$4. \text{Crank effort } (F_t) = F_c \sin(\theta + \beta)$$

$$= \frac{F}{\cos \beta} [\sin(\theta + \beta)]$$

$$5. \text{Torque on crank } (T) = F_t \times \text{radius}(r)$$



Flywheel:



Change in energy of flywheel

$$= (\Delta E)_{\max} = \frac{1}{2} I (\omega_{\max}^2 - \omega_{\min}^2)$$

$$(\Delta E)_{\max} = I \omega^2 C_s = E_{\max} - E_{\min}$$

$$\omega = \frac{\omega_{\max} + \omega_{\min}}{2}$$

$$C_s = \frac{\omega_{\max} - \omega_{\min}}{\omega}$$

= Coefficient speed fluctuation

C_E = Coefficient of energy fluctuation

$$= \frac{E_{\max} - E_{\min}}{(WD)_{\text{cycle}}}$$

$$(WD)_{\text{cycle}} = T_{\text{mean}} \times \theta_{\text{cycle}}$$

WD = work done

Chapter 8: BALANCING

• Balancing

♦ Static Balancing (Balancing of forces) $\sum \vec{F} = 0$

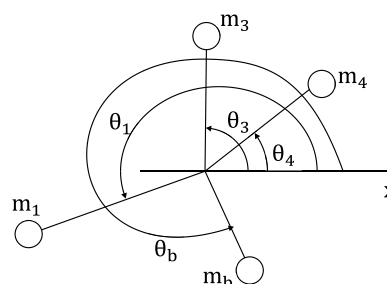
♦ Dynamic Balancing

$$\sum \vec{F} = 0$$

$$\sum \vec{M}_R = 0$$

Balancing of several masses rotating in

Same plane:



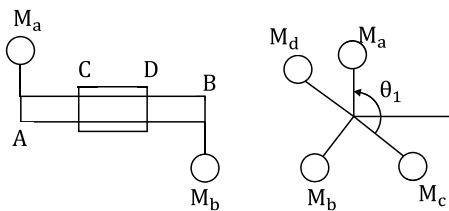
m_b = Balancing mass

$$\sum F_H = 0, \sum F_V = 0$$

$$\sum mr \cos \theta = 0$$

$$\sum mr \sin \theta = 0$$

Balancing of several masses rotating in different plane:



$$\sum F = 0; \sum M = 0$$

Table can be made and get the answer

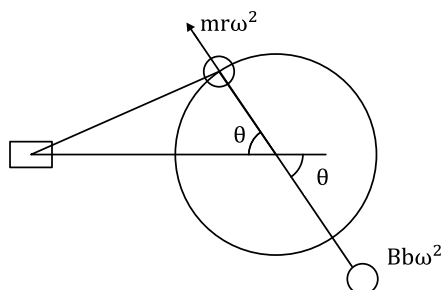
Balancing of Reciprocating Masses:

$$F_{\text{Inertia on Piston}} = mr\omega^2 \left[\cos \theta + \frac{\cos 2\theta}{n} \right]$$

Only primary force is balanced \Rightarrow which is maximum at $\theta = 0^\circ$ and 180°

2nd force is maximum 4 times in one rotation and negligible at moderate speeds.

To Balance primary force a balancing mass is attached opposite to crank.



B = Balance mass

b = Radial distance of balance mass

$$mr\omega^2 \cos \theta = Bb\omega^2 \cos \theta$$

$$mr = Bb$$

We will go for partial Balancing

$$cmr = Bb$$

$F_{\text{unbalance}}$ (along horizontal direction)

$$= (1 - c)mr\omega^2 \cos \theta$$

$F_{\text{unbalance}}$ (\perp^r to line of stroke)

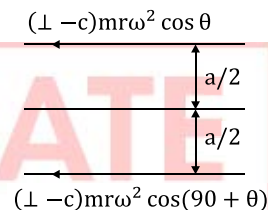
$$= cmr\omega^2 \sin \theta$$

$$F_R = \sqrt{((F_{\text{unbalance}})_H)^2 + ((F_{\text{unbalance}})_{\perp r})^2}$$

$$\text{For minimum } F_R = \frac{dF_R}{dc} = 0$$

Resultant unbalance is minimum when $c = 1/2$.

Balancing of Locomotive:



1. **Tractive Force (F_t):**

$$(1 - c)mr\omega^2 \cos \theta + (1 - c)mr\omega^2 \cos(90 + \theta)$$

F_t is max at $\theta = 135^\circ$ and 315°

2. **Swaying Couple (S):**

$$S = (1 - c)mr\omega^2 \cos \theta \left(\frac{a}{2} \right) - (1 - c)mr\omega^2 \cos(90 + \theta) \times \frac{a}{2}$$

$$S = (1 - c)mr\omega^2 \left(\frac{a}{2} \right) (\cos \theta + \sin \theta)$$

S_{max} at $\theta = 45^\circ$ and 225°

3. **Hammer Blow:**

It is unbalanced vertical force due to balancing mass (B) at radius (b).

$$P = Bb\omega^2 \sin \theta = cmr\omega^2 \sin \theta$$

P_{max} at $\theta = 90^\circ$ and 270°

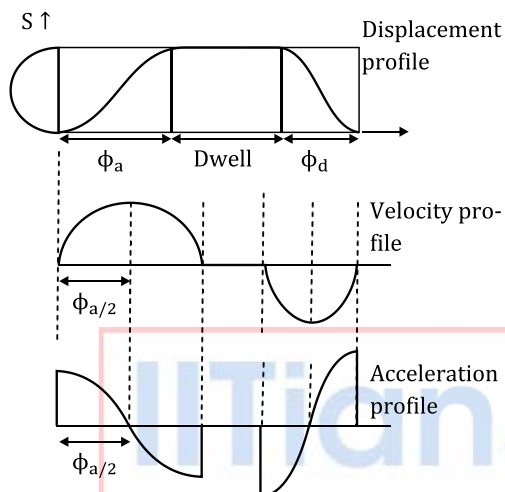
Chapter 9: CAMS & FOLLOWER

Types of motion of Follower

1. SHM (Simple Harmonic Motion):

ϕ_a = Angle of ascent

ϕ_d = Angle of descent

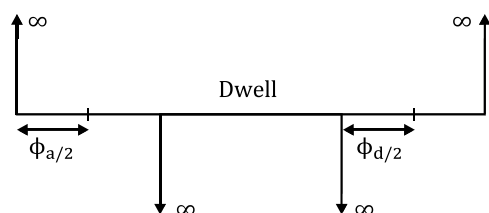


$$(\text{Displacement}) y = \frac{h}{2} \left(1 - \cos \left(\frac{\pi \theta}{\phi_a} \right) \right)$$

$$V = \frac{\pi h \omega}{2 \phi_a} \sin \left(\frac{\pi \theta}{\phi_a} \right)$$

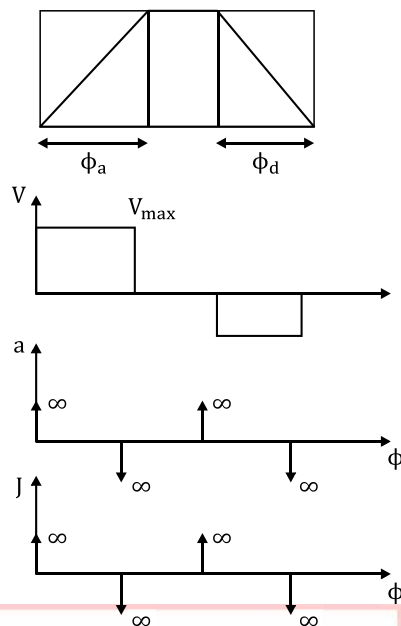
$$a = \frac{\pi^2 h \omega^2}{2 \phi_a^2} \left[\cos \left(\frac{\pi \theta}{\phi_a} \right) \right]$$

Jerk profile



$$J = \frac{da}{dt}$$

2. Uniform Velocity (Worst motion)

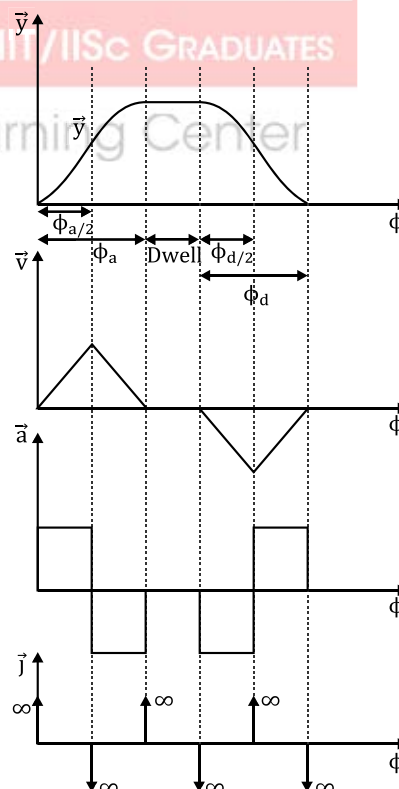


$$y = \frac{h\theta}{\phi_a}$$

$$V = \frac{h\omega}{\phi_a}$$

$$a = 0, J = 0$$

3. Uniform Acceleration:

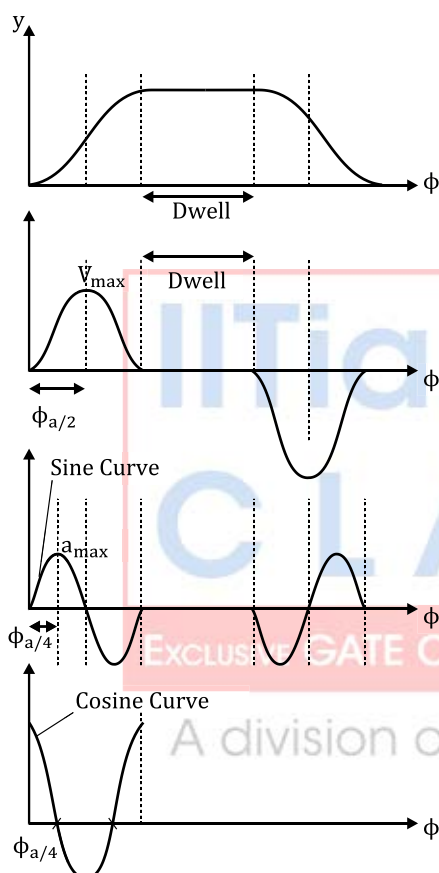


$$y = \frac{2h\theta^2}{\phi_a^2}$$

$$V = \frac{4h\theta\omega}{\phi_a^2}$$

$$a = \frac{4h\omega}{\phi_a^2}$$

4. Cycloidal Motion (Best Motion):



$$y = \frac{h}{\pi} \left(\frac{\pi\theta}{\phi_a} - \frac{1}{2} \sin \left(\frac{2\pi\theta}{\phi_a} \right) \right)$$

$$V = \frac{h\omega}{\phi_a} \left(1 - \cos \left(\frac{2\pi\phi}{\phi_a} \right) \right)$$



NOTE:

$$V_{(\max)\text{cycloidal}} > V_{\text{SHM}} > V_{v=\text{constant}}$$

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