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Space Dynamics
(GATE Aerospace) by
Mr Dinesh Kumar (IIT Madras Fellow)

SPACE DYNAMICS

Topics to study –

- **Gravitation field**
- **Kepler's law**
- **Mechanics of orbital trajectory**
- **Orbits**

Ref: -

- **Introduction to flight by Anderson**
- **NPTEL- Space Technology https://nptel.ac.in/courses/101/106/101106046/**

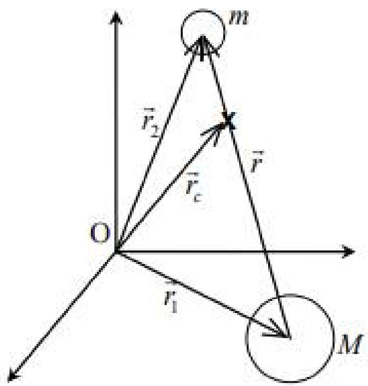
Some constants –

- **Universal gravitation constant, G = 6.67$\times 10^{-11}\ \frac{N-m^2}{kg^2}$**
- **μ = GM = 3.98$\times 10^{14}\ \frac{m^3}{s^2}$**
- **Earth radius, R_e = 6371 km**
- **Gravity acceleration, g_o = 9.81 $\frac{m}{s^2}$**
- **Mass of the Earth, M_e = 5.98$\times 10^{24}$ kg**

1. GRAVITATION FIELD

1.1 Newton’s law of Gravitation

Force between two objects of masses m_1 and m_2 is inversely proportional to the squire of the distance between two centers.



F $\propto \frac{1}{r^2}$

→It is also proportional to product of masses m_1 and m_2

F $\propto m_1 m_2$

F $\propto \frac{m_1 m_2}{r^2}$

F = $\boldsymbol{\frac{G m_1 m_2}{r^2}}$

G = universal gravitational const.

G = 6.67× $10^{-11}\ \frac{N-m^2}{kg^2}$

If one of the mass is earth mass (M)

F = $\boldsymbol{\frac{GMm}{r^2}}$

Where, M = 5.98× 10^{24} kg

Radius of the earth, R_e = 6371 km ≅ 6400 km

μ = GM = 3.98× $10^{14}\ \frac{m^3}{s^2}$

1.2 Gravitational potential energy (U)

The potential energy (GPE) is zero when distance between two masses is infinite (∞).

Below the distance of ∞, GPE is negative.

U(r) = - $\frac{Gm_1m_2}{r}$

r = separation between particles

$m_1\ and\ m_2$ = masses of two objects

→If a particle is brought from a position to new position under gravitational force, change in P.E. is given by

U_f - U_i = - $\int \vec{F} dr$

1.3 Gravitational potential (V)

It is the gravitational potential energy per mass.

$$V_B - V_A = \frac{U_B - U_A}{m}$$

1.4 Gravitational potential field ($\vec{E}$)

It is the intensity of gravitational force per unit mass at a point.

$$\vec{E} = \frac{\vec{F}}{m}$$

The intensity of gravitational force per unit mass at earth surface is known as gravity acceleration (g).

$$\vec{E} = \frac{\vec{F}}{m} = \frac{GMm}{r^2 m} = \frac{GM}{r^2} = \vec{g}$$

1.5 The variation of g from earth surface

- **Above earth at an altitude h**

F = $\frac{GMm}{(R+h)^2}$

∵ g = $\frac{F}{m}$

F = mg

g = $\frac{GM}{R^2(1+\frac{h}{R})^2}$

If h = 0, then g = g_o = 9.81 m/s^2

g = $\frac{g_o}{(1+\frac{h}{R})^2}$ = $g_o\left(1+\frac{h}{R}\right)^{-2} \cong g_o$(1-2h/R)

Here g decreases with height.

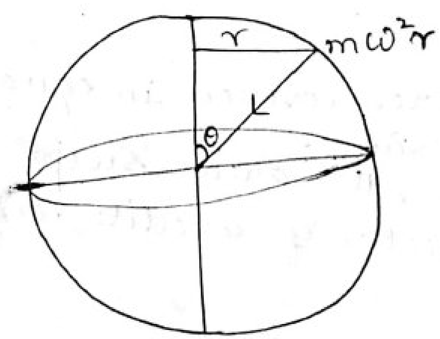
- **Below earth surface**

Gravitational potential force, F = $\frac{GMm}{(R)^3}$(R-h) (h→ depth)

g = $\frac{\vec{F}}{m} = \frac{GM}{(R)^2}$(1-h/R)

Here g decreases with depth up to zero.

1.6 Effect on gravity due to rotation of earth



Centrifugal force = $m\omega^2 r \sin\theta$

g' = $\frac{F}{m}$

F = mg - m$\omega^2 r \sin\theta$ =

$\frac{F}{m}$ = g - $\omega^2 r \sin\theta$

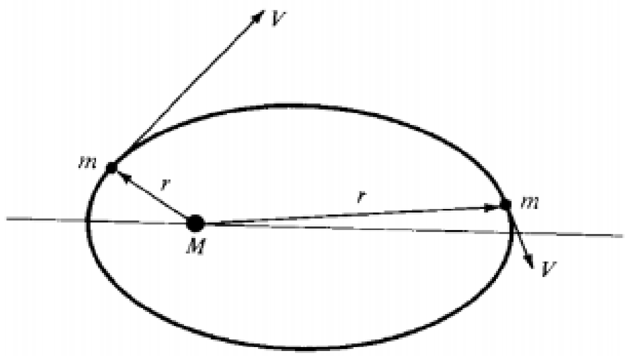
g' = g - $\omega^2 r \sin\theta$

So g is maximum at poles as centrifugal force is zero and minimum at equator as centrifugal force is maximum.

2. Kepler's law

- **First law**

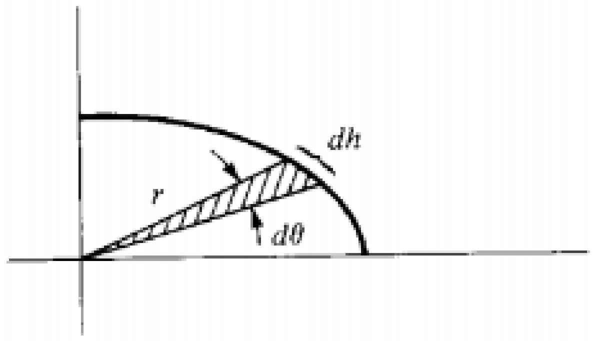
A satellite or object describes an elliptical path around its centre of attraction.



Its path depends on velocity.

- **Second law**

The area swept out by radius vector of object in equal time remains same.



dA = $\frac{1}{2}.r.dh$

dh = r$d\theta$

dA = $\frac{1}{2}.r^2.d\theta$

$\frac{dA}{dt} = \frac{1}{2}.r^2.\frac{d\theta}{dt}$

$\frac{\mathrm{dA}}{dt} = \frac{1}{2}.r^2.\omega = \frac{1}{2}.r^2.\dot{\theta} = \frac{h}{2}$

→ Angular momentum is constant under conservative forces.

m$r^2\dot{\theta}$ = const.

$r^2\dot{\theta}$ = const. = h = $\vec{r} \times \vec{v}$

So, $\frac{\mathrm{dA}}{dt}$ = const.

- **Third law**

The square of time period is proportional to cube of semi-major axis.

$T^2 \propto a^3$

$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$

3. Mechanics of orbital trajectory

- **Lagrange equation of motion**

Kinetic energy of the body

T = T $(q_1, q_2, q_3, q'_1, q'_2, q'_3)$

Potential energy of the body

$\emptyset = \emptyset\ (q_1, q_2, q_3)$

$q_1, q_2, q_3 \rightarrow$ Coordinates

Lagrange's function

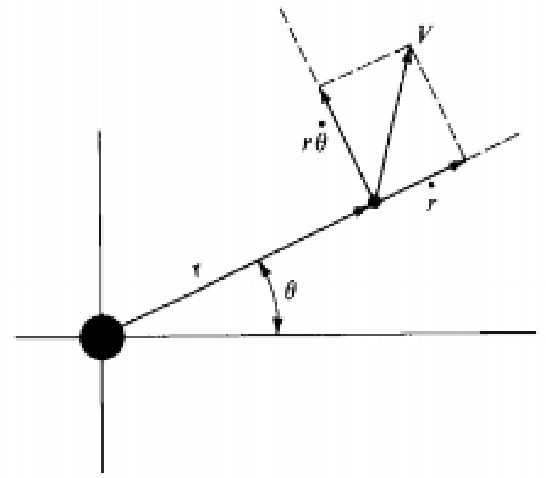
$$\boldsymbol{\beta} = \mathbf{T} - \emptyset$$

$$\frac{d}{dt}\left(\frac{\partial\beta}{\partial\dot{q}_1}\right) - \frac{\partial\beta\prime}{\partial q_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial\beta}{\partial\dot{q}_2}\right) - \frac{\partial\beta\prime}{\partial q_2} = 0$$

$$\frac{d}{dt}\left(\frac{\partial\beta}{\partial\dot{q}_3}\right) - \frac{\partial\beta\prime}{\partial q_3} = 0$$

- **Orbit equation**



Kinetic energy, q = $\frac{1}{2}mv^2$

$v^2 = \dot{r}^2 + (r\dot{\theta})^2$

T = $\frac{1}{2}m(\dot{r}^2 + (r\dot{\theta})^2) = \frac{1}{2}m(\dot{r}^2 + r^2\,\dot{\theta}^2)$

Potential energy

$\emptyset = -\frac{GMm}{r}$

$\rightarrow \beta = T - \emptyset \quad = \frac{1}{2}m(\dot{r}^2 + r^2\,\dot{\theta}^2) + \frac{\mu m}{r}$ ... (1)

<u>For θ co-ordinate</u>

$\frac{d}{dt}\left(\frac{\partial \beta}{\partial \dot{\theta}}\right) - \frac{\partial \beta}{\partial \theta} = 0$ .... (2)

By equation (1),

$\frac{d}{dt}\left(\frac{\partial \beta}{\partial \dot{\theta}}\right) = \frac{d}{dt}(mr^2\dot{\theta})$

Angular momentum = $mr^2\dot{\theta}$ = const.

$r^2\dot{\theta} = \frac{c}{m} = h$ (angular momentum per mass is constant)

So, $\frac{d}{dt}\left(\frac{\partial \beta}{\partial \dot{\theta}}\right) = \frac{d}{dt}(mr^2\dot{\theta}) = 0$

By equation (1),

$\frac{\partial \beta}{\partial \theta} = 0$

<u>For r co-ordinate</u>

$\frac{d}{dt}\left(\frac{\partial \beta}{\partial \dot{r}}\right) - \frac{\partial \beta}{\partial r} = 0$ .... (3)

By equation (1),

$\frac{\partial \beta}{\partial \dot{r}} = m\dot{r}$

$\frac{\partial \beta}{\partial r} = mr\dot{\theta}^2 - \frac{m\mu}{r^2}$

From equation (3),

$\frac{d}{dt}(m\dot{r}) - mr\dot{\theta}^2 + \frac{m\mu}{r^2} = 0$

$m\ddot{r} + \frac{m\mu}{r^2} - mr\dot{\theta}^2 = 0$

$\ddot{r} - \frac{r^4\dot{\theta}^2}{r^3} + \frac{\mu}{r^2} = 0$

$\ddot{r} - \frac{h^2}{r^3} + \frac{\mu}{r^2} = 0$ ..... (4) (h $= r^2\dot{\theta}$ = specific angular momentum)

This is differential equation of 2nd order.

Sol for above equation is given by

$$r = \frac{1}{\frac{\mu}{h^2} + A\cos(\theta - c)} = \frac{\frac{h^2}{\mu}}{1 + A.\left(\frac{h^2}{\mu}\right).\cos(\theta - c)} = \frac{P}{1 + e.\cos(\theta - c)} \quad \left(p = \frac{h^2}{\mu}\right)$$

Here A and C (phase angle) are constant depends on initial condition.

And $A.\left(\frac{h^2}{\mu}\right) = e$ = eccentricity

$$\text{So, } r = \frac{\frac{h^2}{\mu}}{1 + e.\cos(\theta - c)} \quad .... (5)$$

- **The type of orbit is decided by e**

(1) If $e = 0 \rightarrow$ path is circular

(2) If $e < 1 \rightarrow$ path is elliptical

(3) If $e = 1 \rightarrow$ path is parabolic

(4) If $e > 1 \rightarrow$ path is hyperbolic

- **Orbital energy**

It is the summation of K.E. and P.E.

$$\underbrace{\left(\frac{1}{2}\dot{r}^2+\frac{1}{2}r^2\dot{\theta}^2\right)}_{kinetic\ energy/mass}+\underbrace{\left(-\frac{\mu}{r}\right)}_{potential\ energy/mass}=const.=\underbrace{E_T}_{total\ energy/mass}\quad (Orbital\ Energy)$$

or $E_T=\frac{1}{2}V^2-\frac{\mu}{r}=const.$, where $V^2=\frac{1}{2}\dot{r}^2+\frac{1}{2}r^2\dot{\theta}^2$

Now the constant value of E_T can be evaluated at $\theta=0$, where $r=r_{min}$ and $\dot{r}=0$

$E_T=\frac{1}{2}r_{min}^2\dot{\theta}^2-\frac{\mu}{r_{min}}$, also $h=r^2\dot{\theta}=const.=r_{min}^2\dot{\theta}$ and $r_{min}=\frac{h^2/\mu}{1+e}$

$$\therefore \quad E_T = \frac{1}{2} r_{\min}{}^2 \dot{\theta}^2 - \frac{\mu}{r_{\min}} = \frac{1}{2}\frac{h^2}{r_{\min}^2} - \frac{\mu}{r_{\min}} = \frac{\mu(e-1)}{2r_{\min}} = -\frac{\mu^2(1-e^2)}{2h^2}$$

In General

$$E_T = -\frac{\mu^2(1-e^2)}{2h^2} = \frac{1}{2} r^2 \dot{\theta}^2 - \frac{\mu}{r}$$

Eccentricity can be written in terms of orbit energy per unit mass

$$e = \sqrt{1 + \frac{2h^2 E_T}{\mu^2}}$$

When,

$e<1 \quad then \quad E_T<0 \qquad -\frac{\mu}{r}\left(\frac{Potential\ Energy}{mass}\right)>\frac{1}{2}V^2\left(\frac{Kinetic\ Energy}{mass}\right)$, the trajectory is an ELLIPSE (Kepler's first law)

$e=1 \quad then \quad E_T=0 \qquad -\frac{\mu}{r}\left(\frac{Potential\ Energy}{mass}\right)=\frac{1}{2}V^2\left(\frac{Kinetic\ Energy}{mass}\right)$, the trajectory is a PARABOLA

$e>1 \quad then \quad E_T>0 \qquad -\frac{\mu}{r}\left(\frac{Potential\ Energy}{mass}\right)<\frac{1}{2}V^2\left(\frac{Kinetic\ Energy}{mass}\right)$, the trajectory is a HYPERBOLA

Escape velocity

Orbit Energy

$E_T = \frac{1}{2}v^2 - \frac{\mu}{r}$

At escape velocity, the kinetic energy is converted into potential energy.

$E_T = \frac{1}{2}v^2 - \frac{\mu}{r} = 0$

$V_e = \sqrt{\frac{2GM}{r}} = \sqrt{2g_0 r}$

Example1. What will be the escape velocity, if R is doubled and mass remains same?

Sol.

(I) if mass remains same

$V_{esc} = \sqrt{2g_o R}$

g = $\frac{GM}{(R+h)^2} = \frac{GM}{(R+R)^2} = \frac{GM}{(2R)^2} = \frac{g_o}{4}$ (h = R)

$V_{esc,f} = \sqrt{2.\frac{g_o}{4}.2R} = \frac{V_{esc,i}}{\sqrt{2}}$

(II) If density remains same

M = ρV

$M_1 = \rho \times \frac{4}{3}\pi R^3$

$M_2 = \rho \times \frac{4}{3}\pi (2R)^3 = 8M_1$ (R→ $2R$)

$g_{new} = \frac{8GM}{(2R)^2} = 2g_0$

$V_{esc,f} = \sqrt{2.2g_0.2R} = 2V_{esc,i}$

4. Orbits

4.1 For circular orbit

e = 0, for circular orbit

$E_T = -\frac{\mu^2}{2h^2}$

$E_T = \frac{1}{2}v^2 - \frac{\mu}{r}$

$-\frac{\mu^2}{2h^2} = \frac{1}{2}v^2 - \frac{\mu}{r}$

For Circular orbit $r = \frac{h^2}{\mu}$

$-\frac{\mu}{2r} = \frac{1}{2}v^2 - \frac{\mu}{r}$

$v = \sqrt{\frac{\mu}{r}} = \frac{\mu}{h}$

- So velocity for circular orbit, $V_0 = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{GM}{r}}$

Escape velocity from circular orbit

$$\Delta V_{escape} = \sqrt{\frac{2\mu}{r}} - \sqrt{\frac{\mu}{r}}$$

- **Time period**

$$T = \frac{2\pi a}{V}$$

$$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}}$$

$$T = \frac{2\pi}{\sqrt{GM}} R^{3/2}$$

$$T^2 = \frac{4\pi^2}{GM} R^3$$

4.2 For parabolic orbit

Case II: Parabola, $E_T = 0 \quad or \quad e = 1$

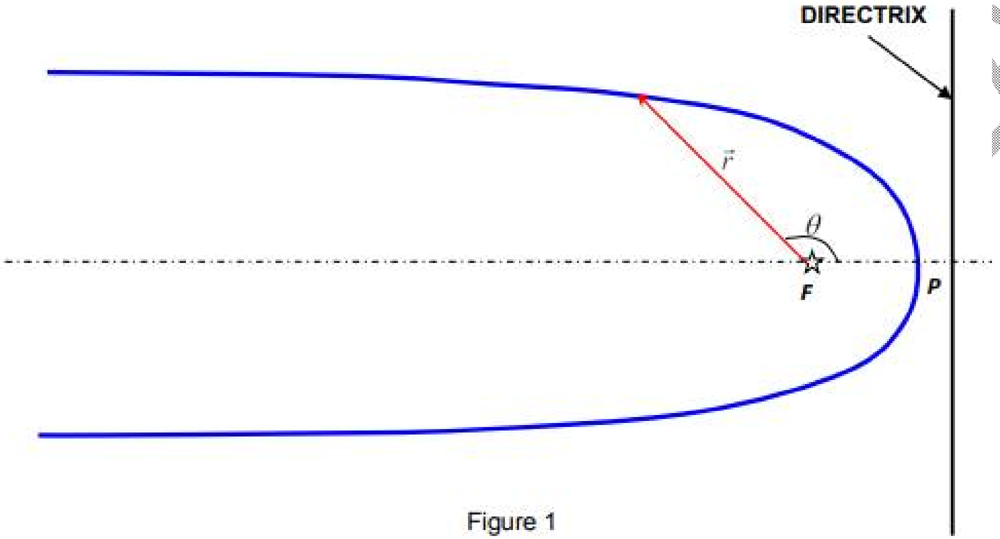


Figure 1

e = 1, for parabolic orbit

$$e = \sqrt{1 + \frac{2h^2 E_T}{\mu^2}} = 1$$

$E_T = 0$

So, $E_T = \frac{1}{2}v^2 - \frac{\mu}{r} = 0$

$v = \sqrt{\frac{2\mu}{r}}$ (escape velocity)

For parabolic trajectory

$r = \frac{\frac{h^2}{\mu}}{1+\cos\theta}$

At $\theta = 0$

$r_p = \frac{h^2}{2\mu}$ (perigee)

$V_p = V_{max} = \sqrt{\frac{2\mu}{r_p}} = \sqrt{\frac{2\mu . 2\mu}{h^2}} = \frac{2\mu}{h}$

4.3 For an elliptical orbit

Case I: Ellipse, $E_T < 0 \quad or \quad e < 1 \ (for\ e = 0, circle)$

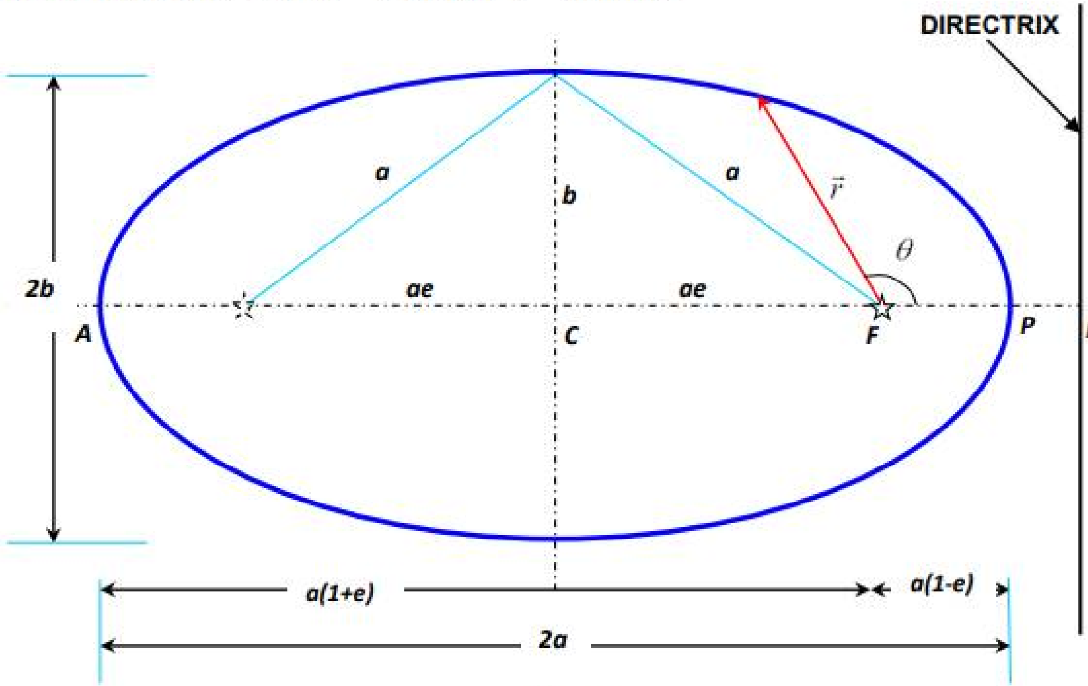


Figure 1

e < 1, for elliptical orbit

a = semi – major axis

b = semi – minor axis

$r_p + r_a$ = 2a

r_{max} = apogee = r_a

r_{min} = perigee = r_p

r_a - r_p = 2ae

r_a = a(1+ e)

r_p = a(1- e)

$$\frac{r_p}{r_a} = \frac{(1-e)}{(1+e)}$$

$$e = \frac{r_a - r_p}{r_a + r_p}$$

e = $\sqrt{1-\frac{b^2}{a^2}}$ from geometry

For Elliptical trajectory

r = $\frac{\frac{h^2}{\mu}}{1+\text{e}.\cos\theta}$

r_p = $\frac{\frac{h^2}{\mu}}{1+e}$

r_a = $\frac{\frac{h^2}{\mu}}{1-e}$

$\rightarrow \frac{h^2}{\mu}$ = r_p(1+e)

$\frac{h^2}{\mu}$= a(1-e)(1+e)

$\frac{h^2}{\mu}$= a(1-e^2)

We know

$$e = \sqrt{1 + \frac{2h^2 E_T}{\mu^2}}$$

Then orbit energy

$$E_T = -\left[\frac{\mu^2(1-e^2)}{2h^2}\right]$$

$$= -\frac{\mu}{2a} \qquad \left\{\frac{h^2}{\mu} = a(1-e^2)\right\}$$

$$E_T = \frac{1}{2}v^2 - \frac{\mu}{r}$$

$$\frac{v^2}{2} = \frac{\mu}{r} - \left[\frac{\mu^2(1-e^2)}{2h^2}\right]$$

$$= \frac{\mu}{r} - \frac{\mu}{2a}$$

$$v = \sqrt{\frac{\mu}{a}\left(\frac{2a}{r} - 1\right)}$$

Velocity at perigee r → r_p

, $V_p = \sqrt{\frac{\mu}{a}\left(\frac{(1+e)}{(1-e)}\right)} = \sqrt{\frac{2\mu r_a}{r_p(r_a+r_p)}}$

Velocity at apogee r → r_a

, $V_a = \sqrt{\frac{\mu}{a}\left(\frac{(1-e)}{(1+e)}\right)} = \sqrt{\frac{2\mu r_p}{r_a(r_a+r_p)}}$

$\frac{V_p}{V_a} = \frac{(1+e)}{(1-e)}$

Escape velocity from elliptical orbit

$\Delta V_{escape} = \sqrt{\frac{2\mu}{r}} - \sqrt{\frac{\mu}{a}\left(\frac{2a}{r} - 1\right)}$

- **Time period**

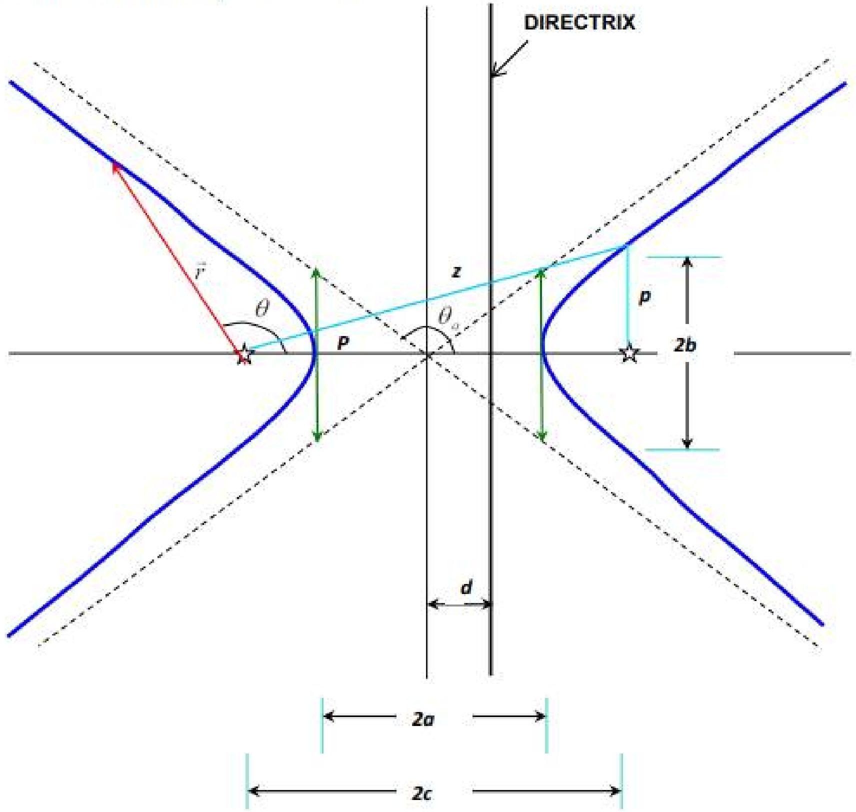
$T = \frac{A}{\frac{dA}{dt}} = \frac{\pi ab}{h/2}$

$$T = \frac{\pi a^2\sqrt{1-e^2}}{\sqrt{a(1-e^2)\frac{\mu}{2}}} = \frac{2\pi}{\sqrt{\mu}} a^{3/2}$$

$$T^2 = \frac{4\pi^2}{\mu} a^3$$

4.4 For hyperbolic orbit

Case III: Hyperbola, $E_T > 0 \quad or \quad e > 1$



e > 1, for hyperbolic orbit

P.E. < K.E.

$$e = \sqrt{1 + \frac{2h^2 E_T}{\mu^2}}$$

$$E_T = -\left[\frac{\mu^2(1-e^2)}{2h^2}\right]$$

For hyperbolic trajectory

$$r = \frac{\frac{h^2}{\mu}}{1+e.\cos\theta}$$

$$e = \sqrt{1 + \frac{b^2}{a^2}} \text{ (for hyperbolic orbit)}$$

$$r_{min.} = r_p = \frac{\frac{h^2}{\mu}}{1+e} = a(e-1)$$

$$a = \frac{\frac{h^2}{\mu}}{e^2-1}$$

Orbital energy = $- \frac{\mu^2(1-e^2)}{2h^2} = \frac{\mu}{2a}$

$$E_T = \frac{1}{2}v^2 - \frac{\mu}{r}$$

$$\frac{\mu}{2a} = \frac{1}{2}v^2 - \frac{\mu}{r}$$

$$v = \sqrt{\frac{\mu}{a}\left(\frac{2a}{r} + 1\right)}$$

Velocity at Perigee

$$V_p = \sqrt{\frac{\mu}{a}\left(\frac{(e+1)}{(e-1)}\right)}$$

Summary

	Ellipse	Parabola	Hyperbola
E_T	$-\mu/2a$	0	$\mu/2a$
V	$\sqrt{\frac{\mu}{a}\left(\frac{2a}{r}-1\right)}$	$\sqrt{\frac{2\mu}{r}}$	$\sqrt{\frac{\mu}{a}\left(\frac{2a}{r}+1\right)}$
h	$\sqrt{\mu a\left(1-e^2\right)}$	$\sqrt{2\mu r_p}$	$\sqrt{\mu a\left(e^2-1\right)}$
r_p	$a(1-e)$	$h^2/2\mu$	$a(e-1)$
e	$\frac{\sqrt{a^2-b^2}}{a}$	1	$\frac{\sqrt{a^2+b^2}}{a}$

GATE QUESTIONS

Gate 2020

Q.No. 11 For hyperbolic trajectory of a satellite of mass m having velocity V at a distance r from the center of earth (G: gravitational constant, M: mass of earth), which one of the following relations is true?

(A) $\frac{1}{2}mV^2 > \frac{GMm}{r}$

(B) $\frac{1}{2}mV^2 < \frac{GMm}{r}$

(C) $\frac{1}{2}mV^2 = \frac{GMm}{r}$

(D) $\frac{1}{2}mV^2 < \frac{2GMm}{r}$

Q.No. 43 The ratio of tangential velocities of a planet at the perihelion and the aphelion from the sun is 1.0339. Assuming that the planet's orbit around the sun is planar and elliptic, the value of eccentricity of the orbit is ____________ (*round off to three decimal places*).

Gate 2019

Q.42 The product of earth's mass (M) and the universal gravitational constant (G) is $GM = 3.986\times10^{14}$ m^3/s^2. The radius of earth is 6371 km. The minimum increment in the velocity to be imparted to a spacecraft flying in a circular orbit around the earth at an altitude of 4000 km to make it exit earth's gravitational field is ________ km/s (*round off to 2 decimal places*).

Q.9 The tangential velocity component '*V*' of a spacecraft, which is in a circular orbit of radius '*R*' around a spherical Earth (μ = GM → gravitational parameter of Earth) is given by the following expression.

(A) $V = \sqrt{\frac{\mu}{2R}}$ (B) $V = \sqrt{\frac{\mu}{R}}$ (C) $V = \frac{2\pi}{\sqrt{\mu}} R^{\frac{3}{2}}$ (D) $V = \frac{2\pi}{\sqrt{\mu}} R^{\frac{2}{3}}$

Q.10 Equation of the trajectory of a typical space object around any planet, in polar coordinates (r, θ) (i.e. a general conic section geometry), is given as follows. (h is angular momentum, μ is gravitational parameter, e is eccentricity, r is radial distance from the planet center, θ, is angle between vectors $\vec{e}$ and $\vec{r}$.

(A) $r = \frac{\left(h^2/\mu\right)}{1-e\cos\theta}$ (B) $r = \frac{\left(h^2/\mu\right)}{e-\cos\theta}$

(C) $r = \frac{\left(h^2/\mu\right)}{1+e\cos\theta}$ (D) $r = \frac{\left(h^2/\mu\right)}{e+\cos\theta}$

Q.11 In an elliptic orbit around any planet, the location at which a spacecraft has the maximum angular velocity is

(A) apoapsis.
(B) periapsis.
(C) a point at +45° from periapsis.
(D) a point at –90° from apoapsis.

Q.44 A spacecraft forms a circular orbit at an altitude of 150 km above the surface of a spherical Earth. Assuming the gravitational parameter, $\mu = 3.986 \times 10^{14}$ m^3/s^2 and radius of earth, $R_E = 6{,}400$ km, the velocity required for the injection of the spacecraft, parallel to the local horizon, is ________________ (accurate to two decimal places).

Gate 2017

Question Number : 14 Correct : 1 Wrong : 0

The period of revolution of earth about the sun is 365.256 days, approximately. The semi-major axis of the earth's orbit is close to 1.4953×10^{11} m. The semi-major axis of the orbit of Mars is 2.2783×10^{11} m. The period of revolution of Mars, about the sun, is ______________ Earth days (in three decimal place)

Gate 2016

Q.55 A satellite is injected at an altitude of 350 km above the Earth's surface, with a velocity of 8.0 km/s parallel to the local horizon. (Earth radius=6378 km, μ_E (GM=Gravitational constant × Earth mass) $= 3.986\times10^{14}\ m^3s^{-2}$). The satellite

(A) forms a circular orbit.
(B) forms an elliptic orbit.
(C) escapes from Earth's gravitational field.
(D) falls back to earth.

Gate 2015

Question Number : 45 Question Type : MCQ

A planetary probe is launched at a speed of 200 km/s and at a distance of 71,400 km from the mass center of its nearest planet of mass $1.9 \times 10^{28} kg$. The universal gravitational constant, $G = 6.67 \times 10^{-11} \frac{m^3}{kg\ s^2}$. The ensuing path of the probe would be

(A) elliptic (B) hyperbolic (C) parabolic (D) circular

Gate 2014

Q.25 Considering R as the radius of the moon, the ratio of the velocities of two spacecraft orbiting moon in circular orbit at altitudes R and $2R$ above the surface of the moon is ________.

Gate 2011

Q.42 An elliptic orbit has its perigee at 400 km above the Earth's surface and apogee at 3400 km above the Earth's surface. For this orbit, the eccentricity and semi-major axis respectively are (assume radius of Earth = 6400 km)

(A) 0.18 and 8300 km
(B) 0.18 and 1900 km
(C) 0.22 and 8300 km
(D) 0.22 and 1900 km

Gate 2010

Q.15 The angular momentum, about the centre of mass of the earth, of an artificial satellite in a highly elliptical orbit is :

(A) a maximum when the satellite is farthest from the earth
(B) a constant
(C) proportional to the speed of the satellite
(D) proportional to the square of the speed of the satellite

Gate 2009

Q.32 The acceleration due to gravity on the surface of Mars is 0.385 times that on earth, and the diameter of Mars is 0.532 times that of earth. The ratio of the escape velocity from the surface of Mars to the escape velocity from the surface of earth is approximately

(A) 0.453 (B) 0.205 (C) 0.851 (D) 0.724

Gate 2008

Q.4 To transfer a satellite from an elliptical orbit to a circular orbit having radius equal to the apogee distance of the elliptical orbit, the speed of the satellite should be

(A) increased at the apogee
(B) decreased at the apogee
(C) increased at the perigee
(D) decreased at the perigee

Q.21 Which of the following quantities remains constant for a satellite in an elliptical orbit around the earth?

(A) Kinetic energy
(B) Product of speed and radial distance from the center of the earth
(C) Rate of area swept by the radial vector from the center of the orbit
(D) Rate of area swept by the radial vector from the center of the earth

Q.22 A planet is observed to be at its slowest when it is at a distance r_1 from the sun and at its fastest when it is at a distance r_2 from the sun. The eccentricity e of the planet's orbit is given by

(A) $e = \frac{r_1}{r_2}$

(B) $e = \frac{r_1 - r_2}{r_1 + r_2}$

(C) $e = \frac{r_2}{r_1}$

(D) $e = \frac{r_1 + r_2}{r_1 - r_2}$

Q.31 The velocity required for a spacecraft to escape earth's gravitational field depends on

(A) the mass of the spacecraft
(B) the distance between earth's center and the spacecraft
(C) the earth's rotational speed about its own axis
(D) the earth's orbital speed

Gate 2007

Q.24 The earth's radius is 6.37×10^6 m and the acceleration due to gravity on its surface is $9.81\ m/s^2$. A satellite is in a circular orbit at a height of 6.30×10^5 m above the earth's surface. The minimum additional speed it needs to escape from the earth's gravitational field is

(A) 3.66×10^3 m/s (B) 3.12×10^3 m/s (C) 3.27×10^3 m/s (D) 3.43×10^3 m/s