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Strength of Materials
(GATE Aerospace and GATE Mechanical) by
Mr Dinesh Kumar

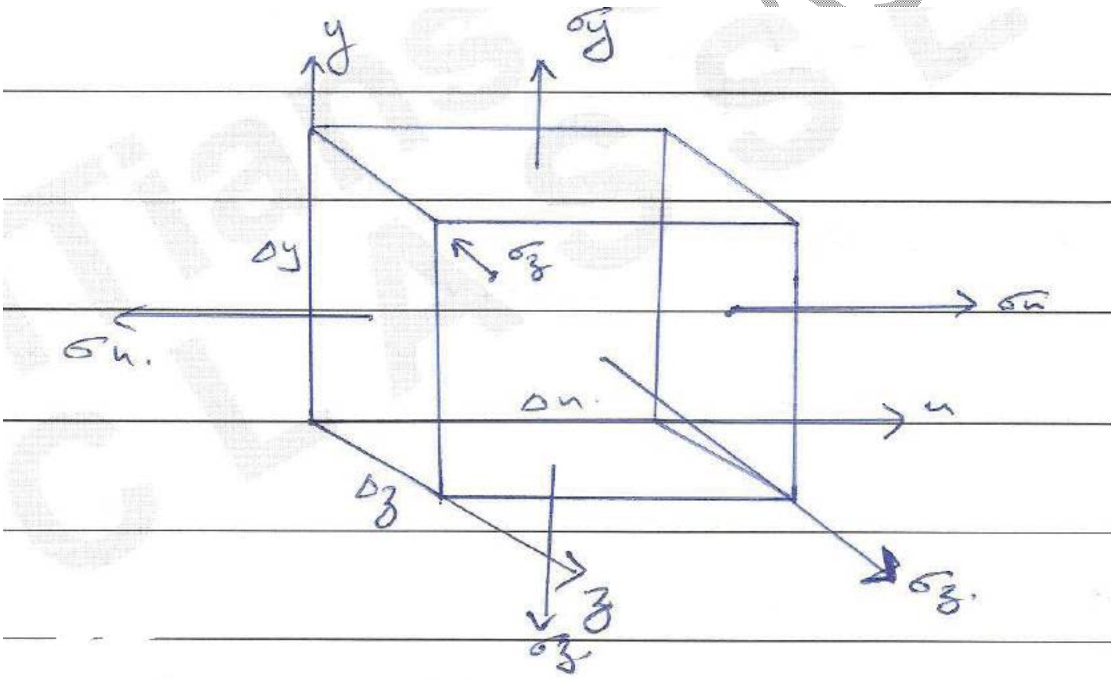
Volumetric Strain or Dilatations

- Volumetric strain occurs only due to normal stress.
- Shear stress causes only distortion and has no change in volume.

Change in volume to original volume due applied normal stresses is known as volumetric strain.

$$e = \frac{change\ in\ volume}{origional\ volume} = \frac{\Delta V}{V}$$

Consider a triaxial stress system as shown in the figure



Final length of cuboids in x, y and z directions are given by

$$\Delta x' = (1+\epsilon_x)\,\Delta x$$

$$\Delta y' = (1+\epsilon_y)\,\Delta y$$

$$\Delta z' = (1+\epsilon_z)\,\Delta z$$

Final volume of cuboids

$$\Delta V' = \Delta x'.\Delta y'.\Delta z'$$

Initial volume of cuboids

$$V = \Delta x.\Delta y.\Delta z$$

Volumetric strain

$$e = \frac{V' - V}{V}$$

$$e = \frac{(1+\epsilon_x)\Delta x(1+\epsilon_y)\Delta y(1+\epsilon_z)\Delta z - \Delta x\Delta y\Delta z}{\Delta x\Delta y\Delta z}$$

Neglecting the higher order term of small quantities

$$\mathbf{e = 1 + \epsilon_x + \epsilon_y + \epsilon_z - 1}$$

$$\mathbf{e = \epsilon_x + \epsilon_y + \epsilon_z}$$

Substituting strain in terms of stress

$$e = \frac{(1-2\nu)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

For hydrostatic loading (pressure),

$$\rightarrow \sigma_x = \sigma_y = \sigma_z = -P \text{ (if it is inwards)}$$

$$e = \frac{-3(1-2\nu)P}{E}$$

$$K = \frac{E}{3(1-2\nu)}$$

Here, K is known as Bulk modulus of elasticity and is measure of compressibility of solid

$$\therefore e = -\frac{P}{K}$$

Value of ν is limited between, $-1 \leq \nu \geq 0.5$

Upper limited is decided by Bulk modulus and lower limit is decided by Modulus of rigidity G

Problems

1.

The Poisson's ratio for a perfectly incompressible linear elastic material is

(A) 1 (B) 0.5 (C) 0 (D) infinity

Sol^n compressibility of a solid is measured by bulk modulus.

$$K = \frac{E}{3(1-2\nu)}$$

for perfectly linear elastic materials

$K = \infty$

So, $\frac{1}{0} = \frac{E}{3(1-2\nu)}$

$\nu = 0.5$. Ans [B]

2.

The value of Poisson's ratio at which the shear modulus of an isotropic material is equal to the bulk modulus is

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{6}$

(D) $\frac{1}{8}$

Sol^n We know
Modulus of rigidity or Shear Modulus

$$G = \frac{E}{2(1+\nu)}$$

Bulk modulus

$$K = \frac{E}{3(1-2\nu)}$$

Given $G = K$

$$\frac{E}{2(1+\nu)} = \frac{E}{3(1-2\nu)}$$

$$3 - 6\nu = 2 + 2\nu$$

$$1 = 8\nu$$

$$\boxed{\nu = 1/8}$$ Ans (D)

3.

A cube made of a linear elastic isotropic material is subjected to a uniform hydrostatic pressure of 100 N/mm^2. Under this load, the volume of the cube shrinks by 0.05%. The Young's modulus of the material, $E = 300\ GPa$. The Poisson's ratio of the material is _____.

Sol^n Given Hydrostatic pressure $P = 100\ N/mm^2$
Volume of cube shrink by 0.05%
Volumetric strain $e = -\frac{0.05}{100} = -0.0005$
Young's modulus $E = 300\ GPa$
We know $e = \frac{-3(1-2\nu)}{E}p$

$$-0.0005 = \frac{-3(1-2\nu)}{300 \times 10^3} \times 100$$

$$0.5 = (1-2\nu)$$

v= 0.25

4.

A rod is subjected to a uni-axial load within linear elastic limit. When the change in the stress is 200 MPa, the change in the strain is 0.001. If the Poisson's ratio of the rod is 0.3, the modulus of rigidity (in GPa) is ________________

Sol^n Given change in stress $\Delta\sigma = 200$ MPa

change in strain $\Delta\epsilon = 0.001$

Poisson's ratio $= 0.3$

We know

Young's modulus $E = \frac{\Delta\sigma}{\Delta\epsilon}$

$= \frac{200}{0.001}$

$= 200$ GPa

Modulus of rigidity $G = \frac{E}{2(1+\nu)}$

$= \frac{200}{2(1+0.3)}$

$= \frac{100}{1.3} = 76.92$ GPa

5.

If the Poisson's ratio of an elastic material is 0.4, the ratio of modulus of rigidity to Young's modulus is _______

Sol^n Given poisson's ratio $\nu = 0.4$

We know $G = \frac{E}{2(1+\nu)}$

Ratio of $\frac{G}{E} = \frac{1}{2(1+\nu)} = \frac{1}{2.8}$

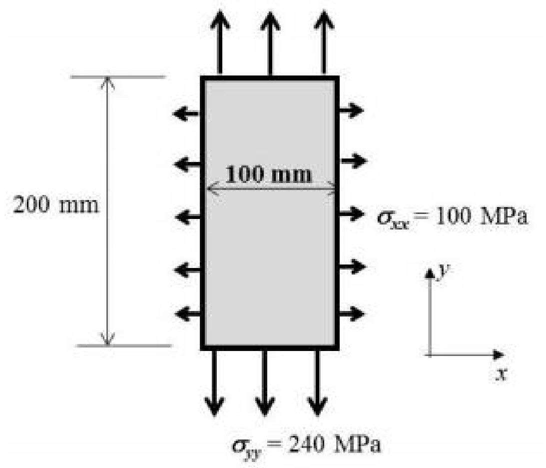
$= 0.357$

6.

The number of independent elastic constants required to define the stress-strain relationship for an isotropic elastic solid is ______

7.

A thin plate with Young's modulus 210 GPa and Poisson's ratio 0.3 is loaded as shown in the figure. The change in length along the y-direction is __________ mm (*round off to 1 decimal place*).



Q.7 Given $E = 210$ GPa, $l_x = 100$ mm, $\sigma_{xx} = 100$ MPa

$\nu = 0.3$, $l_y = 200$ mm, $\sigma_{yy} = 240$ MPa

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu \sigma_{xx}}{E}$$

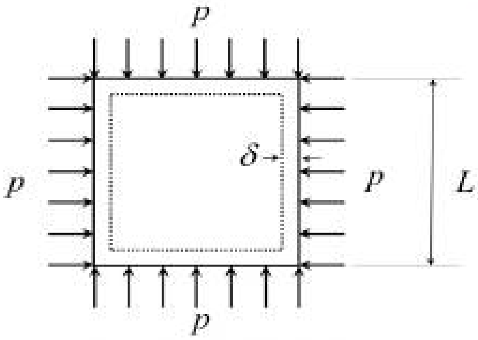
$$\frac{\Delta l_y}{l_y} = \frac{1}{E}(\sigma_{yy} - \nu \sigma_{xx}) = \frac{1}{210 \times 10^3}(240 - 0.3 \times 100)$$

$$\frac{\Delta l_y}{l_y} = 0.001$$

$$\Delta l_y = 0.001 \times 200 = 0.2 \text{ mm}$$

8.

A square plate of dimension $L \times L$ is subjected to a uniform pressure load $p = 250$ MPa on its edges as shown in the figure. Assume plane stress conditions. The Young's modulus $E = 200$ GPa.



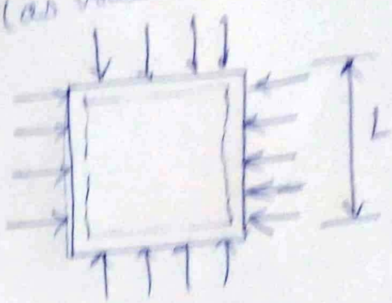
The deformed shape is a square of dimension $L - 2\delta$. If $L = 2$ m and $\delta = 0.001$ m, the Poisson's ratio of the plate material is __________

Sol^n Given $P = 250$ MPa

$E = 200$ GPa

$L' = L - 2\delta$

$L = 2$ m, $\delta = 0.001$ m



Here $\varepsilon_x = \varepsilon_y = \frac{1}{E}(P - \nu P)$

$$\frac{\Delta L'}{L} = \frac{P(1-\nu)}{E}$$

$$\frac{L' - L}{L} = \frac{P}{E}(1-\nu)$$

$$\frac{L - 2\delta - L}{L} = \frac{P}{E}(1-\nu)$$

$$\frac{2 \times 0.001}{2} = \frac{250}{200 \times 10^3}(1-\nu)$$

$$(1-\nu) = \frac{200}{250}$$

$$1 - \nu = 0.8$$

$$\boxed{\nu = 0.2}$$

Thermal Stresses

Mechanical + Thermal loading for a 3-D stress system

Strain due to thermal loading will be same in all the directions.

$$\epsilon_{xT} = \epsilon_{yT} = \epsilon_{zT} = \alpha \Delta T$$

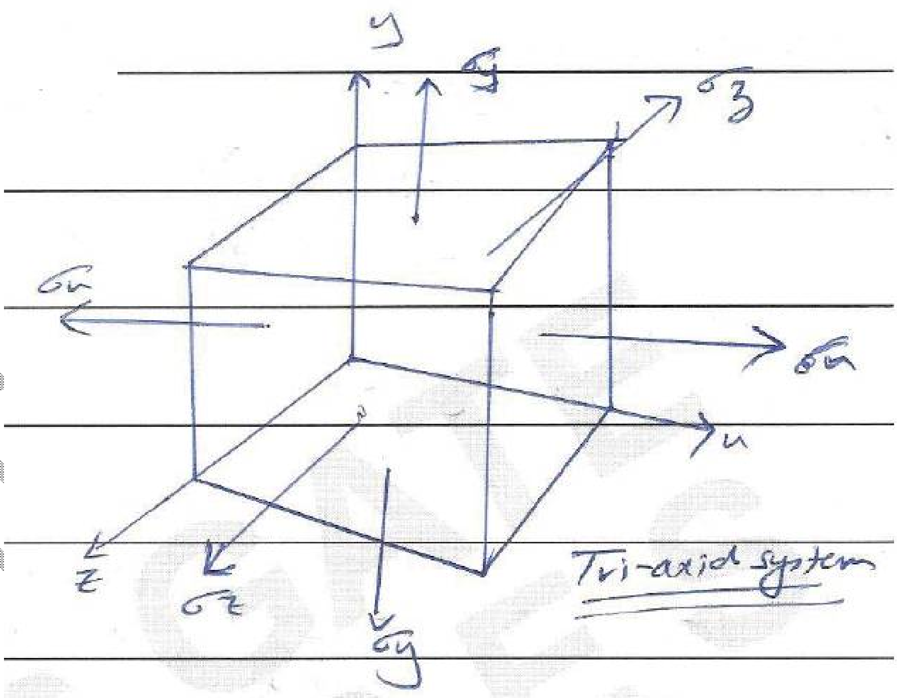
Total strain will be summation of thermal strain + mechanical strain.

Mechanical strain: - Mechanical strain for a triaxial stress system is given by

$$\in_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\in_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$\in_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$



Thermal strain:-

$$\epsilon_{xT} = \epsilon_{yT} = \epsilon_{zT} = \alpha\Delta T$$

If component is subjected to both mechanical and thermal loading,

$$\in = \in_M + \in_T$$

So,

$$\epsilon_x = \epsilon_{xM} + \epsilon_{xT}$$

$$\in_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) + \alpha\Delta T \quad \text{............} \quad (1)$$

$$\in_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) + \alpha\Delta T \quad \text{............} \quad (2)$$

$$\in_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) + \alpha\Delta T \quad \text{............} \quad (3)$$

Case (1) only thermal loading and there is no constrain (free expansion in all directions)

$$\epsilon_x = \epsilon_y = \epsilon_z = \epsilon_T = \alpha\Delta T$$

$$\sigma_x = \sigma_y = \sigma_z = 0$$

Case (2) only thermal loading and constraint in x direction (case of bar)

$$\sigma_x \neq 0, \ \sigma_y = \sigma_z = 0$$

$$\in_x = 0 \ \in_y = \in_z \neq 0$$

From equation (1), total strain is direction is zero

$$0 = \frac{\sigma_x}{E} - \frac{\nu}{E}(0 + 0) + \alpha\Delta T$$

$$\sigma_{xT} = -E\alpha\Delta T$$

From equation (1) and (3),

$$\in_y = \in_z = -\frac{\nu}{E}(\sigma_x) + \alpha\Delta T$$

$$= -\frac{\nu}{E}(-E\alpha\Delta T) + \alpha\Delta T$$

$$\in_y = \in_z = (1+\nu)\,\alpha\Delta T$$

Case (3) only thermal loading and constraint in x & y directions

$$\sigma_x = \sigma_y \neq 0, \ \sigma_z = 0$$

$$\in_x = \in_y = 0 \ \in_z \neq 0$$

From equations (1) & (2),

$$0 = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y) + \alpha\Delta T$$

&

$$0 = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x) + \alpha\Delta T$$

So $\quad \sigma_{xT} = \sigma_{yT} = -\frac{E\alpha\Delta T}{1-\nu}$

And from equation (3),

$$\epsilon_z = \left(\frac{1+\nu}{1-\nu}\right)\alpha\Delta T$$

Case (4) only thermal loading and constraint in x, y & z directions

$$\sigma_x = \sigma_y = \sigma_z \neq 0$$
$$\epsilon_x = \epsilon_y = \epsilon_z = 0$$

So from equations (1), (2) and (3),

$$\rightarrow \sigma_x = \sigma_y = \sigma_z = -\frac{E\alpha\Delta T}{1-2\nu}$$
$$\rightarrow \epsilon_x = \epsilon_y = \epsilon_z = 0$$

Problems

1.

In a general case of a homogeneous material under thermo-mechanical loading the number of distinct components of the state of stress is

(A) 3 (B) 4 (C) 5 (D) 6

2.

A steel cube, with all faces free to deform, has Young's modulus, E, Poisson's ratio, ν, and coefficient of thermal expansion, α. The pressure (hydrostatic stress) developed within the cube, when it is subjected to a uniform increase in temperature, ΔT, is given by

(A) 0 (B) $\frac{\alpha(\Delta T)E}{1-2\nu}$ (C) $-\frac{\alpha(\Delta T)E}{1-2\nu}$ (D) $\frac{\alpha(\Delta T)E}{3(1-2\nu)}$

3.

A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is υ, the thermal stress developed in the cube due to heating is

(A) $-\frac{\alpha(\Delta T)E}{(1-2\upsilon)}$ (B) $-\frac{2\alpha(\Delta T)E}{(1-2\upsilon)}$ (C) $-\frac{3\alpha(\Delta T)E}{(1-2\upsilon)}$ (D) $-\frac{\alpha(\Delta T)E}{3(1-2\upsilon)}$

4.

A steel bar of rectangular cross-section is heated uniformly and the rise in the temperature is ΔT. The Young's modulus is E, the Poisson's ratio is ν and the coefficient of thermal expansion is α. The bar is completely restrained in the axial direction and lateral directions.

(a) The thermal stress developed in the bar along the axial direction is

(A) $-\dfrac{E\alpha\Delta T}{1+2\nu}$ (B) $-E\alpha\Delta T$ (C) $-\dfrac{E\alpha\Delta T}{1-2\nu}$ (D) $-\dfrac{E\alpha\Delta T\nu}{1-2\nu}$

(b) Assume that the bar is allowed to deform freely in the lateral directions, while keeping the axial direction restrained. The percentage change in the magnitude of axial thermal stress for $\nu = 0.25$ is

(A) 0 (B) 25 (C) 50 (D) 100

[GATE XE 2010]

Sol^n (a) When bars is restrained in all direction

$\sigma_{xT_1} = \dfrac{-E\alpha\Delta T}{1-2\nu}$

(b) When bar is restrained is only in x direction

$\sigma_{xT_2} = -E\alpha\Delta T$

% change in axial thermal stress

$= \left(\dfrac{\sigma_{xT_2} - \sigma_{xT_1}}{\sigma_{xT_1}}\right)\times 100$ $\quad \sigma_{xT_1} = -2E\alpha\Delta T$

$= \left(\dfrac{-E\alpha\Delta T + 2E\alpha\Delta T}{2E\alpha\Delta T}\right)\times 100$

=50%

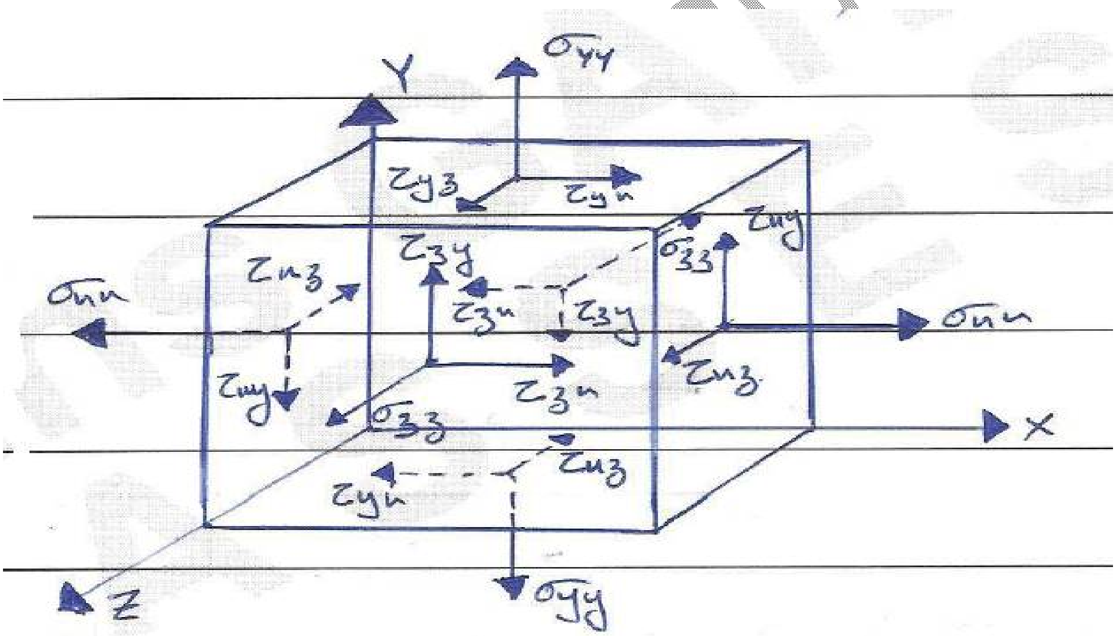
- **Strain energy**

When a body structure undergoes deformation, strain energy is stored in the structure. It is the summation of volumetric strain energy and shear strain energy.

$$U = U_v + U_s$$

<u>Strain energy density:</u>

Strain energy per unit volume is known as strain energy density.
Consider a 3D stress system as shown in the figure



Strain energy density due to normal stress

$\rightarrow u_v = \frac{1}{2}(\sigma_x \in_x + \sigma_y \in_y + \sigma_z \in_z)$

$\rightarrow u_v = \frac{1}{2E}[\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\vartheta(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)]$

$\rightarrow u_v = \frac{E}{2(1+\vartheta)(1-2\vartheta)}[(1-\vartheta).(\in_x^2 + \in_y^2 + \in_z^2) + 2\vartheta(\in_x\in_y + \in_y\in_z + \in_z\in_x)]$

Strain energy density due to shear stresses:

$\rightarrow u_s = \frac{1}{2}(\tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx})$

$\rightarrow u_s = \frac{G}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{zx}^2)$

⇛Strain energy = strain energy density×volume

Strain energy density due to hydrostatic loading

$u_v = \frac{3}{2E}[1-2\vartheta)\ p^2$

Here p is pressure loading

Problem

A solid steel sphere (E = 210 GPa, ν = 0.3) is subjected to hydrostatic pressure p such that its volume is reduced by 0.4%.

(a) Calculate the pressure p.

(b) Calculate the volume modulus of elasticity K for the steel.

(c) Calculate the strain energy U stored in the sphere if its diameter is d = 150 mm.

(a) p = 700 MPa; (b) K = 175 GPa;
(c) U = 2470 J

Sol

$E = 210$ GPa $\quad \nu = 0.3$

Hydrostatic Pressure. V_0 = Initial volume

$\Delta V = 0.004V_0$

Dilatation: $e = \dfrac{\Delta V}{V_0} = 0.004$

(a) Pressure

$$e = \frac{3\sigma_0(1-2\nu)}{E}$$

or $$\sigma_0 = \frac{Ee}{3(1-2\nu)} = 700 \text{ MPa}$$

Pressure $p = \sigma_0 = 700$ MPa ⟵

(b) Volume modulus of elasticity

$$K = \frac{\sigma_0}{e} = \frac{700 \text{ MPa}}{0.004} = 175 \text{ GPa} \quad \longleftarrow$$

(c) Strain energy (d = diameter)

$d = 150$ mm $\quad r = 75$ mm

From Eq. (7-57b) with $\sigma_x = \sigma_y = \sigma_z = \sigma_0$:

$u = \dfrac{3(1-2\nu)\sigma_0^2}{2E} = 1.40$ MPa

$V_0 = \dfrac{4\pi r^3}{3} = 1767 \times 10^{-6}$ m^3

$U = uV_0 = 2470$ N · m $= 2470$ J ⟵

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