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Strength of Materials
(GATE Aerospace Engineering and GATE Mechanical Engineering)

By

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Thermal Stresses and Strains

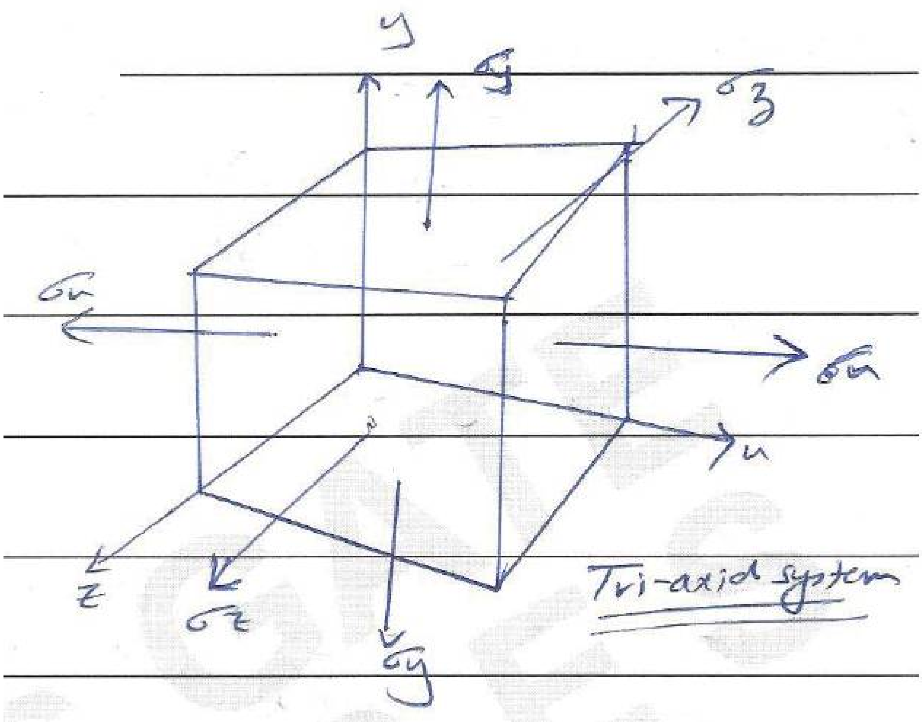
To understand thermal stresses and strains well consider a combined Mechanical + Thermal loading for a 3-D stress system.

Strain due to thermal loading is independent of c/s properties and will be same in all the directions.
It depends only on thermal coefficient of expansion and thermal loading.

Thermal strain:-

$$\epsilon_{xT} = \epsilon_{yT} = \epsilon_{zT} = \alpha \Delta T$$

Mechanical strain: - Mechanical strain for a triaxial stress system is given by



$$\epsilon_{xM} = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\epsilon_{yM} = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$\epsilon_{zM} = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

Total strain under combined loading will be summation of thermal strain and mechanical strain.
If component is subjected to both mechanical and thermal loading,

$$\in = \in_M + \in_T$$

So,

$$\in_x = \epsilon_{xM} + \epsilon_{xT}$$

$$\in_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) + \alpha\Delta T \qquad \text{............ (1)}$$

$$\in_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) + \alpha\Delta T \qquad \text{............ (2)}$$

$$\in_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) + \alpha\Delta T \qquad \text{............ (3)}$$

Case (1) only thermal loading and there is no constrain (free expansion in all directions)

$$\in_x = \in_y = \in_z = \epsilon_T = \alpha\Delta T$$

$$\sigma_x = \sigma_y = \sigma_z = 0$$

Case (2) only thermal loading and constraint in x direction (case of bar)

$$\sigma_x \neq 0, \ \sigma_y = \sigma_z = 0$$

$$\in_x = 0 \ \in_y = \in_z \neq 0$$

From equation (1), total strain is direction is zero

$$0 = \frac{\sigma_x}{E} - \frac{\nu}{E}(0 + 0) + \alpha\Delta T$$

$$\sigma_{xT} = -E\alpha\Delta T$$

From equation (2) and (3),

$$\in_y = \in_z = -\frac{\nu}{E}(\sigma_x) + \alpha\Delta T$$

$$= -\frac{\nu}{E}(-E\alpha\Delta T) + \alpha\Delta T$$

$$\in_y = \in_z = (1+\nu)\,\alpha\Delta T$$

Case (3) only thermal loading and constraint in x & y directions

$$\sigma_x = \sigma_y \neq 0, \ \sigma_z = 0$$

$$\epsilon_x = \epsilon_y = 0 \quad \epsilon_z \neq 0$$

From equations (1) & (2),

$$0 = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y) + \alpha\Delta T$$

&

$$0 = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x) + \alpha\Delta T$$

$$\sigma_{xT} = \sigma_{yT} = -\frac{E\alpha\Delta T}{1-\nu}$$

And from equation (3),

Strain in z direction

$$\epsilon_z = \left(\frac{1+\nu}{1-\nu}\right) \alpha\Delta T$$

Case (4) only thermal loading and constraint in x, y & z directions

$$\sigma_x = \sigma_y = \sigma_z \neq 0$$
$$\epsilon_x = \epsilon_y = \epsilon_z = 0$$

So from equations (1), (2) and (3),

$$\rightarrow \sigma_x = \sigma_y = \sigma_z = -\frac{E\alpha\Delta T}{1-2\nu}$$

Problems

1.

In a general case of a homogeneous material under thermo-mechanical loading the number of distinct components of the state of stress is

(A) 3 (B) 4 (C) 5 (D) 6

Sol. D

2.

A steel cube, with all faces free to deform, has Young's modulus, E, Poisson's ratio, ν, and coefficient of thermal expansion, α. The pressure (hydrostatic stress) developed within the cube, when it is subjected to a uniform increase in temperature, ΔT, is given by

(A) 0 (B) $\frac{\alpha(\Delta T)E}{1-2\nu}$ (C) $-\frac{\alpha(\Delta T)E}{1-2\nu}$ (D) $\frac{\alpha(\Delta T)E}{3(1-2\nu)}$

Sol. A

There is no constrain (free expansion in all directions)

$$\epsilon_x = \epsilon_y = \epsilon_z = \epsilon_T = \alpha\Delta T$$
$$\sigma_x = \sigma_y = \sigma_z = 0$$

3.

A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is υ, the thermal stress developed in the cube due to heating is

(A) $-\frac{\alpha(\Delta T)E}{(1-2\upsilon)}$ (B) $-\frac{2\alpha(\Delta T)E}{(1-2\upsilon)}$ (C) $-\frac{3\alpha(\Delta T)E}{(1-2\upsilon)}$ (D) $-\frac{\alpha(\Delta T)E}{3(1-2\upsilon)}$

Sol. A

Here is only thermal loading and constraint in x, y & z directions

$$\sigma_x = \sigma_y = \sigma_z \neq 0$$
$$\epsilon_x = \epsilon_y = \epsilon_z = 0$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) + \alpha\Delta T \qquad \text{............} \quad (1)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z) + \alpha\Delta T \qquad \text{............} \quad (2)$$

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) + \alpha\Delta T \qquad \text{............} \quad (3)$$

So from equations (1), (2) and (3),

$$\rightarrow \sigma_x = \sigma_y = \sigma_z = -\frac{E\alpha\Delta T}{1-2\nu}$$

4.

A steel bar of rectangular cross-section is heated uniformly and the rise in the temperature is ΔT. The Young's modulus is E, the Poisson's ratio is ν and the coefficient of thermal expansion is α. The bar is completely restrained in the axial direction and lateral directions.

(a) The thermal stress developed in the bar along the axial direction is

(A) $-\frac{E\alpha\Delta T}{1+2\nu}$ (B) $-E\alpha\Delta T$ (C) $-\frac{E\alpha\Delta T}{1-2\nu}$ (D) $-\frac{E\alpha\Delta T\nu}{1-2\nu}$

(b) Assume that the bar is allowed to deform freely in the lateral directions, while keeping the axial direction restrained. The percentage change in the magnitude of axial thermal stress for $\nu = 0.25$ is

(A) 0 (B) 25 (C) 50 (D) 100

[GATE XE 2010]

Sol.

(a)

When bar is restrained in the axial and lateral directions then thermal stress developed in the bar along the axial direction is

$$\sigma_{xT1} = \frac{-E\alpha\Delta T}{1-2\nu} =$$

$$= \frac{-E\alpha\Delta T}{1-2*0.25}$$

$$-2\ E\alpha\Delta T$$

(b)

When bar is restrains is only in x-direction then stress is

$$\sigma_{xT2} = -E\alpha\Delta T$$

So % change in axial thermal stress = $\left(\frac{\sigma_{xT2}-\sigma_{xT1}}{\sigma_{xT1}}\right)\times 100\%$

$= \left(\frac{-E\alpha\Delta T - (-2E\alpha\Delta T)}{(-2E\alpha\Delta T)}\right)\times 100\%$

$= 50\%$