AIRCRAFT SRUCRURE

TOPICS:-

- **Unsymmetrical Bending**
- **Torsion of thin walled closed and open section**
- **Shear flow due to non-uniform bending of thin walled structures**
- **Shear Centre**

1. Unsymmetrical Bending

As we know simple bending equation,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

If the cross section of the beam is not symmetrical about any axis or applied load is not acting through plane of symmetry than bending will be unsymmetrical

Consider a beam cross section as shown in fig,

1.1 SIGN CONVENTION

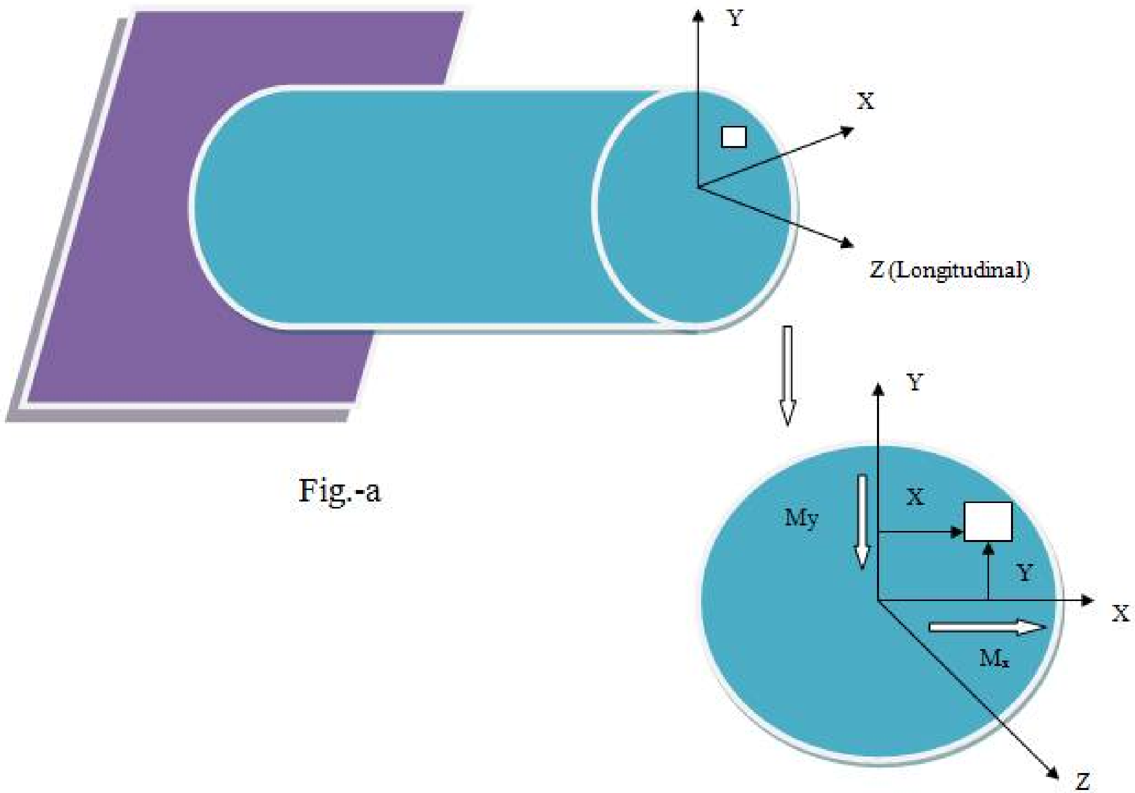
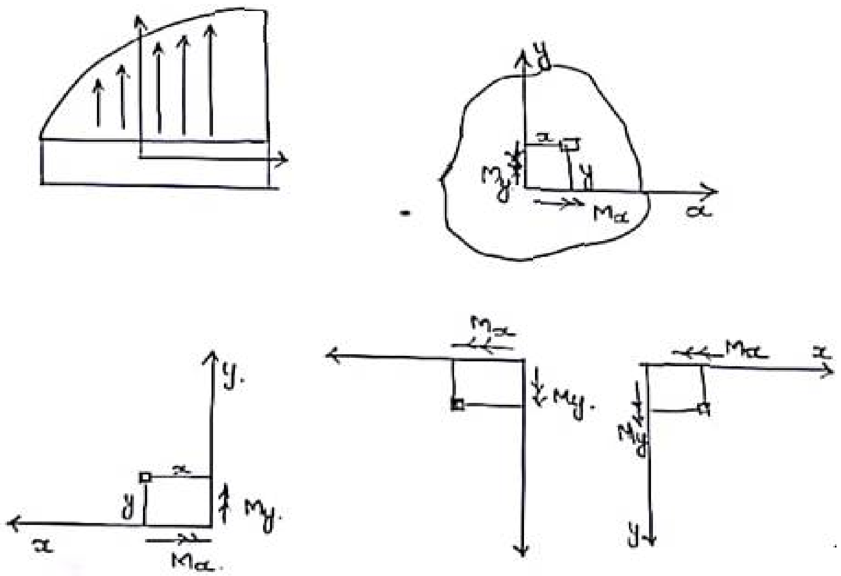


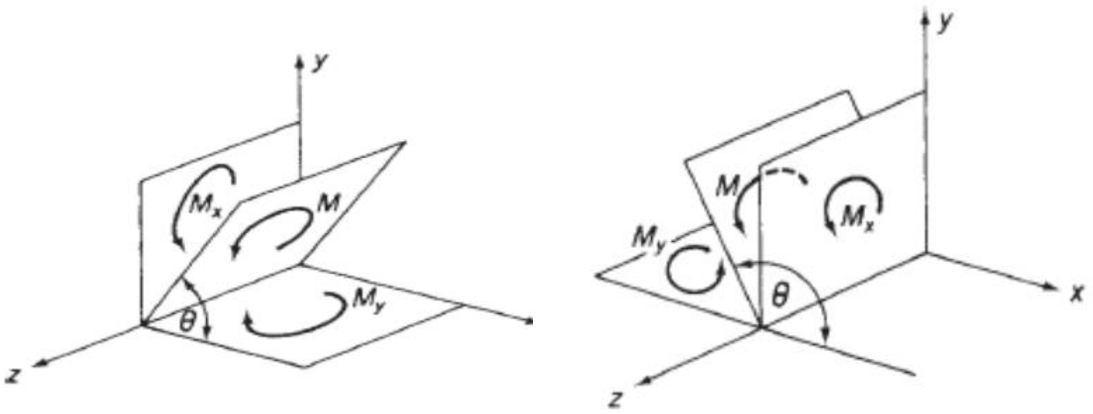
Fig.-a

- To produce to same effect or same kind of stress (compressive or tension), moment need to follow each other.



1.2 Moments in inclined plane

- The moment in YZ plane is always about X- axis.
- The moment in XZ plane is always about Y- axis.



(a) $\theta < 90°$

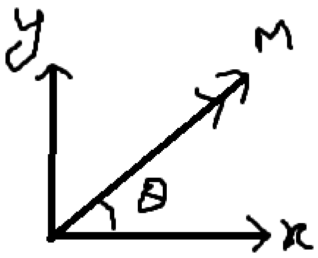
(b) $\theta > 90°$

- Resolution of bending moments sign depending on the size of θ. In both cases, for the sense of M shown
- $M_x = M sin\theta$

- $M_y = Mcos\theta$

  This gives,
- For $\theta < \frac{\pi}{2}, M_x$ and M_y positive (fig (a)) and for $\theta > \frac{\pi}{2}, M_x$ positive and M_y negative (fig (b)).

1.3 Moments about inclined axis

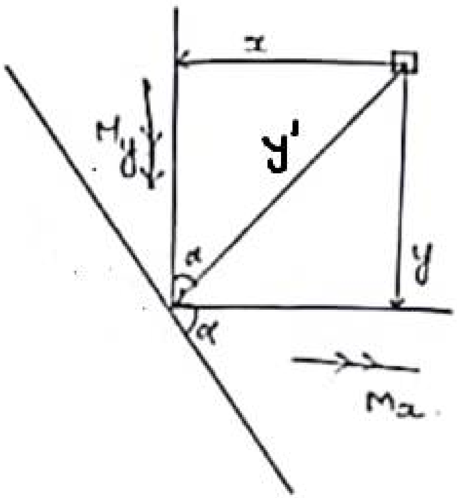


Resolving Bending Moment along x and y axis

- $M_x = Mcos\theta$
- $M_y = -Msin\theta$

  For all values of θ

1.4 Direct stress due to unsymmetrical bending



Here

$y' = x sin\alpha + y cos\alpha$ (From transformation)

Strain on a small element at a distance y' from Centroid

$$\varepsilon = \frac{y'}{R}$$

$$\varepsilon = \frac{x sin\alpha + y cos\alpha}{R}$$

$$\sigma_z = E\varepsilon$$

$$\sigma_z = \frac{E}{R}(x sin\alpha + y cos\alpha)$$

Stress resultant

Force resultant

$$\int \boldsymbol{\sigma_z dA = 0}$$ (No axial load on cross section of beam under bending)

$$\int y'dA = 0$$ (So, N.A. passes through Centroid of c/s)

Moment resultant

$$M_x = \int \sigma_z \, ydA$$

$$M_y = \int \sigma_z xdA$$

$$M_x = \frac{E}{R}[sin\alpha \int xydA + cos\alpha \int y^2 dA]$$

$$= \frac{E}{R}[sin\alpha I_{xy} + cos\alpha I_{xx}]$$

Similarly

$$M_y = \frac{E}{R}[cos\alpha I_{xy} + sin\alpha I_{yy}]$$

This is matrix form

$$\begin{bmatrix} M_x \\ M_y \end{bmatrix} = \frac{E}{R}\begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}\begin{Bmatrix} cos\alpha \\ sin\alpha \end{Bmatrix}$$

$$\begin{Bmatrix} cos\alpha \\ sin\alpha \end{Bmatrix} = \frac{E}{R}\begin{bmatrix} M_x \\ M_y \end{bmatrix}\begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}^{-1}$$

$$\begin{Bmatrix} cos\alpha \\ sin\alpha \end{Bmatrix} = \frac{R}{E(I_{xx}I_{yy} - I_{xy}{}^2)}\begin{bmatrix} I_{yy} & -I_{xy} \\ -I_{xy} & I_{xx} \end{bmatrix}\begin{Bmatrix} M_x \\ M_y \end{Bmatrix}$$

$$cos\alpha = \frac{R}{E(I_{xx}I_{yy} - I_{xy}{}^2)}\left(I_{yy}M_x - I_{xy}M_y\right)$$

$$sin\alpha = \frac{R}{E(I_{xx}I_{yy} - I_{xy}{}^2)}\left(-I_{xy}M_x + I_{xx}M_y\right)$$

$$\sigma_z = \frac{E}{R}(x sin\alpha + y cos\alpha)$$

$$\boldsymbol{\sigma_z} = \frac{(I_{xx}M_y - I_{xy}M_x)}{(I_{xx}I_{yy} - {I_{xy}}^2)}\boldsymbol{x} + \frac{(I_{yy}M_x - I_{xy}M_y)}{(I_{xx}I_{yy} - {I_{xy}}^2)}\boldsymbol{y}$$

$$\sigma_z = \frac{M_y}{I_{yy}}x + \frac{M_x}{I_{xx}}y$$

Above equation is for complex bending when cross section is symmetric but applied bending moment is about an inclined axis.

If about y axis, $M_y = 0$ $\qquad \sigma_z = \frac{M_x}{I_{xx}}y$

If about x axis, $M_x = 0$ $\qquad \sigma_z = \frac{M_y}{I_{yy}}y$

Above equation is for simple bending when cross section is symmetric but applied bending moment is about either x- axis or y- axis.

1.4 Position of the neutral axis

About neutral axis bending stress is zero

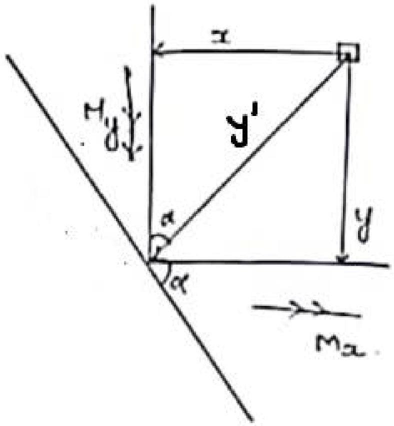
$$\sigma_z = k_1 x + k_2 y$$

$$0 = k_1 x + k_2 y$$

$$k_1 = \frac{(I_{xx}M_y - I_{xy}M_x)}{(I_{xx}I_{yy} - I_{xy}{}^2)}$$

$$k_2 = \frac{(I_{yy}M_x - I_{xy}M_y)}{(I_{xx}I_{yy} - I_{xy}{}^2)}$$

The angle mode by the Neutral axis



$$0 = k_1 x + k_2 y$$

$$k_1 x = -k_2 y$$

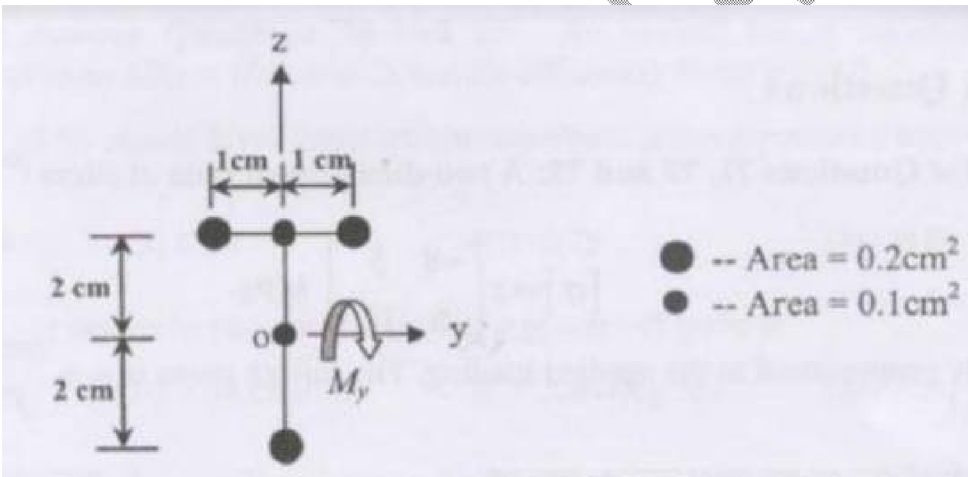
From above diagram

$$tan\alpha = \frac{x}{y} = -\frac{k_2}{k_1}$$

Here α is measure from y axis and is in clockwise direction.

Problem-1)

An idealized thin walled cross-section of the beam and perspective areas of boom are as shown in a bending moment M_y is acting on the cross-section the ratio of magnitude of normal stress in the top boom that of bottom boom. Here flange is 2cm and web is 4cm.



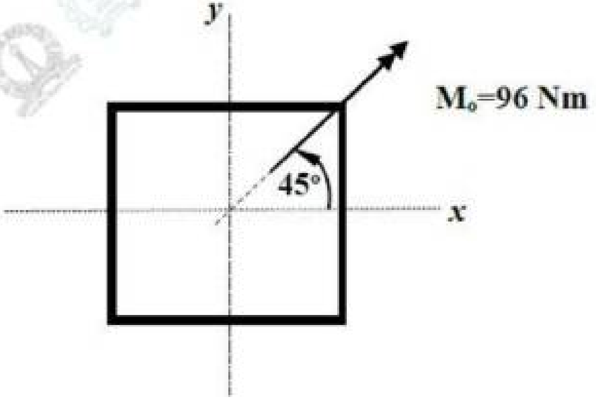
(a). $\frac{5}{11}$ (b). $\frac{2}{5}$

(c). 1 (d). $\frac{5}{2}$ **Gate: 2008, Q-67**

Sol:-

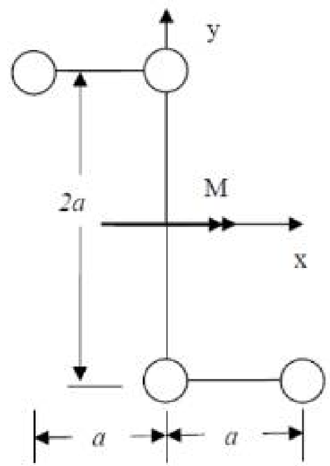
Problem-2)

The maximum normal stress in MN/m^2 for the thin walled beam of square cross section of outer dimension 120 mm x 120 mm and wall thickness 1 mm under the action of moment M_o=96 Nm as shown in the figure is _________ (in three decimal places).



Problem-3)

The idealized cross-section of a beam is comprised of four identical booms connected by shear webs. The beam is subjected to a bending moment *M* as shown in the figure. The inclination of the neutral axis to the x-axis in degrees is



(A) 45 CW (B) 45 CCW (C) 26.6 CW (D) 63.4 CCW

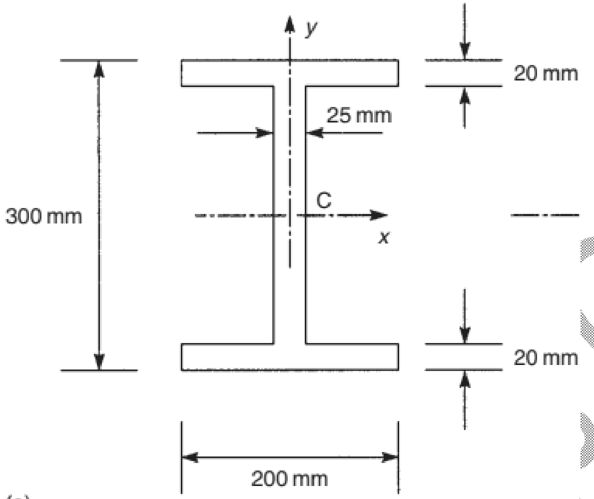
Sol:-

Problem-4)

A thin-walled tube with external radius of 100 mm and wall thickness of 2 mm, is fixed at one end. It is subjected to a compressive force of 1 N acting at a point on the circumference parallel to its length. The maximum normal stress (in kPa) experienced by the structure is ____________ (accurate to two decimal places).

Example 16.1

The cross-section of a beam has the dimensions shown in Fig. 16.6(a). If the beam is subjected to a negative bending moment of 100 kN m applied in a vertical plane, determine the distribution of direct stress through the depth of the section.



(a)

Example 16.2

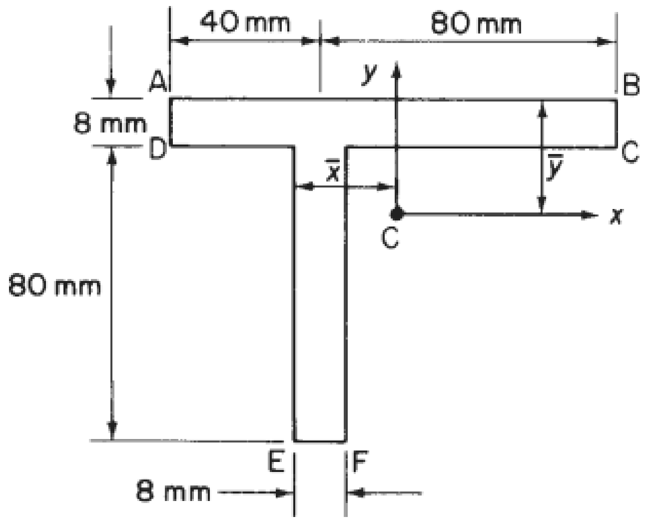
Now determine the distribution of direct stress in the beam of Example 16.1 if the bending moment is applied in a horizontal plane and in a clockwise sense about Cy when viewed in the direction yC.

Example 16.3

The beam section of Example 16.1 is subjected to a bending moment of 100 kN m applied in a plane parallel to the longitudinal axis of the beam but inclined at 30° to the left of vertical. The sense of the bending moment is clockwise when viewed from the left-hand edge of the beam section. Determine the distribution of direct stress.

Example 16.4

A beam having the cross-section shown in Fig. 16.13 is subjected to a bending moment of 1500 N m in a vertical plane. Calculate the maximum direct stress due to bending stating the point at which it acts.

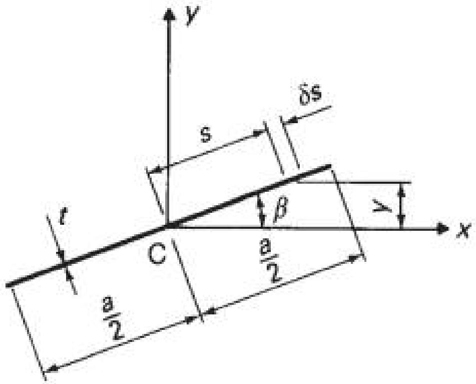


$$I_{xx} = 2\int_0^{a/2} ty^2\,\mathrm{d}s = 2\int_0^{a/2} t(s\sin\beta)^2\,\mathrm{d}s$$

from which

$$I_{xx} = \frac{a^3 t \sin^2\beta}{12}$$

Bending of open and closed, thin-walled beams



Similarly

$$I_{yy} = \frac{a^3 t \cos^2 \beta}{12}$$

The product second moment of area is

$$I_{xy} = 2 \int_0^{a/2} txy \, \mathrm{d}s$$

$$= 2 \int_0^{a/2} t(s \cos \beta)(s \sin \beta) \, \mathrm{d}s$$

which gives

$$I_{xy} = \frac{a^3 t \sin 2\beta}{24}$$

$$I_{xx} = \int_0^{\pi r} t y^2 \, \mathrm{d}s$$

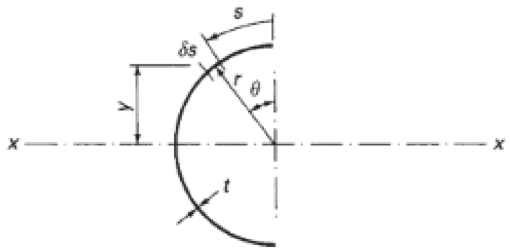


Fig. 16.33 Second moment of area of a semicircular section.

Expressing y and s in terms of a single variable θ simplifies the integration, hence

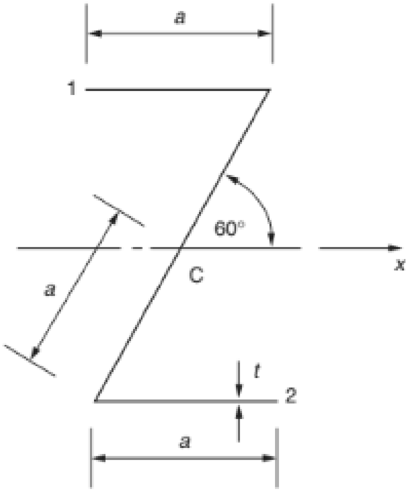
$$I_{xx} = \int_0^{\pi} t (r \cos\theta)^2 r \, \mathrm{d}\theta$$

from which

$$I_{xx} = \frac{\pi r^3 t}{2}$$

P.16.6 The thin-walled beam section shown in Fig. P.16.6 is subjected to a bending moment M_x applied in a negative sense. Find the position of the neutral axis and the maximum direct stress in the section.

Ans. NA inclined at 40.9° to Cx. $\pm 0.74\ M_x/ta^2$ at 1 and 2, respectively.



P.16.8 A uniform thin-walled beam has the open cross-section shown in Fig. P.16.8. The wall thickness t is constant. Calculate the position of the neutral axis and the maximum direct stress for a bending moment $M_x = 3.5$ N m applied about the horizontal axis Cx. Take $r = 5$ mm, $t = 0.64$ mm.

Ans. $\alpha = 51.9°$, $\sigma_{z,\max} = 101$ N/mm^2.