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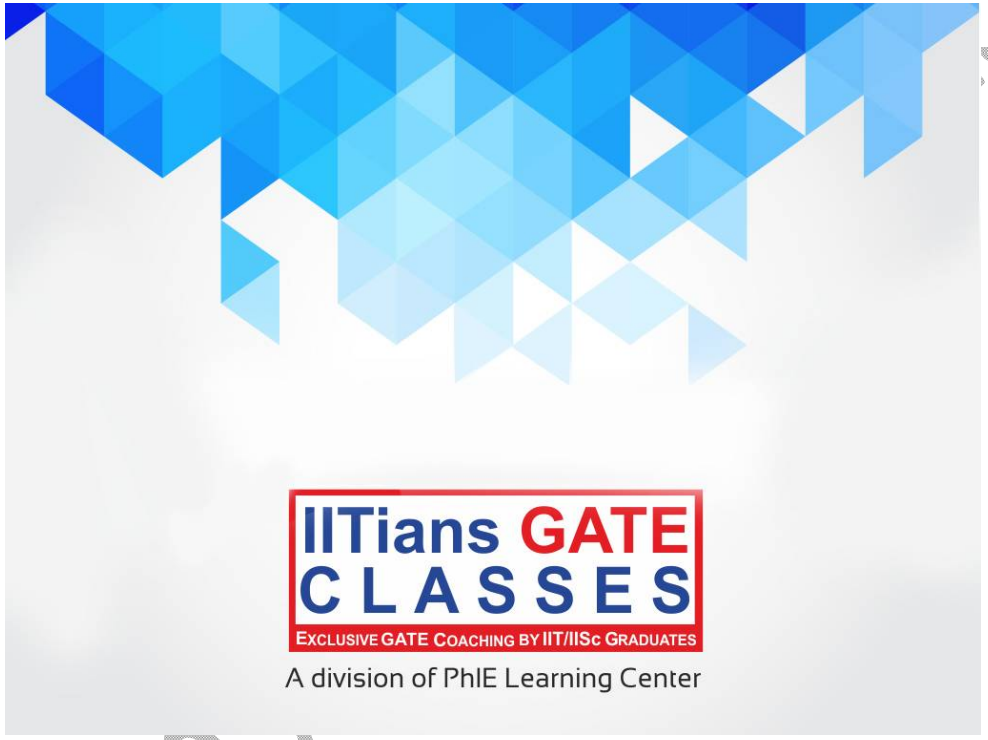

Aircraft Structures
(GATE Aerospace) by
Mr Dinesh Kumar (IIT Madras Fellow)

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Torsional Shear Flow

Torsion of thin walled closed section
Bredth Betho Theory

Assumption:-

- Warping displacement are freely permitted
- $\tau_{zs}$is presents and every other stress component is zero
- τ_{zs} doesn't vary along thickness direction

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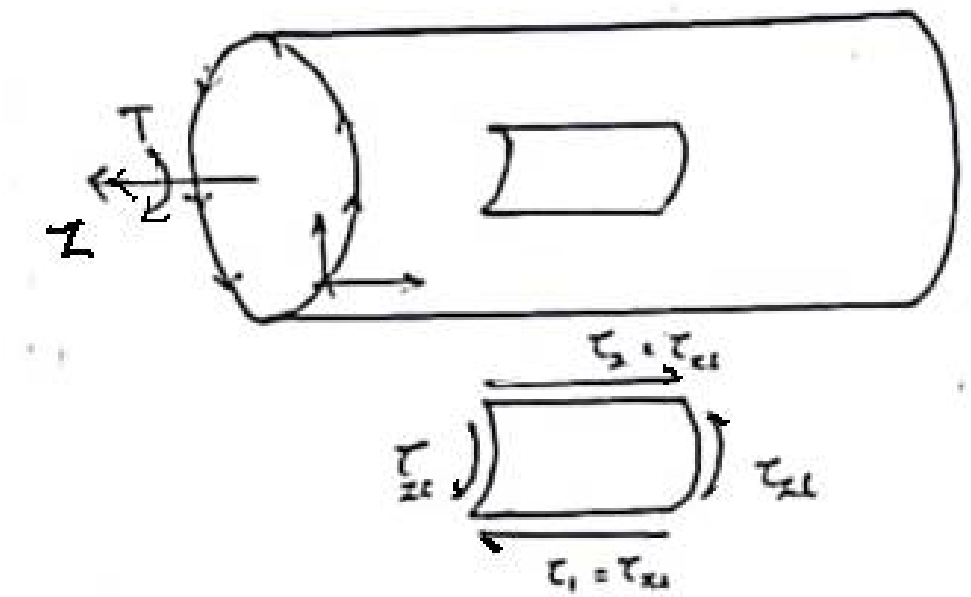

Force equilibrium along Z-axis,

$$-\tau_1 t_1 dz + \tau_2 t_2 dz = 0$$

Here, $q = \tau * t$ is a shear flow due to torsion and it is remains constant.

$$dT = qds.\rho$$

$$T = \int dT = \int q\rho ds = 2\int q\frac{1}{2}\rho ds$$

$$T = 2\int q\, dA$$

$$T = 2q\int dA$$

$$T = 2Aq$$

$$q = \frac{T}{2A}$$

Angle of Twist

Shear strain energy stored in the structure

$$= \frac{1}{2}T\theta$$

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$$= \int \frac{1}{2} \tau \gamma \, volume$$

$$\frac{1}{2} T\theta' = \int \frac{1}{2} \frac{\tau^2}{G} (tds * 1)$$

$$\frac{1}{2} T\theta' = \int \frac{\tau^2}{2G} tds$$

$$\theta' = \int \frac{\tau^2}{TG} tds$$

$$\theta' = \int \frac{q^2}{tTG} ds$$

$$\boldsymbol{\theta' = \int \frac{q}{2AGt} ds = \frac{1}{2AG} \int \frac{q}{t} ds = \frac{T}{4A^2G} \int \frac{1}{t} ds}$$

θ' = Angle of twist per unit length

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Torsional Constant (J)

$$\theta' = \frac{T}{4A^2G}\int \frac{1}{t}ds$$

$$J = \frac{T}{G\theta'}$$

$$J = \frac{4A^2}{\int \frac{ds}{t}}$$

Here Area A is area of c/s not the material area

Torsional Rigidity

$$GJ = \frac{T}{\theta'} \quad ; \quad J = \frac{4A^2}{\int \frac{ds}{t}}$$

$GJ = \frac{4A^2}{\int \frac{ds}{Gt}} \rightarrow torstonal\ Rigidity$ $J = I_P \rightarrow$ **for circular crossection= $2\pi r^3 t$**

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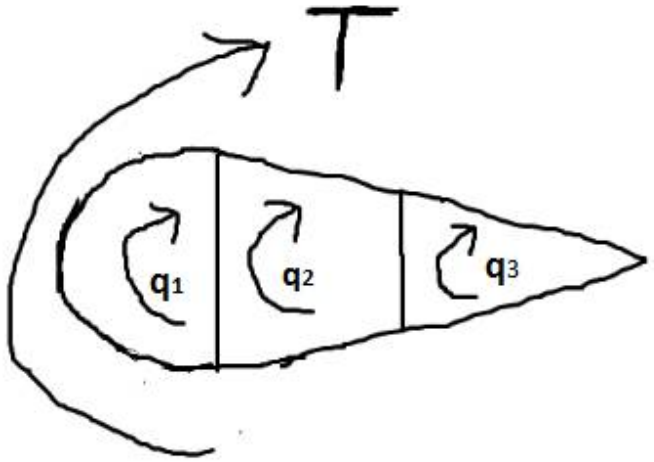
Torsion for multi cell



Bredth Betho Equation

T = $2A_1q_1$+$2A_2q_2$+$2A_3q_3$ ..... (1)

Compatibility equation

$\theta'_1 = \theta'_2 = \theta'_3$ ..... (2)

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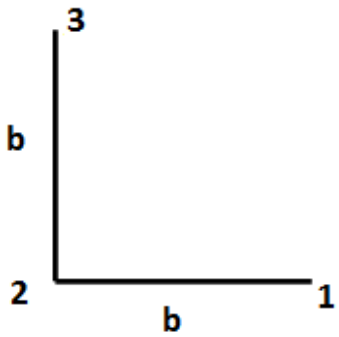
Torsion of thin walled open section

As we know,

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$



- **Torsion constant J** $= \sum \frac{bt^3}{3}$ **/** $\int \frac{t^3 ds}{3}$
- $\tau_{max} = \frac{T}{J} t$

  Here thickness is t

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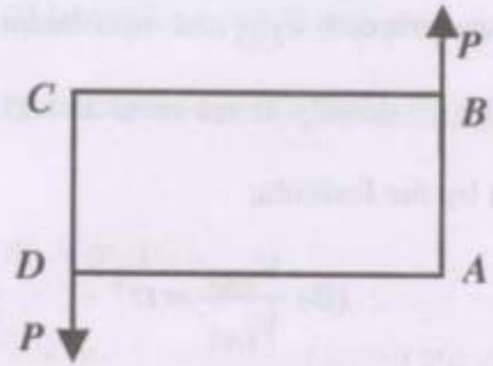
Problem:-

(1).

In a thin walled rectangular tube subjected to equal and opposite forces P as shown in the figure, the shear stress along leg AB is



(A) zero
(B) constant non-zero
(C) varies linearly
(D) varies parabolically

Sol:-

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(2).

A thin walled tube of circular cross-section with mean radius r has a central web which divides it into two symmetric cells as shown. A torque M is acting on the section. The shear flow q in the central web is

(A) $q = \frac{M}{2\pi r^2}$ (B) $q = 0$ (C) $q = \frac{M}{4\pi r^2}$ (D) $q = \frac{M}{\pi r^2}$

Sol:-

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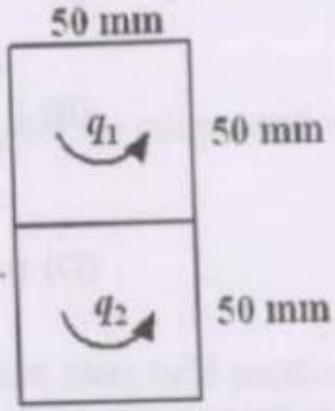
(3).

A 2-celled tube with wall thickness 0.5 mm is subjected to a torque of 10 N-m. The resulting shear flows in the two cells are q_1 and q_2 as shown below.



The torque balance equation (Bredt-Batho formula) for this section leads to

(A) $q_1 - q_2 = 2000$ N/m

(B) $q_1 + 2q_2 = 2000$ N/m

(C) $q_1 + q_2 = 2000$ N/m

(D) $2q_1 + q_2 = 2000$ N/m

Sol.

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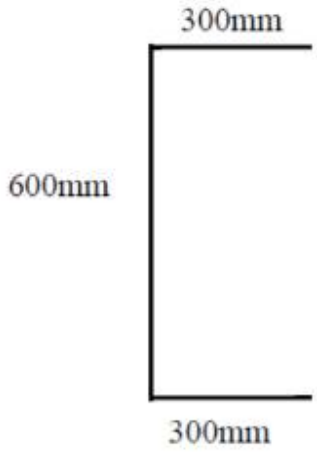
(4).

The torsion constant J of a thin-walled closed tube of thickness t and mean radius r is given by

(A) $J = 2\pi rt^3$ (B) $J = 2\pi r^3 t$ (C) $J = 2\pi r^2 t^2$ (D) $J = 2\pi r^4$

(5).

The channel section of uniform thickness 2mm shown in the figure is subjected to a torque of 10 Nm. If it is made of a material with shear modulus of 25 GPa, the twist per unit length in radians/m is _____



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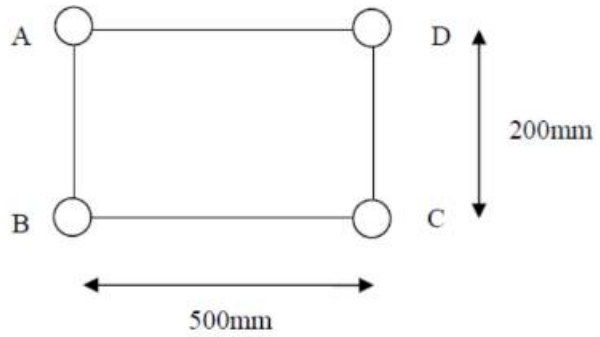
(6).

The stiffened cross-section of a long slender uniform structural member is idealized as shown in the figure below. The lumped areas at A, B, C and D have equal cross-sectional area of 3 cm^2. The webs AB, BC, CD and DA are each 5 mm thick. The structural member is subjected to a twisting moment of 10 kNm. The magnitudes of the shear flow in the webs, q_{AB}, q_{BC}, q_{CD}, and q_{DA} in kN/m are, respectively



(A) 20, 20, 20, 20

(B) 0, 0, 50, 50

(C) 40, 40, 0, 0

(D) 50, 50, 50, 50

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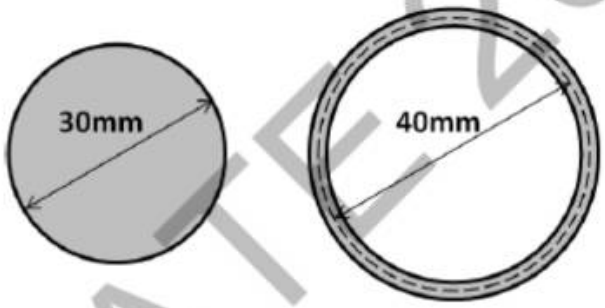
(7).

There are 2 designs proposed for a shaft of length *l*, with a torque carrying capacity of *T*.*Design I* is a solid circular cross-section shaft of diameter30*mm*. *Design II* is a thin-walled circular shaft of average diameter 40*mm*. Thickness of the wall in *Design II* has to be determined such that maximum shear stress is the same in both the designs for the given torque *T*(so that same material can be used for manufacturing both the shafts). Ratio of mass of shaft using *Design I*to the mass of shaft using *Design II* is



Design I **Design II**

(A) 2.68 (B) 5.36 (C) 1.79 (D) 3.58

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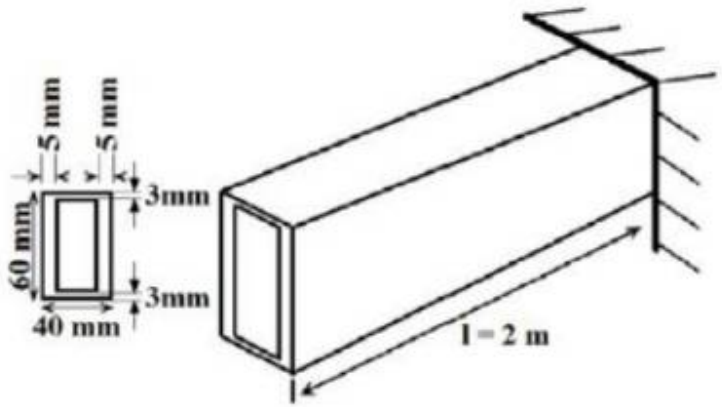
(8).

The thin rectangular tube shown below is made of a material with shear modulus, $G = 80\ GPa$. The shear flow is calculated based on the mid-thickness dimensions. If the free end is allowed to twist no more than $0.0727\ rad$, then the maximum torque (in Nm) which the tube can be subjected to at its free end is _____.



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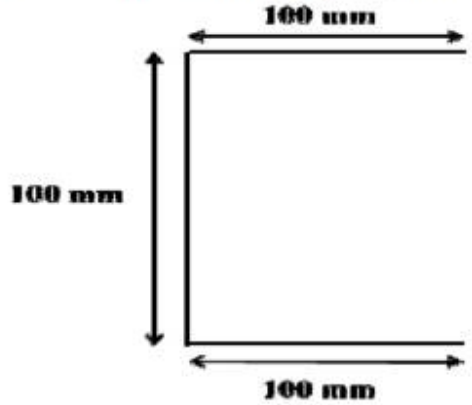
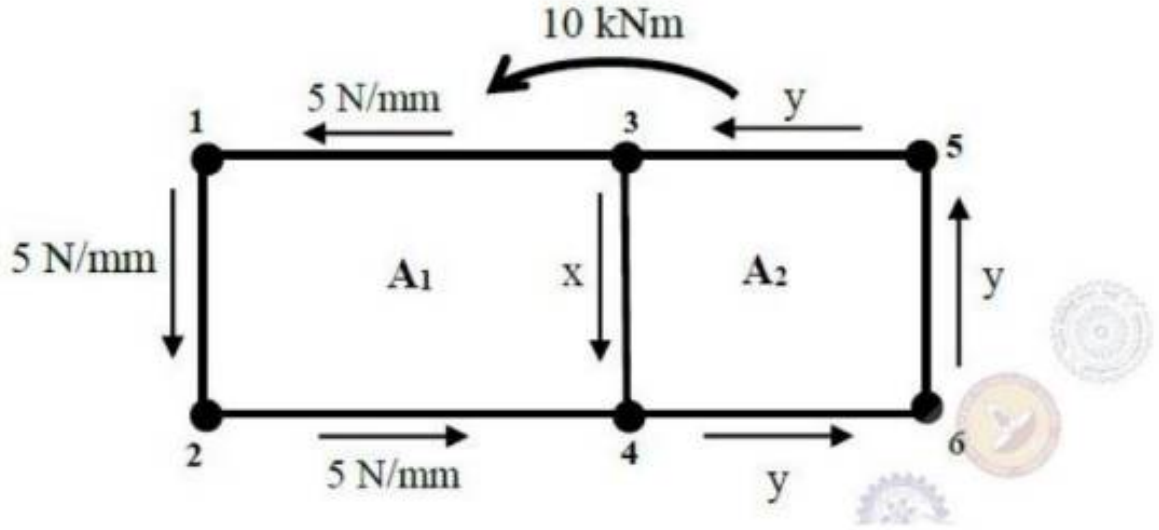
(9).

A channel section shown in the figure has uniform thickness. It is subjected to an anticlockwise torque of 62.5×10^3 Nmm. The maximum possible thickness of the channel section, such that the shear stress induced in it does not exceed 100 N/mm^2, is ____ mm.



(10).

The idealized cross section of a thin-walled wing box structure shown in the figure is subjected to an anticlockwise torque of 10 kNm. The corresponding shear-flow distribution under this loading condition is shown in the figure. The area of each cell is $A_1 = 300x10^3 mm^2$ and $A_2 = 250x10^3 mm^2$. The ratio of the unknowns $\frac{x}{y}$ is given by _____ (in three decimal places).



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(11).

Q.No. 24 A thin walled beam of constant thickness shown in the figure is subjected to a torque of 3.2 kNm. If the shear modulus is 25 GPa, the angle of twist per unit length is _____ rad/m (*round off to three decimal places*).