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Mechanical Vibration
(GATE Aerospace and GATE Mechanical) by
Mr Dinesh Kumar (IIT Madras Fellow)

Mechanical Vibration

Topic to study

- **Undamped free vibration of single degree of freedom**
- **Damped free vibration of single degree of freedom**
- **Forced vibration of single degree of freedom**
- **Free vibration of second degree of freedom**
- **Free vibration for continuous system**

Ref: -

- **Mechanical vibration by S S Rao**
- **Mechanical vibration by V P Singh**
- **Mechanical vibration by Grover**

Fundamental of vibration

A body is said to vibrate if it has to – and – fro motion.

Swinging of pendulum on either side of a mean position takes place under the action of gravity. When the pendulum swings through the mid position, its centre of mass is at the lowest point and it possesses only kinetic energy. At each extremely of its swing, it has only potential energy. In the absence of any friction, the motion continues infinitely.

Usually vibration happens due to elastic forces. Whenever a body is displayed from its equilibrium position, work is done on the elastic constraints of the forces on the body and is stored as strain energy. Now if the body is released, the internal force causes the body to move towards its equilibrium position. Due to internal molecular friction vibration dies after some time.

Free (Natural) vibration – Elastic vibrations in which there is no friction and external forces after the initial releases of the body are known as a free or natural vibration.

Damped vibrations – When the energy of a vibrating system is gradually dissipated by friction and other resistances. The vibrations are said to be damped. The vibrations gradually cause and the system rests in its equilibrium position.

Forced vibrations – When a repeated force continuously acts on a system, the vibrations are said to be forced. The frequency of the vibrations is that of the applied force and is independent of their own natural frequency of vibrations.

Periodic motion – A motion which repeat after equal interval time.

Ex - Simple harmonic motion (SHM)

Time period - Time to taken for complete one cycle.

Frequency (f) – No. of cycle per unit time

$$\omega = 2\pi f \quad \left(\frac{rad}{sec}\right)$$

Amplitude – The maximum displacement of a vibrating body from the mean position.

Natural frequency (ω_n) –

- It is a dynamic characteristic of system.

- It is a load independent.
- It is the frequency of free vibration system.

Resonance –

- When frequency of system becomes equal to natural frequency of system.
- Amplitude of vibration becomes excessive.

Damping - The resistance to the motion of vibrating system.

Degree of freedom – The number of independent co-ordinates required to specify the complete motion of system known as a degree of freedom.

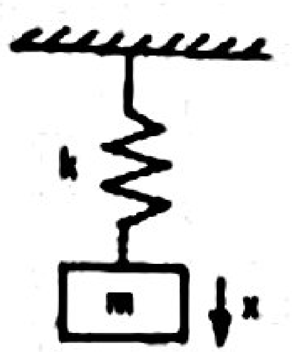


Fig (a) Single degree of freedom

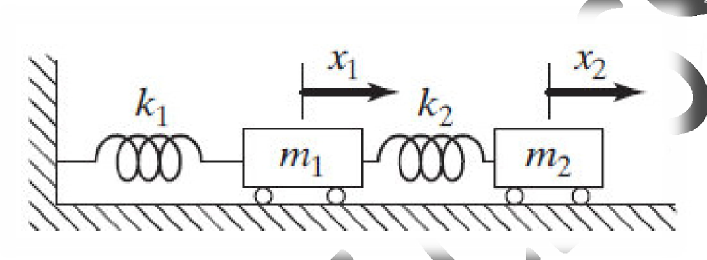


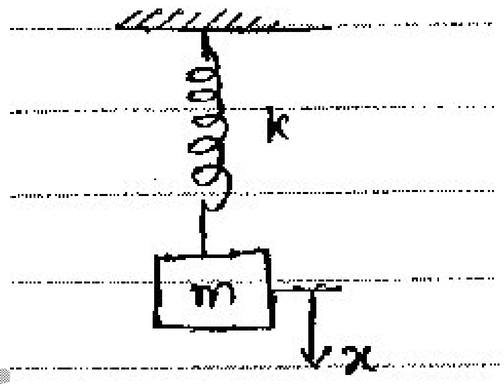
Fig (b) Second degree of freedom

Continuous System- Degree of freedom for continuous system is infinite. Example cantilever beam

Undamped free vibration of single degree of freedom

- **Newton's Method :-**

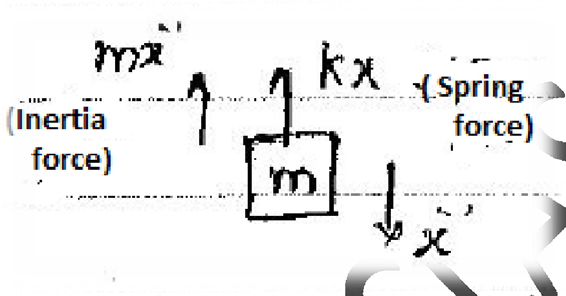
Considering spring mass system in vertical position, the body is displayed from its equilibrium position downward.



For initial state equilibrium,

mg = kΔ

Where Δ = state deflection due to mass



When deflected by x then equation can be written as,

$m\ddot{x}$ + kx = 0 .... (1)

This can be written as,

$\ddot{x} + \frac{k}{m}x = 0$ .... (1a)

This is the equation of simple harmonic and is analogous to,

$\ddot{x} + \omega_n^2 x = 0$

By comparing above equation with equation (1a), we can find

$$\omega_n = \sqrt{\frac{k}{m}}$$

By putting $\frac{k}{m}$ from initial static equilibrium,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\Delta}}$$

Equation of motion for

$m\ddot{x}$ + kx = 0

Can be found by putting, x = Acos $\omega_n t$ + Bsin $\omega_n t$

Which can be written as a

x = Xsin($\omega_n t$ + ∅) .... (2)

Putting this in equation of motion also can be found,

$\omega_n = \sqrt{\frac{k}{m}}$ .... (3)

Frequency of vibration,

$f_n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad H_z$

Unknown value of X and Ø can be found from initial condition.

Initial conditions

If the motion is started by displacing the mass through a distance x_0 and giving a velocity V_0, then the solution of equation (2) can be find as,

t = 0, x = x_0 ; $\dot{x} = V_0$

x = Xsin($\omega_n t$ + Ø)

x_0 = Xsin Ø

$\dot{x}$ = Xω_n cos($\omega_n t$ + Ø)

V_0 = Xω_n cos Ø

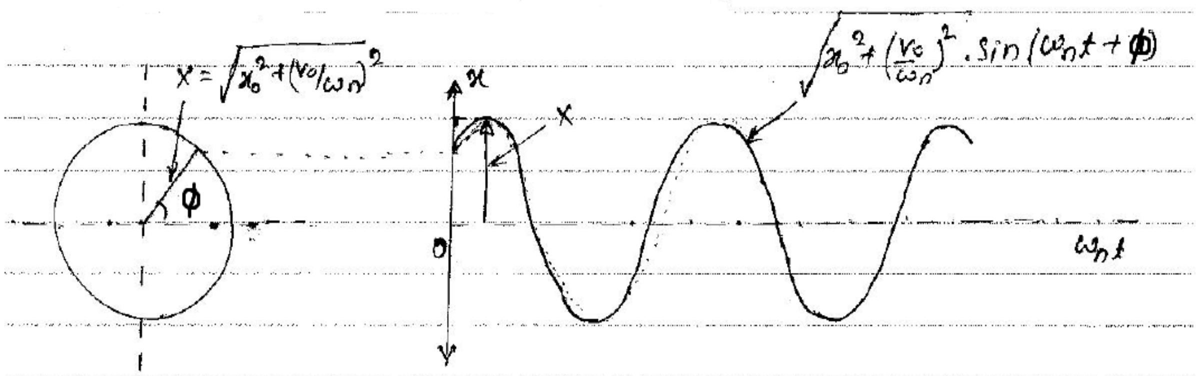
$\rightarrow x_0{}^2 + \left(\frac{V_0}{X\omega_n}\right)^2 = X^2$

$$X = \sqrt{\left[x_0^{\,2} + \left(\frac{V_0}{X\omega_n}\right)^2\right]}$$

$$\tan\emptyset = \frac{x_0}{V_0/\omega_n}$$

$$\emptyset = \tan^{-1}\left[\frac{x_0}{V_0/\omega_n}\right]$$

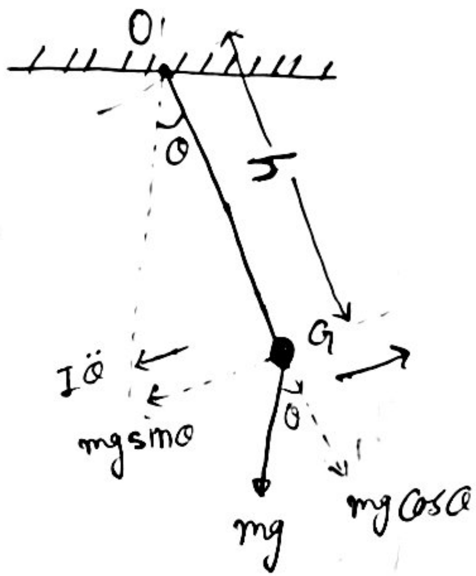
Solution can be plotted graphically



- **Vibration of different vibrating system**

- **The compound pendulum**

Let Pendulum is displayed by an angle θ of small value.



Here G is the centre of gravity.

Restoring torque = mgsin $\theta \times h$ = mgh. θ

Inertial torque $= I\ddot{\theta}$

$\therefore I\ddot{\theta}$ + mgh. θ = 0

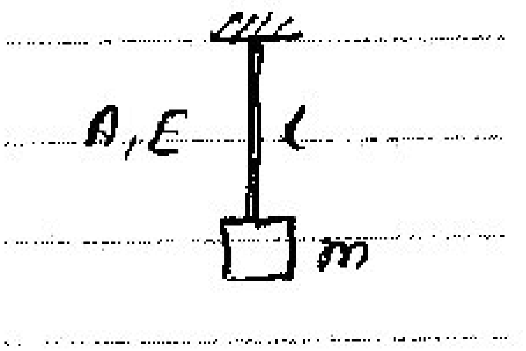
$$\omega_n = \sqrt{\frac{mgh}{I}}$$

- **Single concentrated load**

In case of bar, shaft and beams of negligible mass carrying a concentrated mass, the force is proportional to the deflection of the mass from the equilibrium position and the relation derived for natural frequency of vibration holds goods.

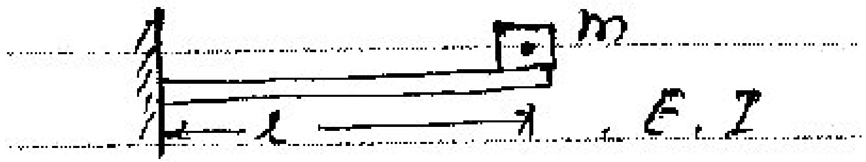
$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}}$$

(a) Rod of length l suspended vertically. A mass m is suspended at the free end.



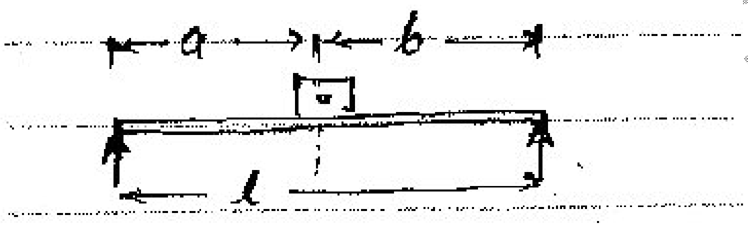
Static deflection, $\Delta = \frac{mgl}{AE}$

(b) Cantilever beam supporting a concentrated mass



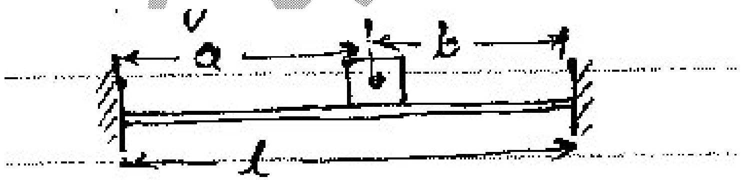
$$\Delta = \frac{mgl^3}{3EI}$$

(c) Simply supported beam carrying a concentrated mass



$$\Delta = \frac{mga^2b^2}{3EIl}$$

(d) For a beam fixed at both end, carrying a concentrated mass



$$\Delta = \frac{mga^3b^3}{3EIl^3}$$

Problem 1.)

The number of natural frequencies of an elastic beam with cantilever boundary conditions is

(A) 1 (B) 3 (C) 1000 (D) Infinite

Problem 2.)

A 1 kg mass attached to a spring elongates it by 16mm. The mass is then pulled from its equilibrium position by 10mm and released from rest. Assuming the acceleration due to gravity of 9.81 m/s^2, the response of the mass in mm is given by

(A) $x = 10 \sin 24.76t$
(B) $x = 10 \cos 24.76t$
(C) $x = \sin 16t$
(D) $x = 10 \cos 16t$

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Problem 3.)

Given that $\ddot{x}+3x=0$, and $x(0)=1$, $\dot{x}(0)=0$, what is $x(1)$?

(A) -0.99 (B) -0.16 (C) 0.16 (D) 0.99

Problem 4.)

One end of a linear spring is attached to a fixed support and a mass of 2 kg hangs from it at the other end. A force of 4 N causes a displacement of 0.02m. The mass is pulled down a distance of 0.04 m from its static equilibrium position and released with zero velocity

(a) The natural frequency of vibration is
(A) 1 rad/s (B) 1.59 rad/s (C) 5 rad/s (D) 10 rad/s

(b) The magnitude of velocity when the body has moved half way towards the static equilibrium position from its initial position is
(A) 0.212 m/s (B) 0.346 m/s (C) 0.4 m/s (D) 1.0 m/s

[XE GATE 2008]

Problem 5.)

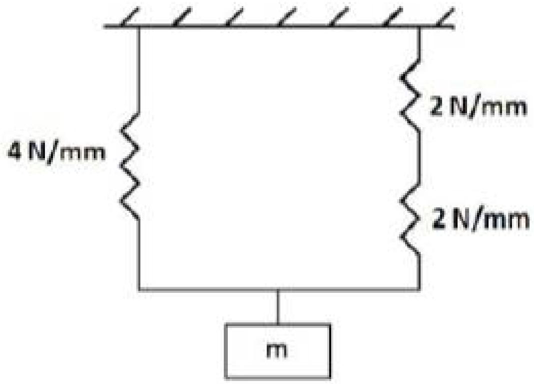
The natural frequency of a spring-mass system on earth is ω_n. The natural frequency of this system on the moon ($g_{moon} = g_{earth}/6$) is

(A) ω_n (B) $0.408\omega_n$ (C) $0.204\omega_n$ (D) $0.167\omega_n$

[ME GATE 2010]

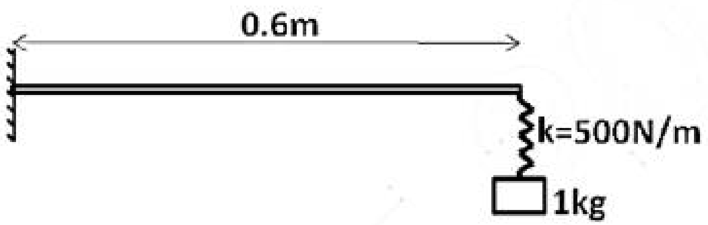
Problem 6.)

The effective stiffness of the spring-mass system as shown in the figure below is ____ N/mm.



Problem 7.)

1kg mass is hanging from a spring of stiffness $500 N/m$ attached to a massless, symmetric beam of length $0.6m$, moment of inertia about the bending axis $I = 8.33 \times 10^{-10} m^4$ and Young's modulus $E = 210 GPa$ as shown in the figure. The fundamental natural frequency (in *rad/s*) of the system is



(A) 3.24 (B) 20.36 (C) 22.36 (D) 3.56

[AE GATE 2014]

Problem 8.)

A cantilever beam of negligible mass is 0.6 m long. It has a rectangular cross-section of width 8 mm and thickness 6 mm and carries a tip mass of 1.4 kg. If the natural frequency of this system is 10 rad/s, Young's modulus of the material of the beam in GPa is ________

Problem 9.)

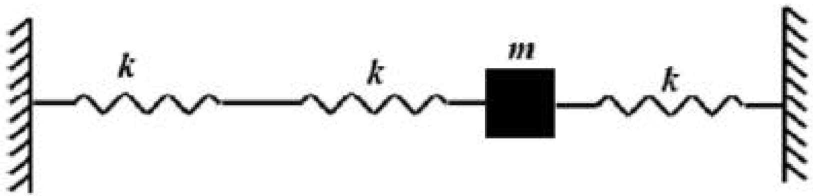
Consider a single degree of freedom spring-mass system of spring stiffness k_1 and mass m which has a natural frequency of 10 rad/s. Consider another single degree of freedom spring-mass system of spring stiffness k_2 and mass m which has a natural frequency of 20 rad/s. The spring stiffness k_2 is equal to

(A) k_1 (B) $2k_1$ (C) $\frac{k_1}{4}$ (D) $4k_1$

[AE GATE 2011]

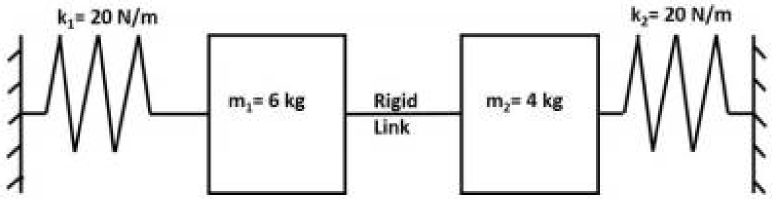
Problem 10.)

In the spring-mass system, shown in the figure, mass $m = 3$ kg and the spring stiffness $k = 20$ kN/m. The natural frequency of the system is _______ Hz (*round off to the nearest integer*).



Problem 11.)

The natural frequency (in rad/s) of the spring-mass system shown in the figure below is _____ (accurate to one decimal place).



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Problem 12.)

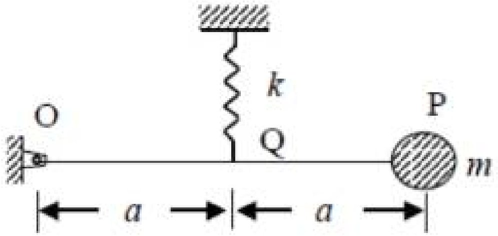
A 0.5 kg mass is suspended vertically from a point fixed on the Earth by a spring having a stiffness of 5 N/mm. The static displacement (in mm) of the mass is _____.

Problem 13.)

The static deflection of a spring under gravity, when a mass of 1 kg is suspended from it, is 1 mm. Assume the acceleration due to gravity g =10 m/s^2. The natural frequency of this spring-mass system (in rad/s) is______________

Problem 14.)

Figure shows a single degree of freedom system. The system consists of a massless rigid bar OP hinged at O and a mass m at end P. The natural frequency of vibration of the system is



(A) $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{4m}}$

(B) $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

(C) $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(D) $f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

[ME GATE 2015]

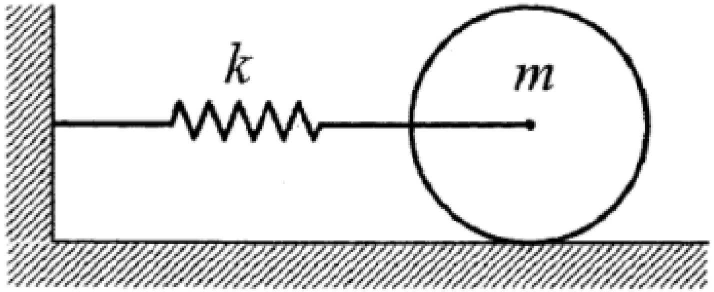
Problem 15.)

A uniform cross-section rigid rod of mass m and length l, is hinged at its upper end and suspended like a pendulum. Its natural frequency for small oscillations is

(A) $\sqrt{\frac{g}{2l}}$ (B) $\sqrt{\frac{g}{l}}$ (C) $\sqrt{\frac{2g}{l}}$ (D) $\sqrt{\frac{3g}{2l}}$

Problem 16.)

A disc of mass m is attached to a spring of stiffness k as shown in the figure. The disc rolls without slipping on a horizontal surface. The natural frequency of vibration of the system is

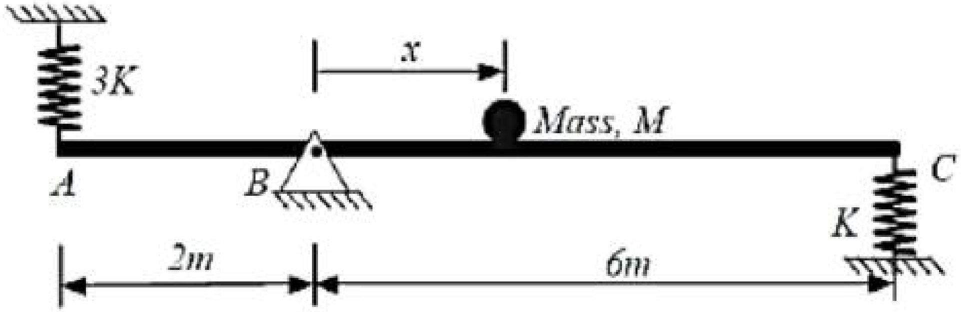


(A) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{3k}{2m}}$

[ME GATE 2011]

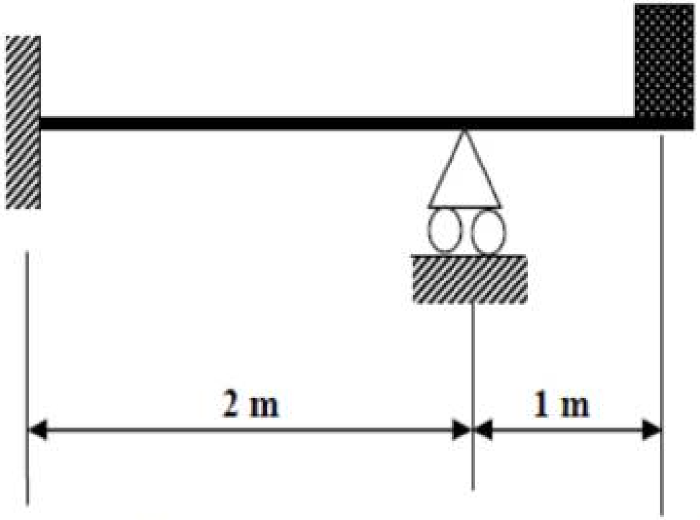
Problem 17.)

Consider the system shown below. Mass M is fixed to the rod AC at a distance x from the hinge point at B. Two springs of stiffness $3K$ and K are attached to the rod at points A and C, respectively. The natural frequency of angular oscillation of the system about B is 20 rad/s. Assume the rod to be rigid and massless. **Magnitude of x** (in metres) is ________.
($M = 30$kg, and $K = 1$kN/m).



Problem 18.)

The vibrating system shown in the figure carries a mass of 10 kg at the free end, where the static deflection is 1 mm. This system is to be replaced by an equivalent vibrating spring mass system having equivalent mass of 2 kg (assume g =10 m/s^2). The natural frequency (in rad/s) and the stiffness (in kN/m) of the equivalent system respectively are



(A) 10 and 20 (B) 20 and 100 (C) 100 and 20 (D) 1000 and 20

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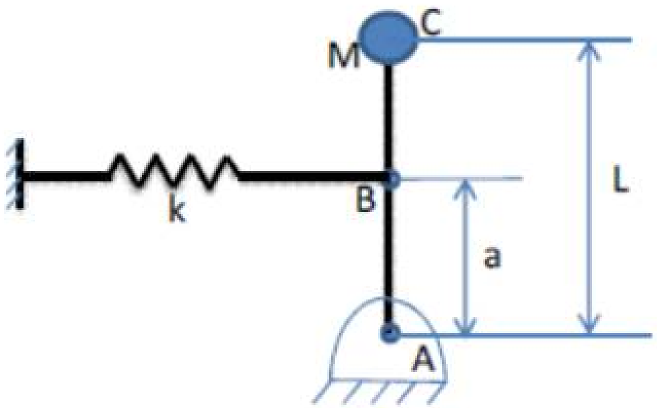
Problem 19.)

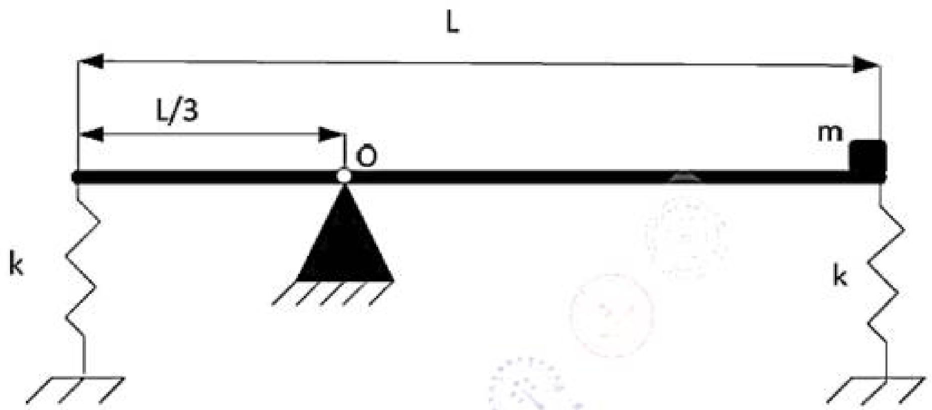
A rigid massless rod ABC is hinged at A and carries a point mass M (in kg) at C. Point B is connected to a linear spring with spring constant k (in N/m) as shown in the figure. The length AB and AC area and L, respectively. Neglecting the effect of gravity, the natural frequency of this spring-mass system in rad/s is



(A) $\sqrt{\frac{kL^2}{Ma^2}}$ (B) $\sqrt{\frac{ka^2}{ML^2}}$ (C) $\sqrt{\frac{kL}{Ma}}$ (D) $\sqrt{\frac{ka}{ML}}$

Problem 20.)

A thin uniform rigid bar of length L and mass M is hinged at point O, located at a distance of $\frac{L}{3}$ from one of its ends. The bar is further supported using springs, each of stiffness k, located at the two ends. A particle of mass $m = \frac{M}{4}$ is fixed at one end of the bar, as shown in the figure. For small rotations of the bar about O, the natural frequency of the system is

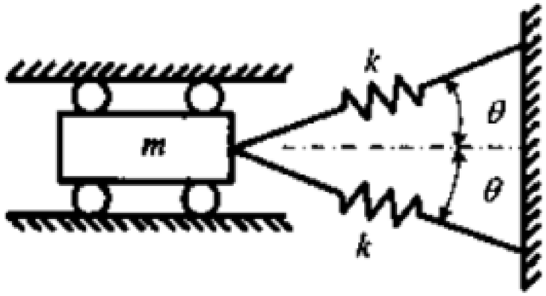


(A) $\sqrt{\frac{5k}{M}}$ (B) $\sqrt{\frac{5k}{2M}}$ (C) $\sqrt{\frac{3k}{2M}}$ (D) $\sqrt{\frac{3k}{M}}$

[ME GATE 2017]

Problem 21.)

A spring-mass system shown in the figure is vibrating with very small amplitude. The natural frequency of the system is



(A) $\sqrt{\frac{k}{m}}$ (B) $\sqrt{\frac{2k}{m}}$ (C) $\sqrt{\frac{2k\cos\theta}{m}}$ (D) $\sqrt{\frac{2k\cos^2\theta}{m}}$

[XE GATE 2010]

Problem 22.)

The natural frequency of the system suspended by two identical springs of stiffness k as shown in the figure is given by $\omega_n = a\sqrt{\frac{k}{m}}$ for small displacement. Both the springs make an angle of 45° with the horizontal. The value of a is________ (in two decimal places).