IITians GATE CLASSES
Exclusive GATE Coaching by IIT/IISc Graduates
A division of PhIE Learning Center

QUICK REVISION
FORMULA SHEET
for
GATE-AE VIBRATIONS

Table Of Content

OUR ACHIEVERS

GATE-2022 AE

SUBHROJYOTI BISWAS
IIEST, SHIBPUR
AIR - 4

SANJAY. S
AMRITA UNIV, COIMBATORE
AIR - 7

AKILESH . G
HITS, CHENNAI
AIR - 7

D. MANOJ KUMAR
AMRITA UNIV, COIMBATORE
AIR - 10

DIPAYAN PARBAT
IIEST, SHIBPUR
AIR - 14

And Many More ....

GATE-2021 AE

NILADRI PAHARI
IIEST, SHIBPUR
AIR - 1

VISHAL .M
MIT, CHENNAI
AIR - 2

SHREYAN .C
IIEST, SHIBPUR
AIR - 3

VEDANT GUPTA
RTU, KOTA
AIR - 5

SNEHASIS .C
IIEST, SHIBPUR
AIR - 8

And Many More ....

OUR PSU JOB ACHIEVERS

FATHIMA J
(MIT, CHENNAI)
HAL DT ENGINEER 2022

MOHAN KUMAR .H
(MVJCE, BANGALORE)
HAL DT ENGINEER 2022

ARATHY ANILKUMAR NAIR
(AMRITA UNIV, COIMBATORE)
HAL DT ENGINEER 2021

SADSIVUNI TARUN
(SASTRA TANJORE)
HAL DT ENGINEER 2021

VIGNESHA .M
(MIT, CHENNAI)
MRS E-II CRL BEL

RAM GOPAL SONI
(GVIET, PUNJAB)
CEMILAC LAB, DRDO

VIBRATIONS

MECHANICAL VIBRATIONS

Basic:

Free (Natural) Vibration:

Elastic vibrations in which there is no friction and external forces after the initial releases of the body are known as a free or natural vibration.

Damped Vibrations:

When the energy of a vibrating system is gradually dissipated by friction and other resistances. The vibrations are said to be damped. The vibrations gradually cause and the system rests in its equilibrium position.

Forced Vibrations:

When a repeated force continuously acts on a system, the vibrations are said to be forced. The frequency of the vibrations is that of the applied force and is independent of their own natural frequency of vibrations.

Periodic Motion:

A motion which repeats after equal interval time. **Example:** Simple harmonic motion (SHM)

Time Period:

Time to taken for complete one cycle.

Frequency (f) – No. of cycle per unit time

$\omega = 2\pi f (rad/sec)$

Amplitude:

The maximum displacement of a vibrating body from the mean position.

Natural Frequency (ω_n):

- It is a dynamic characteristic of system.
- It is a load independent.
- It is the frequency of free vibration system.

Resonance:

When frequency of system becomes equal to natural frequency of system.

Amplitude of vibration becomes excessive.

Damping

The resistance to the motion of vibrating system.

1. **Equation of Stiffness of a Spring:**

For A helical Spring

$$k_{equivalent} = k_{eq} = \frac{d^4 G}{8D^3 n}$$

d – Wire diameter

D – Coil diameter

G – Shear modulus

n – Number of turns

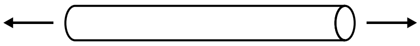
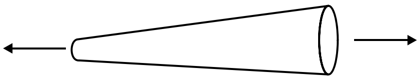
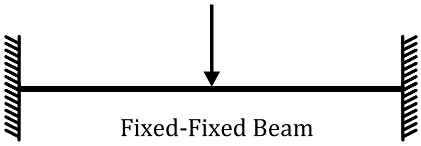
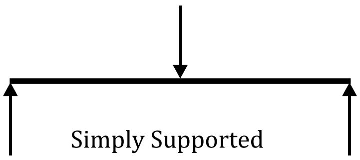
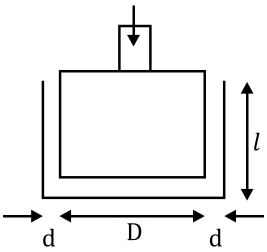
2. **Springs in Series:**

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}$$

Springs in parallel

$$k_{eq} = k_1 + k_2 + k_3 + \cdots + k_n$$

3. **Equivalent Springs of System**

System	Formula of Stiffness
 Rod Under Axial Load	$k = \frac{EA}{l}$
 Tapered Rod	$k = \frac{\pi EDd}{4l}$
 Fixed-Fixed Beam	$k = \frac{192EI}{l^3}$
 Cantilever Beam	$k = \frac{3EI}{l^3}$
 Simply Supported	$k = \frac{48EI}{l^3}$
 Axial motion of Piston in a cylinder (Dashpot damper).	$C_{eq} = \frac{\mu 3\pi D^3 l}{4d^3}\left(1 + \frac{2d}{D}\right)$

4. **Combination of Dampers:**

a. Parallel $C_{eq} = C_1 + C_2 + \cdots + C_n$

b. Series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}$

5. **Categories of Vibration:**

a. $m\ddot{x} + kx = 0$; Free undamped

b. $m\ddot{x} + c\dot{x} + kx = 0$; Free damped

c. $m\ddot{x} + kx = F(x)$; Forced undamped

d. $m\ddot{x} + c\dot{x} + kx = F(x)$; Forced damped

6. **Free Vibration:**

Translational Vibration

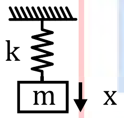
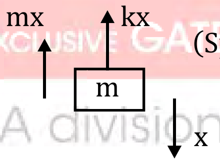


Fig (a) Single degree of freedom

For initial state equilibrium, $mg = k\Delta$

Where Δ = state deflection due to mass



When deflected by x then equation can be written as,

$m\ddot{x} + kx = 0$ .... (1)

This is the equation of simple harmonic and is analogous to,

$\ddot{x} + \omega_n^2 x = 0$

By comparing above equation with equation (1a), we can find

$$\omega_n = \sqrt{\frac{k}{m}}$$

By putting $\frac{k}{m}$ from initial static equilibrium,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\Delta}}$$

Equation of motion for

$m\ddot{x} + kx = 0$

$x = X\sin(\omega_n t + \phi)$ ... ... (2)

Initial Conditions

If the motion is started by displacing the mass through a distance x_0 and giving a velocity V_0, then the solution of equation (2) can be find as,

$t = 0, \quad x = x_0\ ;\ \dot{x} = V_0$

$x = X\sin(\omega_n t + \phi)$

$x_0 = X\sin\phi$

$\dot{x} = X\omega_n\cos(\omega_n t + \phi)$

$V_0 = X\omega_n\cos\phi$

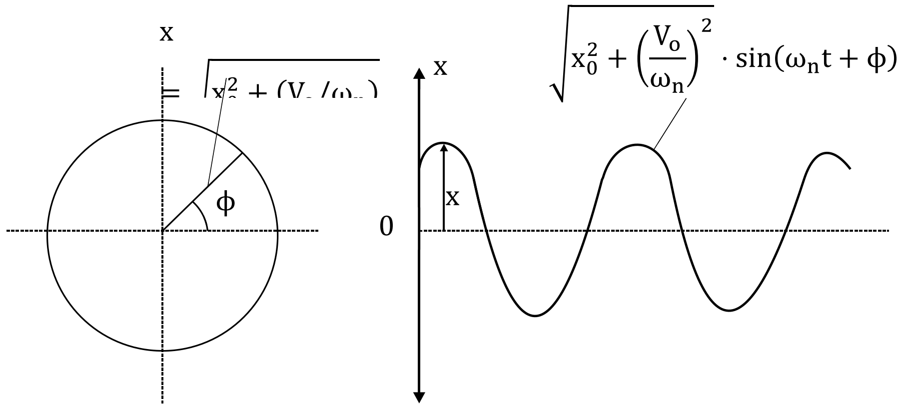
$\rightarrow x_0^2 + \left(\frac{V_0}{\omega_n}\right)^2 = X^2$

$X = \sqrt{\left[x_0^2 + \left(\frac{V_0}{\omega_n}\right)^2\right]}$

$\tan\phi = \frac{x_0}{V_0/\omega_n}$

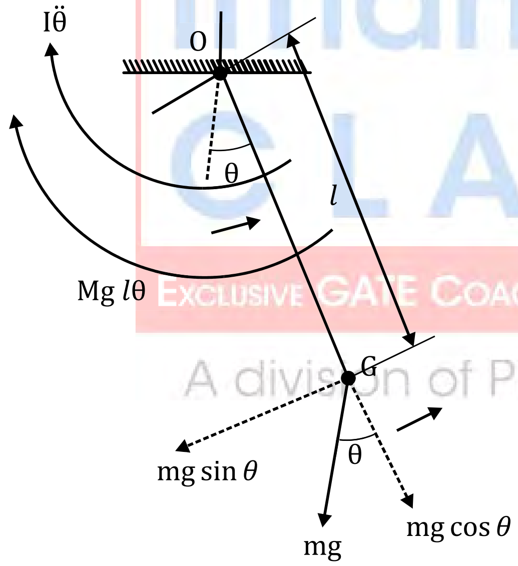
$\phi = \tan^{-1}\left[\frac{x_0}{V_0/\omega_n}\right]$

Solution can be plotted graphically



Rotational Vibration

Let Pendulum is displayed by an angle θ of small value.



Here G is the centre of gravity.

Restoring torque = mgsin θ × l = mgl. θ

Inertial torque = $I\ddot{\theta}$

∴ $I\ddot{\theta}$ + mgl. θ = 0

$$\omega_n = \sqrt{\frac{mgl}{I}}$$

Torsional Vibration

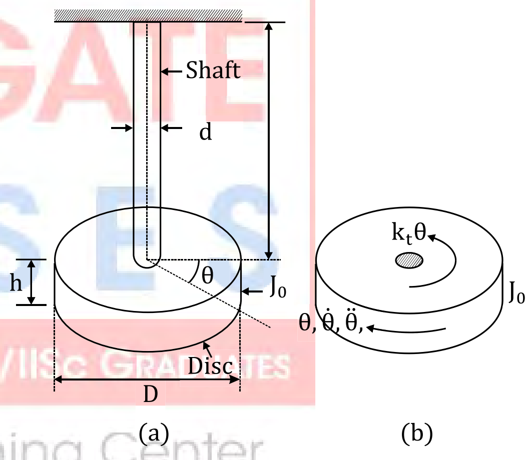


Fig: Torsional Vibration of a Disc

$$J_0\ddot{\theta} + k_t\theta = 0$$

$$\omega_n = \left(\frac{k_t}{J_0}\right)^{1/2}$$

And the period in seconds and frequency of vibration in cycles per second are

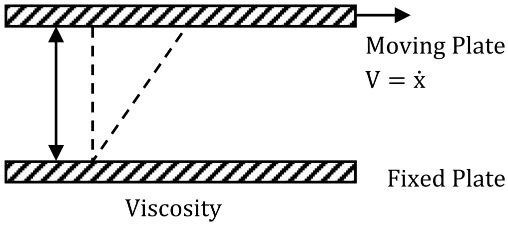
$$\tau_n = 2\pi\left(\frac{J_0}{k_t}\right)^{1/2}$$

$$f_n = \frac{1}{2\pi}\left(\frac{k_t}{J_0}\right)^{1/2}$$

7. Free Damped Vibration:

Types of Damping

- **Viscous Damping**



$$F = \frac{\mu A}{t}\dot{x}$$

Where, A = Area of the plate;

t = Thickness

μ = Coefficient of absolute viscosity of the film

$F = c\,\dot{x}$

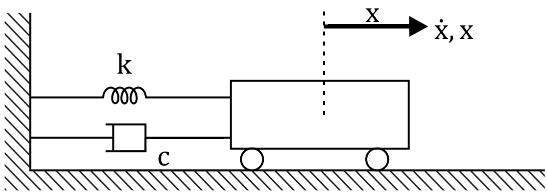
Where, $c = \frac{\mu A}{t}$

Where, c is viscous damping coefficient.

Energy Dissipation in Viscous Damping

$\Delta E = \pi c \omega A^2$

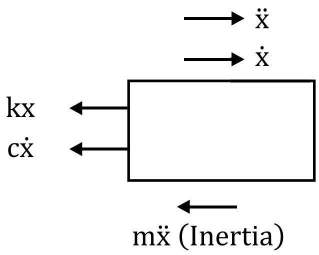
Differential Equation of Damped Free Translational Vibration



The equation of motion,

$m\ddot{x} + c\dot{x} + kx = 0$

Free body diagram



For damped $(m\ddot{x} + c\dot{x} + kx = 0)$

Using $x = Ae^{-st}$

$$S_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$= \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

Critical Damping Coefficient:

$C_c = 2m\omega_n = 2\sqrt{km}$

Natural Frequency:

$\omega_n = \sqrt{k/m}$

Damping Factor or Damping Ratio

$$\xi = \frac{c}{C_c}$$

Now, $\frac{C}{2m} = \frac{C}{C_c} \cdot \frac{C_c}{2m} = \zeta\omega_n$

So,

$$S_{1,2} = \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n$$

If $C > C_c$ or $\zeta > 1$, system is over damped

$C = C_c$ or $\zeta = 1$, system is critically-damped

$C < C_c$ or $\zeta < 1$, system is under-damped

Over Damped System ($\zeta > 1$)

General solution,

$$x = C_1 e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + C_2 e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$$

For Intial condition, $\left.\begin{array}{l} x = X_0 \\ \text{And } \dot{x} = 0 \end{array}\right\} \rightarrow \boxed{\text{At } t = 0}$

Solving for the condition,

$$x = \frac{X_0}{2\sqrt{\zeta^2-1}}\left[\left(-\zeta + \sqrt{\zeta^2-1}\right)e^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + \left(-\zeta - \sqrt{\zeta^2-1}\right)e^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t}\right]$$

Critical Damped System ($\zeta = 1$)

$S_1 = S_2 = -\omega_n$

Solution, $x = (C_1+C_2 t)e^{-\omega_n t}$

For initial condition, $\left.\begin{matrix} x = X_0 \\ \text{And } \dot{x} = 0 \end{matrix}\right\}$ $\rightarrow$ $\boxed{\text{At } t = 0}$

$\rightarrow x = X_0(1 + \omega_n t)\, e^{-\omega_n t}$

- Critical damping is smallest damping for which response is non-oscillatory
- With Critical damping system will have steep fall in displacement

Under – Damped System ($\zeta < 1$)

$$x = \frac{X_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_d t + \phi) \text{ Where, } \phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$\text{Amplitude} = \frac{X_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}$$

Which is seems to decay exponentially with time. Theoretically, the system will never come to rest, although the amplitude of vibration may become infinitely small.

$$\text{Period, } T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\left(\sqrt{1-\zeta^2}\right)}$$

Damped Frequency:

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

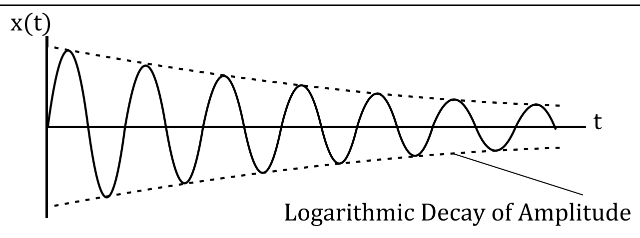
Logarithmic Decrement:

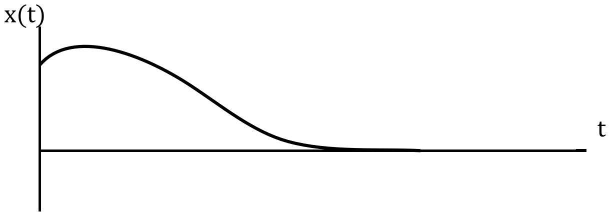
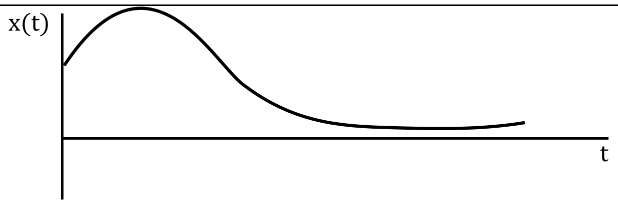
It is the ratio of any two successive amplitudes for an under-damped system vibrating freely is constant and it is a function of damping only

$$\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \approx 2\pi\xi(\xi << 1)$$

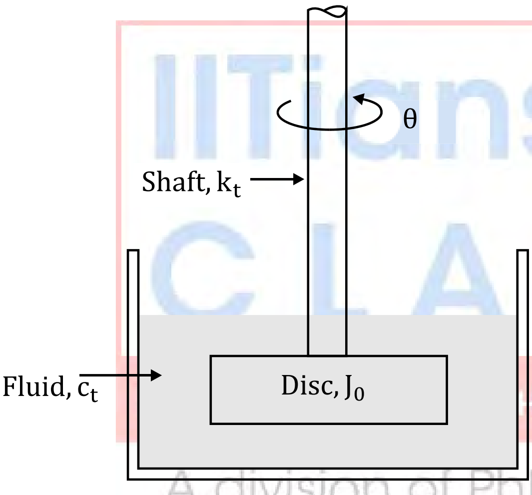
Here, amplitude = n+1

Cycles = n - 0 = n

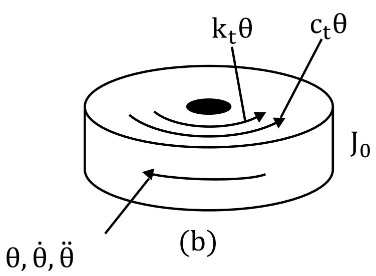
Case	Damping Ratio	Curve
Underdamped	$\xi < 1$	

Critical	$\xi = 1$	
Overdamped	$\xi > 1$	

Differential Equation of Damped Free Torsional Vibration



(a)



(b)

$$J_0\ddot{\theta} + c_t\dot{\theta} + k_t\theta = 0$$

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

$$\omega_n = \sqrt{k_t/J_0}$$

$$\zeta = \frac{c_1}{c_{tc}} = \frac{c_t}{2J_0\omega_n} = \frac{c_t}{2\sqrt{k_tJ_0}}$$

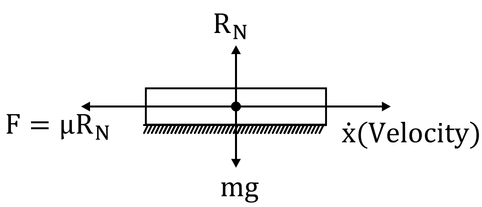
where c_{tc} is the critical torsional damping constant

Note- In case of rotational vibration also equation of motion and other quantities will be like rotational vibration

COULOMB DAMPING

This type of damping occurs when two machine parts rubs against each other, dry or un- lubricated. The damping resistance in the case is practically constant and is independent of the rubbing velocity.

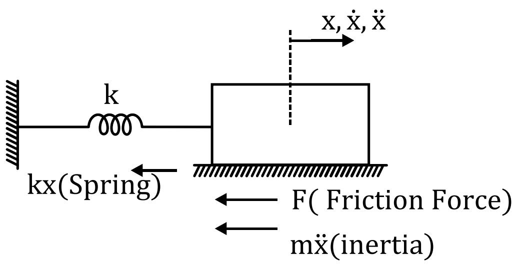
General expression for coulomb damping is:

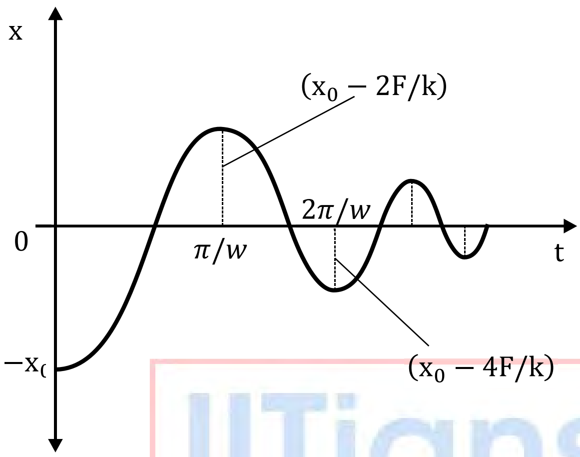


$$\boxed{F = \mu R_N}$$

Where, μ = coefficient of friction

R_N = Normal reaction





1. The equation of motion is nonlinear with Coulomb damping, whereas it is linear with viscous damping.

2. The natural frequency of the system is unaltered with the addition of Coulomb dumping, whereas it is reduced with the addition of viscous damping.

3. The motion is periodic with Coulomb damping, whereas it can be nonperiodic in a viscously damped (overdamped) system.

4. The system comes to rest after some lime with Coulomb damping, whereas the motion theoretically continues forever (perhaps with an infinitesimally small amplitude) with viscous and hysteresis damping.

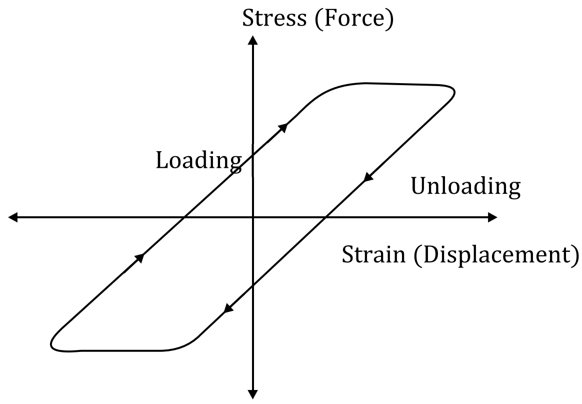
5. The amplitude reduces linearly with Coulomb damping, whereas it reduces exponentially with viscous damping.

6. In each successive cycle, the amplitude of motion is reduced by tire amount 4fiN/k, so the amplitudes at the end of any two consecutive cycles are related:

$$X_n = X_{n-1} - \frac{4\mu N}{k}$$

Structural Damping

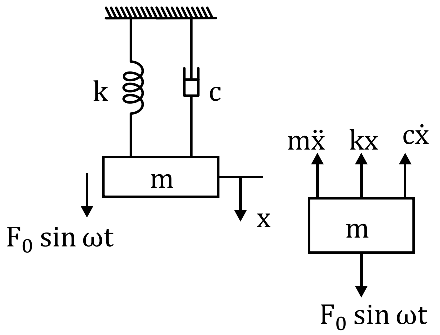
This type of damping is due to the internal friction of the molecules. The stress – strain diagram for vibrating body is not a straight line but forms a hysteresis loop. The area of which represents the energy dissipated due to molecular friction per cycle per unit volume.



The energy loss per cycle,

$E = \pi k \lambda A^2$

8. **Forced Vibration:**



$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$

Above differential equation will have two part in solution

The complementary function is obtained from

$m\ddot{x} + c\dot{x} + kx = 0$

$x_c = A_2 e^{-\zeta\omega_n t} \sin\left[\left(\sqrt{1-\zeta^2}\right)\omega_n t + \phi_2\right]$

If the particular solution is x_p then

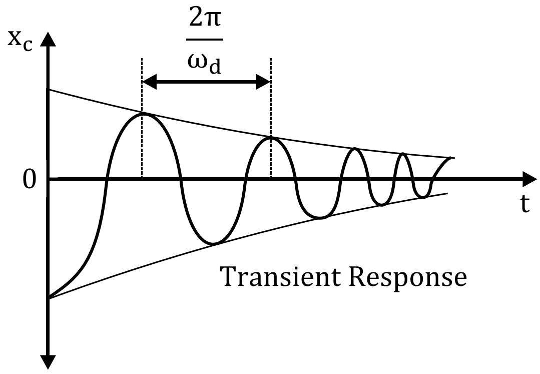
$x_p = X\sin(\omega t - \phi)$

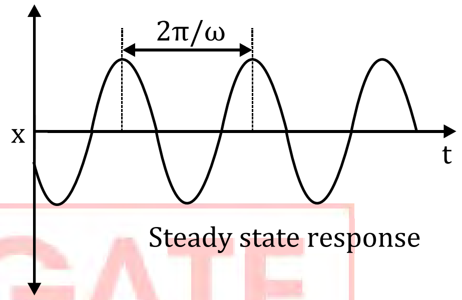
$$\text{Then, } X = \frac{X_{st}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$\phi = \tan^{-1}\left[\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$x = x_c + x_p$

$$= A_2 e^{-\zeta\omega_n t} \sin\left[\left(\sqrt{1-\zeta^2}\right)\omega_n t + \phi_2\right] + \frac{X_{st}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}} \sin(\omega t - \phi)$$





Magnification Factor

The ratio of the steady state amplitude to the zero frequency (static) deflection i.e., $\frac{X}{X_{st}}$ is defined as the magnification factor and is denoted by M.F.

$$M.F. = \frac{X}{X_{st}}$$

$$= \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

At resonance, $\frac{\omega}{\omega_n} = 1$

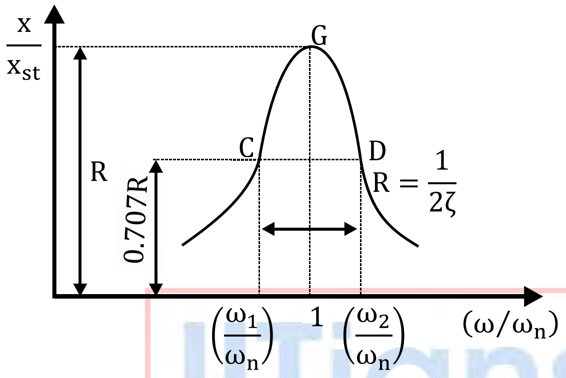
$$M.F. = \frac{X}{X_{st}} = \frac{1}{2\zeta}$$

Quality Factor

The magnification factor at resonance is known as quality factor and is denoted by R.

$$R = \frac{1}{2\zeta}$$

Half power points



→C and D are the half power points.

Assume ω_1 and ω_2 are frequency corresponding to points C and D respectively.

$$\frac{\omega_2 - \omega_1}{\omega_n} = 2\zeta = \text{Band width} = \frac{1}{R}$$

Force Transmissibility

$$F_{tr} = \sqrt{(kX)^2 + (c_\omega X)^2}$$

$$F_{tr} = X\sqrt{k^2 + (c_\omega)^2}$$

$$F_{tr} = \frac{F_o\sqrt{k^2 + (c_\omega)^2}}{\sqrt{(k - m\omega^2)^2 + (c_\omega)^2}}$$

$$T_r = \frac{F_{tr}}{F_o} = \frac{\sqrt{1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

T_r being the transmissibility

Forced Vibration with Rotating and Reciprocating Unbalance

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = m_0 e\omega^2 \sin \omega t$$

$$X = \frac{\frac{m_0 e\omega^2}{k}}{\sqrt{\left(1 - \frac{m\omega^2}{k}\right) + \left(\frac{c\omega}{k}\right)}}$$

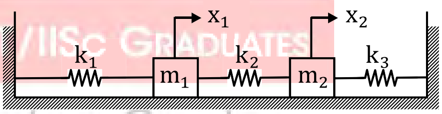
$$\frac{X}{\left(\frac{m_0 e}{m}\right)} = -\frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$\phi = \tan^{-1}\left[\frac{2\zeta\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right]$$

At resonance when $\frac{\omega}{\omega_n} = 1$, we have

$$\frac{X}{\left(\frac{m_0 e}{m}\right)} = \frac{1}{2\zeta}$$

Two-Degree of Freedom



If x_1 and x_2 are degrees of freedom

Let $x_1 > x_2$

Equation of motion for masses

$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = 0$ .... (1)

$m_2\ddot{x}_2 + (k_3 + k_2)x_2 - k_2x_1 = 0$ .... (2)

Let, $x_1 = X_1 \cos(\omega t + \phi)$; $x_2 = X_2 \cos(\omega t + \phi)$

$\{-m_1\omega^2 + (k_1 + k_2)\}X_1 - k_2X_2 = 0$ .... (3)

$-k_2X_1 + \{-m_2\omega^2 + (k_2 + k_3)\}X_2 = 0$ .... (4)

$\rightarrow (m_1 m_2)\omega^4 - \{m_1(k_3 + k_2) + m_2(k_1 + k_2)\}\omega^2 + \{k_1k_2 + k_2k_3 + k_3k_1\} = 0$

Above equation is the equation to calculate natural frequencies

So natural frequencies can be found as

$$\omega_1^2, \omega_2^2 = \frac{1}{2}\left\{\frac{(k_1+k_2)\,m_2 + (k_2+k_3)\,m_1}{m_1 m_2}\right\} \pm \frac{1}{2}\left[\left\{\frac{(k_1+k_2)\,m_2 + (k_2+k_3)\,m_1}{m_1 m_2}\right\}^2 - 4 \times 1 \left\{\frac{(k_1+k_2)\,(k_2+k_3) - k_2^2}{m_1 m_2}\right\}\right]^{1/2}$$

Here determinant is homogenous, only ratio of amplitude of frequencies can be found.

$$r_1 = \frac{X_1^{(1)}}{X_2^{(1)}} = \frac{k_2}{-m_1\omega_1^2 + (k_1+k_2)} = \frac{-m_2\omega_1^2 + (k_2+k_3)}{k_2};$$

(amplitude ratio at frequency ω_1)

$$r_2 = \frac{X_1^{(2)}}{X_2^{(2)}} = \frac{k_2}{-m_1\omega_2^2 + (k_1+k_2)} = \frac{-m_2\omega_2^2 + (k_2+k_3)}{k_2};$$

(amplitude ratio at frequency ω_2)

The normal modes of vibration corresponding to ω_1^2 and ω_2^2 can be expressed.

$$\bar{X}^{(1)} = \begin{Bmatrix} X_1^{(1)} \\ X_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} X_1^{(1)} \\ r_1 X_1^{(1)} \end{Bmatrix}$$

And $\bar{X}^{(2)} = \begin{Bmatrix} X_1^{(2)} \\ X_2^{(2)} \end{Bmatrix} = \begin{Bmatrix} X_1^{(2)} \\ r_2 X_1^{(2)} \end{Bmatrix}$

The vectors $\vec{X}^{(1)}$ and $\vec{X}^{(2)}$, which denote the normal mode of vibration are known as 'mode vectors' of the system.

Motion for mass m_1 can be written as

$x_1(t) = x_1^{(1)}(t) + x_1^{(2)}(t)$; (motion with ω_1 and ω_2)

$$x_1(t) = X_1^{(1)}\cos(\omega_1 t + \phi_1) + X_1^{(2)}\cos(\omega_2 t + \phi_2)$$

Motion for mass m_2 can be written as

$x_2(t) = x_2^{(1)}(t) + x_2^{(2)}(t)$; (motion with ω_1 and ω_2)

$$x_2(t) = X_2^{(1)}\cos(\omega_1 t + \phi_1) + X_2^{(2)}\cos(\omega_2 t + \phi_2)$$

If initial velocities are zero then

$$\left.\begin{aligned} x_1 &= X_1^{(1)}\cos\omega_1 t + X_1^{(2)}\cos\omega_2 t \\ x_2 &= X_2^{(1)}\cos\omega_1 t + X_2^{(2)}\cos\omega_2 t \end{aligned}\right]$$

$$\frac{X_1^{(1)}}{X_2^{(1)}} = \left(\frac{X_1}{X_2}\right)_1$$

$$\frac{X_2^{(2)}}{X_2^{(2)}} = \left(\frac{X_1}{X_2}\right)_2$$

$$\left.\begin{aligned} X_1^{(1)} + X_1^{(2)} &= \text{Initial displacement } m_1 \\ X_2^{(1)} + X_2^{(2)} &= \text{Initial displacement } m_2 \end{aligned}\right]$$

Static And Dynamic Coupling in Vibration

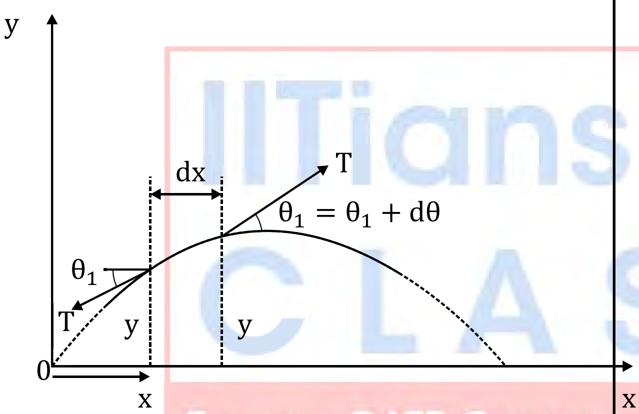
Static Coupling- If non principle diagonal terms in K matrix are non-zero then system is said to be statically coupled.

Dynamics Coupling- If non principle diagonal terms in M and C matrix are non-zero then system is said to be dynamically coupled.

VIBRATION OF CONTINUOUS SYSTEMS

Continuous system has infinite principal mode of vibration and have infinite natural frequencies. **Example:** Vibration of string, bar/rod, beam, shaft etc.

1. **Vibrating String:**



$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} \quad \left(\text{putting, } c^2 = \frac{T}{\rho}\right)$$

$$\because c = \sqrt{\frac{T}{\rho}}$$

= velocity of wave propagation

→This equation is one- dimensional wave equation for lateral vibrations of string.

Sol for gov differential equation

$$y = XT$$

$$y = \sum_{i=1}^{\infty}\left(A_i \sin\frac{\omega_i}{C}x + B_i \cos\frac{\omega_i}{C}x\right)(C_i \sin\omega_i t + D_i \cos\omega_i t)$$

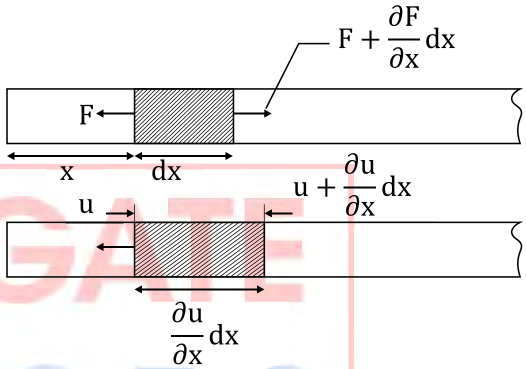
Note: Above equation has 2 boundary conditions and 2 initial conditions

For both ends fixed:

At x = 0 & L → y = 0

$$\rightarrow \omega = \frac{n\pi}{L}\sqrt{\frac{T}{\rho}}$$

2. **Longitudinal Vibration of a Bar:**



$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

$$\text{Where, } c^2 = \frac{E}{\rho} \text{ or } c = \sqrt{\frac{E}{\rho}}$$

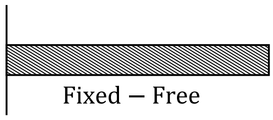
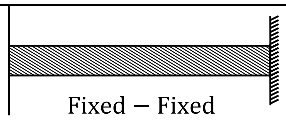
c is the velocity of wave propagation in bar/rod '

Sol for gov differential equation

$$u = XT$$

$$u = \sum_{i=1}^{\infty}\left(A_i \sin\frac{\omega_i}{C}x + B_i \cos\frac{\omega_i}{C}x\right)(C_i \sin\omega_i t + D_i \cos\omega_i t)$$

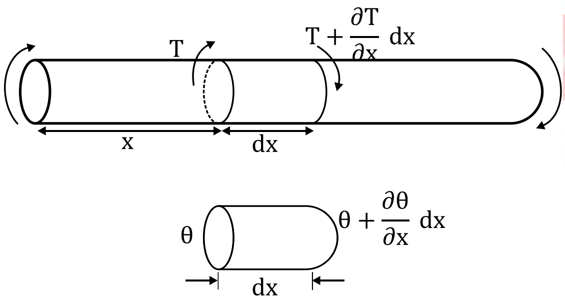
Boundary Conditions and Natural Frequencies:

Condition	Boundary Conditions	Frequency Equation	Mode Shape (Normal Function)	Natural Frequency
 Free – Free	$\frac{\partial u}{\partial x}(0,t)=0$ $\frac{\partial u}{\partial x}(l,t)=0$	$\sin\left(\frac{wl}{c}\right)=0$	$u_n(x)=c_n\cos\left(\frac{n\pi x}{l}\right)$	$\omega_n=\frac{n\pi c}{l}, n=0,1,2,\ldots$
 Fixed – Free	$u(0,t)=0$ $\frac{\partial u}{\partial x}(l,t)=0$	$\cos\left(\frac{\omega l}{c}\right)=0$	$u_n(x)=c_n\frac{(2n+1)\pi x}{2l}$	$\omega_n=\frac{(2n+1)\pi c}{2l}$ $n=0,1,2\ldots$
 Fixed – Fixed	$u(0,t)=0$ $u(l,t)=0$	$\sin\left(\frac{\omega l}{c}\right)=0$	$u_n(x)=C_n\cos\left(\frac{n\pi c}{l}\right)$	$\omega_n=\frac{n\pi c}{l}$ $n=1,2,3\ldots$

Initial Conditions are: $u(x,t=0)=u_o(x)$

$$\frac{\partial u}{\partial t}(x,0)=\dot{u}_t(x,0)=\dot{u}_o(x)$$

3. **Torsional Vibration of Shaft or Rod.**



$$\frac{\partial^2\theta}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2\theta}{\partial t^2}$$

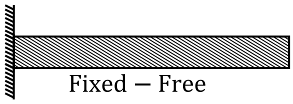
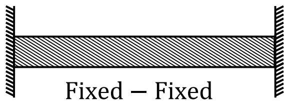
Here, $c=\sqrt{\frac{G}{\rho}}$;

Sol for gov differential equation

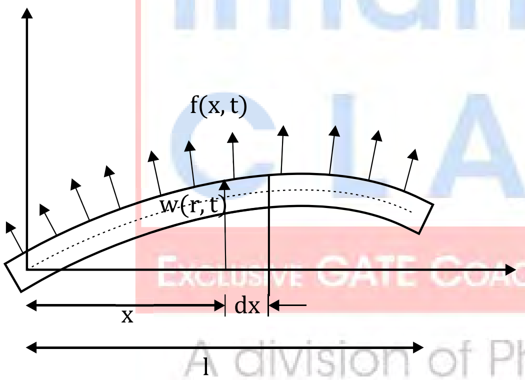
$$\theta(x,t)=X.T$$

$$\theta=\sum_{i=1}^{\infty}\left(A_i\sin\frac{\omega_i}{C}x+B_i\cos\frac{\omega_i}{C}x\right)(C_i\sin\omega_i t+D_i\cos\omega_i t)$$

Boundary Conditions:

Condition	Boundary Conditions	Frequency Equation	Mode Shape	Natural Frequency
 Free – Free	$\frac{\partial\theta}{\partial x}(0,t) = 0$ $\frac{\partial\theta}{\partial x}(l,t) = 0$	$\sin\left(\frac{\omega l}{c}\right) = 0$	$\theta(x) = C_n \cos\left(\frac{n\pi x}{l}\right)$	$\omega_n = \frac{n\pi c}{l}$ $n = 0,1,2,\ldots$
 Fixed – Free	$\theta(0,t) = 0$ $\frac{\partial\theta}{\partial x}(l,t) = 0$	$\cos\left(\frac{\omega l}{c}\right) = 0$	$\theta(x) = C_n \sin\left(\frac{(2n+1)\pi x}{2l}\right)$	$\omega_n = \frac{(2n+1)\pi c}{2l}$ $n = 0,1,2,..$
 Fixed – Fixed	$\theta(0,t) = 0$ $\theta(l,t) = 0$	$\sin\left(\frac{\omega l}{c}\right) = 0$	$\theta(x) = C_n \cos\left(\frac{n\pi x}{l}\right)$	$\omega_n = \frac{n\pi c}{l}$ $n = 1,2,3\ldots$

4. **Lateral Vibration of Beam:**



$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = 0$$

Or

$$C^2\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} = 0$$

$$C = \sqrt{\frac{EI}{\rho A}}$$

Sol for gov differential equation

$\rightarrow$ $w(x,t) = X(x).T(t)$

2nd order derivative w.r.t. 'time.

4th order derivative w.r.t 'x' displacement

$$w(x) = C_1\cos\beta x + C_2\cosh\beta x + C_3\sin\beta x + C_4\sinh\beta x$$

$$\beta = \sqrt{\frac{\omega}{C}}$$

$w(x)$ – Normal mode or characteristic function of beam

ω – Natural frequency

Boundary Conditions

1. **Free End:**

Bending Moment $= EI\frac{\partial^2 w}{\partial x^2} = 0$

Shear force $= \frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right) = 0$

2. **Simply Supported:**

Deflection $= w = 0$

Bending Moment $= EI = \frac{\partial^2 w}{\partial x^2} = 0$

3. **Fixed Ends:**

Deflection = 0;

$$\text{Slope} = \frac{\partial w}{\partial x} = 0$$

Summary of B/C				
	Displacement X	**Slope** $\frac{dX}{dx}$	**B. M.** $\frac{d^2X}{dx^2}$	**S. F.** $\frac{d^3X}{dx^3}$
Hinge/ Simply support	0	-	0	-
Fixed support	0	0	-	-

Beam with different supports

1. **Simply – Simply Supported Beam**

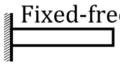
$\sin \beta_n l = 0$

$\beta L = n\pi$

$$\omega = n^2\pi^2 \frac{c}{L^2} = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}$$

$n = 1$, first mode of vibration.

2. **Cantilever Beam**



$(\cos \beta L)\ (\cos h\beta L) = -1$

From above equation natural frequencies can be found

$\beta L = 1.875104$

$$\omega = 1.875^2 \frac{c}{L^2} = \frac{1.875^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}$$

3. **Fixed-Fixed**

$(\cos \beta L)\ (\cos h\beta L) = 1$

From above equation natural frequencies can be found

$\beta L = 4.73$

$$\omega = 4.73^2 \frac{c}{L^2} = \frac{4.73^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}$$

OUR COURSES

GATE Online Coaching

Course Features

- Live Interactive Classes
- E-Study Material
- Recordings of Live Classes
- Online Mock Tests

TARGET GATE COURSE

Course Features

- Recorded Videos Lectures
- Online Doubt Support
- E-Study Materials
- Online Test Series

Distance Learning Program

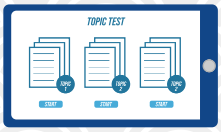
Course Features

- E-Study Material
- Topic Wise Assignments (e-form)
- Online Test Series
- Online Doubt Support
- Previous Year Solved Question Papers

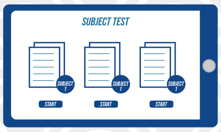
OUR COURSES

Online Test Series

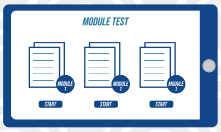
Course Features



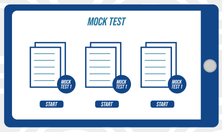
Topic Wise Tests



Subject Wise Tests



Module Wise Tests



Complete Syllabus Tests

More About IGC

SAMPLE STUDY MATERIALS
https://bit.ly/3Wu7lsp

SAMPLE MOCK TESTS
https://bit.ly/3FDyF0f

SAMPLE VIDEO LECTURES
https://bit.ly/3V4xU6j

TEAM IGC GATE
https://bit.ly/3G21khi

OUR TESTIMONIALS
https://bit.ly/3Yp2Hh7

IGC TELEGRAM GROUP
https://bit.ly/3j4koCd

Follow us on:

For more Information Call Us
+91-97405 01604

Visit us
www.iitiansgateclasses.com

IITians GATE CLASSES

Exclusive GATE Coaching by IIT/IISc Graduates

A division of PhIE Learning Center

Announcing

New ***Batches*** *For*

GATE 2024/25

Live Online Classes

AE | ME | CE | EC | EE | IN | CSE

ENROLL NOW

Hurry up !

Limited Seats Available

Batch *Starting From*

19th Feb 2023

For more Information Call Us

+91-97405 01604

Visit us **www.iitiansgateclasses.com**