GATE ECE&EE Coaching By IITians GATE CLASSES

Contents

Boolean Algebra:

Due to historical reasons, digital circuits are called switching circuits, digital circuit functions are called switching functions and the algebra is called switching algebra. The algebraic system known as Boolean algebra named after the mathematician George Boole. George Boole Invented multi-valued discrete algebra (1854) and E. V. Huntington developed its postulates and theorems (1904).

Examples of Huntington 's postulates are given below:

Postulate1.

If X and Y are in set (0, 1) then operations X + Y and X · Y are also in set (0, 1)

Postulate 2.

For each operation there exists an identity element.

$$X + 0 = 0 \qquad and \qquad X \cdot 1 = X$$

Postulate 3.

The operations are commutative

For all x and y ∈ {0,1},

- x + y = y + x
- x . y = y . x

Postulate 4.

The operations are distributive

For all x, y and z ∈ {0,1},

- x + (y . z) = (x + y) . (x + z)
- x . (y + z) = (x . y) + (x . z)

Postulate 5.

For every element x ∈ {0,1} there exists an element ∈ {0,1} (called the complement of x) such that

- $X + \bar{X} = 1$
- $X \cdot \bar{X} = 0$

Postulate 6.
There exist at least two elements x and y ∈ {0,1} such that$X \neq Y$.

Useful properties

Property 1: Special law of 0 and 1

For all $X \in \{0,1\}$

a. $X \cdot 0 = 0$
b. $X + 1 = 1$

Proof:-

$$X \cdot 0 = (X \cdot 0) + 0$$
$$= (X \cdot 0) + (X \cdot \bar{X})$$
$$= X \cdot (0 + \bar{X})$$
$$= X \cdot \bar{X}$$
$$= 0$$

$$X + 1 = (X + 1) \cdot 1$$
$$= (X + 1) \cdot (X + \bar{X})$$
$$= X + (1 \cdot \bar{X})$$
$$= X + \bar{X}$$
$$= 1$$

Property 2: Complement Law

a. The Complement of 0 is 1
b. The complement of 1 is 0

Proof:-

$$X + 0 = X \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(By\ Postulet\ 2)$$
$$By\ replacing\ X\ with\ \bar{0}$$
$$\bar{0} + 0 = \bar{0}$$
$$But,$$
$$\bar{0} + 0 = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(By\ Postulate\ 5)$$
$$\therefore \bar{\mathbf{0}} = \mathbf{1}$$

$$X \cdot 1 = X \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulet\ 2)$$
$$By\ replacing\ X\ with\ \bar{0}$$
$$\bar{1} \cdot 1 = \bar{1}$$
$$But,$$
$$\bar{1} \cdot 1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 5)$$
$$\therefore \bar{\mathbf{1}} = \mathbf{0}$$

Property 2: Idempotency Law

For all $X \in \{0, 1\}$

a. X + X = X
b. X · X = X

Proof:-

$$X + X = (X + X) \cdot 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$$
$$= (X + X) \cdot (X + \bar{X}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 5)$$
$$= X + (X \cdot \bar{X}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 4)$$
$$= X + 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 5)$$
$$= X \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$$

$$X \cdot X = (X \cdot X) + 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$$
$$= (X \cdot X) + (X \cdot \bar{X}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 5)$$
$$= X + (X \cdot \bar{X}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 4)$$
$$= X + 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 5)$$
$$= X \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$$

Property 4: Adjacency Law

For all X and Y $\in \{0, 1\}$

a. $X \cdot Y + X \cdot \bar{Y} = X$
b. $(X + Y) \cdot (X + \bar{Y}) = X$

Proof:-

$$X \cdot Y + X \cdot \bar{Y} = X \cdot (Y + \bar{Y}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 4)$$
$$= X \cdot 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 5)$$
$$= X \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$$

$(X+Y)\cdot(X+\bar{Y}) = X+(Y\cdot\bar{Y})$ ……………………… (By Postulate 4)
$= X+0$ ……………………………………………… (By Postulate 5)
$= X$ ………………………………………………… (By Postulate 2)

Property 5: Absorption Law

For all X and Y ∈ $\{0,1\}$

a. $X+(X\cdot Y)=X$
b. $X\cdot(X+Y)=X$

Proof:-

$X+(X\cdot Y)=(X\cdot 1)+(X\cdot Y)$ ……………………… (By Postulate 2)
$= X\cdot(1+Y)$ ……………………………………… (By Postulate 4)
$= X\cdot 1$ ……………………………………………… (By Postulate 2)
$= X$ ………………………………………………… (By Postulate 2)

$X\cdot(X+Y)=(X+0)\cdot(X+Y)$ ……………………… (By Postulate 2)
$= X+(0\cdot Y)$ ……………………………………… (By Postulate 4)
$= X+0$ ……………………………………………… (By Postulate 2)
$= X$ ………………………………………………… (By Postulate 2)

Property 6: Second law of Absorption.

For all X and Y ∈ $\{0,1\}$

a. $X+(\bar{X}\cdot Y)=X+Y$
b. $X\cdot(\bar{X}+Y)=X\cdot Y$

Proof:-

$X+(\bar{X}\cdot Y)=(X+\bar{X})\cdot(X+Y)$ ……………………… (By Postulate 2)
$= 1\cdot(X+Y)$ ……………………………………… (By Postulate 4)
$= X+Y$ ……………………………………………… (By Postulate 2)

$X\cdot(\bar{X}+Y)=(X\cdot\bar{X})+(X\cdot Y)$ ……………………… (By Postulate 2)
$= 0+(X\cdot Y)$ ……………………………………… (By Postulate 4)
$= X\cdot Y$ ……………………………………………… (By Postulate 2)

Property 7: Consensus law.

For all X, Y, Z $\in \{0,1\}$

a. $X\cdot Y+\bar{X}\cdot Z+Y\cdot Z=X\cdot Y+\bar{X}\cdot Z$
b. $(X+Y)\cdot(\bar{X}+Z)\cdot(Y+Z)=(X+Y)\cdot(\bar{X}+Z)$

Proof:-

$$X\cdot Y+\bar{X}\cdot Z+Y\cdot Z$$
$$=X\cdot Y+\bar{X}\cdot Z+1\cdot Y\cdot Z\ \ldots\ldots (By\ Postulate\ 2)$$
$$=X\cdot Y+\bar{X}\cdot Z+(X+\bar{X})\cdot Y\cdot Z\ \ldots\ldots (By\ Postulate\ 5)$$
$$=X\cdot Y+X\cdot Y\cdot Z+\bar{X}\cdot Z+\bar{X}\cdot Y\cdot Z\ \ldots\ldots (By\ Postulate\ 4)$$
$$=X\cdot Y\cdot(1+Z)+\bar{X}\cdot Z\cdot(1+Y)\ \ldots\ldots (By\ Postulate\ 4)$$
$$=X\cdot Y\cdot 1+\bar{X}\cdot Z\cdot 1\ \ldots\ldots (By\ Postulate\ 2)$$
$$=X\cdot Y+\bar{X}\cdot Z\ \ldots\ldots (By\ Postulate\ 2)$$

$$(X+Y)\cdot(\bar{X}+Z)\cdot(Y+Z)$$
$$=(X+Y)\cdot(\bar{X}+Z)\cdot(0+Y+Z)\ \ldots\ldots (By\ Postulate\ 2)$$
$$=(X+Y)\cdot(\bar{X}+Z)\cdot\left((X\cdot\bar{X})+Y+Z\right)\ \ldots\ldots (By\ Postulate\ 5)$$
$$=(X+Y)\cdot(X+Y+Z)\cdot(\bar{X}+Z)\cdot(\bar{X}+Y+Z)\ \ldots\ldots (By\ Postulate\ 4)$$
$$=(X+Y)+(0\cdot Z)\cdot(\bar{X}+Z)+(0\cdot Y)\ \ldots\ldots (By\ Postulate\ 4)$$
$$=(X+Y)+0\cdot(\bar{X}+Z)+0\ \ldots\ldots (By\ Postulate\ 2)$$
$$=(X+Y)\cdot(\bar{X}+Z)\ \ldots\ldots (By\ Postulate\ 2)$$

Property 8: DeMorgan's law.

For all X, Y $\in \{0,1\}$

a. $\overline{X+Y}=\bar{X}\cdot\bar{Y}$
b. $\overline{X\cdot Y}=\bar{X}+\bar{Y}$

Proof:-

$$(X+Y)\cdot(\bar{X}\cdot\bar{Y})$$
$$=(X\cdot\bar{X}\cdot\bar{Y})+(Y\cdot\bar{X}\cdot\bar{Y})\ \ldots\ldots (By\ Postulate\ 2)$$
$$=0+0=0\ \ldots\ldots (By\ Postulate\ 5)$$

$$(X+Y)+(\bar{X}\cdot\bar{Y})$$
$$=(X+\bar{X}\cdot\bar{Y})+(Y+\bar{X}\cdot\bar{Y})\ \ldots\ldots (By\ Postulate\ 3)$$

$= (X + \bar{X}) + (Y + \bar{Y}) \ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$
$= 1 + 1 = 1 \ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 5)$

Therefore, $(\bar{X} \cdot \bar{Y})$ is the complement of (X+Y)

$(X \cdot Y) \cdot (\bar{X} + \bar{Y})$
$= \big(X \cdot (\bar{X} + \bar{Y})\big) \cdot \big(Y \cdot (\bar{X} + \bar{Y})\big) \ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 3)$
$= X \cdot \bar{Y} \cdot \bar{X} \cdot Y \ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$
$= 0 \cdot 0 = 0 \ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$

$(X \cdot Y) + (\bar{X} + \bar{Y})$
$= (X + \bar{X} + \bar{Y}) \cdot (Y + \bar{X} + \bar{Y}) \ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 3)$
$= (1 + \bar{Y}) + (1 + \bar{X}) \ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 2)$
$= 1 + 1 = 1 \ldots\ldots\ldots\ldots\ldots\ldots (By\ Postulate\ 5)$

Therefore, $(\bar{X} + \bar{Y})$ is the complement of (X · Y)

Duality Theorem:-

To find the dual replace " + " with " · " , " · " with " + ", 1 with 0 and 0 with 1.

a. + → ·

b. · → +

c. 1 → 0

d. 0 → 1

Solved Example:-

1. Find the dual of the $A \cdot (B + C) = A \cdot B + A \cdot C$

Answer:-

To find the dual replace + with ·, · with +, 1 with 0 and 0 with 1.

i.e replacing AND with OR, OR with AND, 1 with 0 and 0 with 1.

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

2. Find dual of the $A + 1 = A$

Answer:-

To find the dual replace + with ·, · with +, 1 with 0 and 0 with 1.
i.e replacing AND with OR, OR with AND, 1 with 0 and 0 with 1.

$$A \cdot 0 = A$$

3. Reduce the expression $\overline{\bar{A}B + \bar{A} + AB}$

Answer:-

$$\overline{\bar{A}B + \bar{A} + AB}$$
$$= \overline{\bar{A}B} \cdot \bar{\bar{A}} \cdot \overline{AB}$$
$$= (A + \bar{B}) \cdot A \cdot (\bar{A} + \bar{B})$$
$$= A \cdot (\bar{A} + \bar{B})$$
$$= A \cdot \bar{B}$$

Exercise

1. Reduce the expression $AB + \bar{A}C + A\bar{B}C(AB + C)$
2. Reduce the expression $(\overline{AB} + \overline{A + B})A\bar{B}$
3. Reduce the expression $\bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$
4. Reduce the expression $\overline{\overline{A\bar{B} + ABC} + A(B + AB)}$
5. Reduce the expression $\overline{A\bar{B}C + \bar{A}BC} + A + BC$
6.

Answer

1. $AB + C$
2. $A\bar{B}$
3. $AB + BC + AC$
4. 0
5. 1