Chapter 4 – Dielectric and Boundary condition

Dielectric

Materials are classified in terms of their electrical properties as conductors and nonconductors. Nonconducting materials are usually referred to as *insulators* or *dielectrics.*

It has the following property

- There is no free charge that can be transported within them to produce conduction current. Instead, all charge is confined to molecular or lattice sites by coulomb forces.
- An applied electric field has the effect of displacing the charges slightly; leading to the formation of ensembles of electric dipoles this effect is called polarization.
- The extent to which the polarization occurs is measured by the relative permittivity, or dielectric constant.

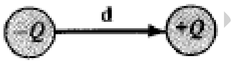
Electric Dipole: An electric dipole is formed when two point charges of equal magnitude but opposite sign are separated by a small distance.

Dipole Moment: Dipole moment is defined as the product of the charge and distance vector from negative charge to positive charge Mathematically it is defined as

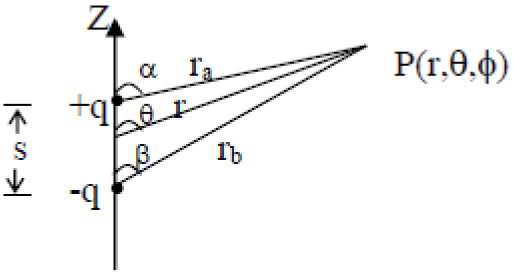
$$\boldsymbol{P} = Q\boldsymbol{d}$$

P is the dipole moment and ***d*** is the distance vector from –Q charge to Q charge.

Type of dielectric



Potential due to a dipole: Let us consider a physical dipole located on Z – axis and the point of observation is P(r, θ, ϕ).



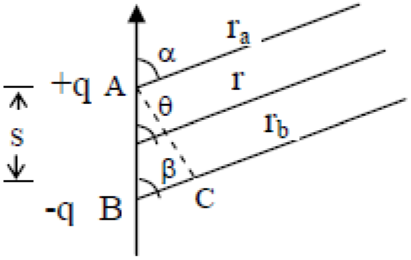
Electric potential at point P due to the charge –q and q is given by

V(P) = V(due to q) + V(due to -q)

$$= \frac{q}{4\pi\epsilon_0 r_a} + \frac{-q}{4\pi\epsilon_0 r_b} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r_b - r_a}{r_b r_a}\right]$$

If the point of observation is far away from the dipole then r_a and r_b will be almost same and can be treated as the parallel lines.



r_a = r_b = r and r_b - r_a = BC
BC = s cosθ

So $V(P) = \frac{q}{4\pi\epsilon_0}\left[\frac{s\cos\theta}{r^2}\right] = p\cos\theta / 4\pi\varepsilon_0 r^2$ (because p = qs)

Electric field intensity due to a Dipole:

$E = -\nabla V$

So, $E = p / (4\pi\varepsilon_0 r^3)[2\cos\theta\, \boldsymbol{r} + \sin\theta\, \boldsymbol{\vartheta}]$ where ***r*** and ***ϑ*** are the unit vector.

$E \propto 1/r^3$

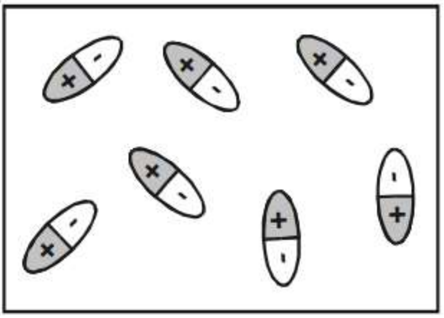
Note:

1. Potential due to an electric dipole $V(p) \propto 1 / r^2$. But for single point charge $V(p) \propto 1 / r$
2. Electric field to an electric dipole $V(p) \propto 1 / r^3$. But for single point charge $V(p) \propto 1 / r^2$

Type of dielectric

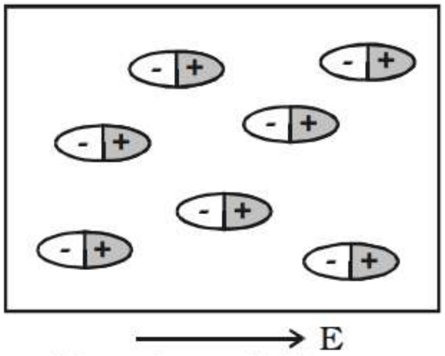
There are two type of dielectric:

1. ***Polar dielectric***: This type of dielectric have a permanent displacement existing between the centers of "gravity" of the positive and negative charges, and each pair of charges acts as a dipole.. They are randomly oriented as shown in fig below. Net dipole moment zero until an electric field is applied. Examples of polar dielectric are water, sulfur dioxide, and hydrochloric acid have built-in permanent dipoles that are randomly oriented



Polar dipole in absence of

Polar dipole in absence of
external electric field



Polar dipole in presence
of external electric field ***E***

2. **Non – Polar Dielectrics:** A nonpolar dielectric does not have the dipole arrangement until after a field is applied. In presence of a the electric field the negative and positive charges shift in opposite directions against their mutual attraction and produce a dipole that is aligned with the electric field. Examples of nonpolar dielectric are hydrogen, oxygen, nitrogen.

Polarization (P)

Quantitatively it is defined as effective dipole moment per unit volume.

If there are N dipoles in a volume Δv of the dielectric, the total dipole moment due to the electric field is

$$Q_1\mathbf{d_1} + Q_2\mathbf{d_2} + \cdots \ldots \ldots . + Q_N\mathbf{d_N} = \sum_{k=1}^{N} Q_k\mathbf{d_k}$$

So according to definition

$$\boldsymbol{P} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} Q_k\mathbf{d_k}}{\Delta v}$$

Susceptibility (χ)

In vacuum electric flux density ***D*** and Electric field intensity ***E*** are related as

$\boldsymbol{D} = \varepsilon_0\boldsymbol{E}$

In presence of a dielectric this relationship changes to

$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$ where ***P*** is the polarization

P is directly proportional to the applied electric field ***E***

$\boldsymbol{P} \propto E$

$P = \chi_e\, \varepsilon_0\boldsymbol{E}$

So $\boxed{\boldsymbol{D} = \varepsilon_0(1 + \chi_e)\boldsymbol{E}}$

D is also defined as $\boxed{D = \varepsilon_0\varepsilon_r\boldsymbol{E} = \varepsilon\boldsymbol{E}}$

where ε_r is the relative permittivity and ε is the permittivity of the given dielectric.

$$\boxed{\chi_e = \varepsilon_r - 1}$$

$$\boxed{\varepsilon = \varepsilon_0\varepsilon_r}$$

The **dielectric constant** (or **relative permittivity)** ε_r, is the ratio of the permittivity of the dielectric to that of free space.

For free space and non-dielectric materials (such as metals) $\varepsilon_r = 1$.

Dielectric Boundary Conditions:

Till now we have seen the electric field in a homogenous medium. If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called *boundary conditions.* With the help of the boundary condition if