IITians GATE CLASSES

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QUICK REVISION

FORMULA SHEET

for

GATE -AE FLUID MECHANICS

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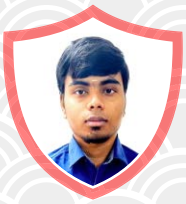

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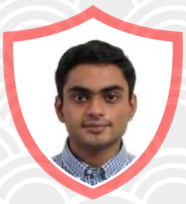

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FLUID MECHANICS

FLUID BASICS

Note:

1. For a static fluid, shear force is zero.
2. Fluids only show resistance to compressive loads.

$$k = \text{Bulk Modulus} = \frac{\text{Hydrostatic stress}}{\text{Volumetric strain}}$$

$$= \frac{-dp}{dV/V}$$

V= initial volume

$$\beta = \frac{1}{k}$$

β = Compressibility

1. **Isothermal Bulk Modulus (k_T) = p**

   Where p is absolute pressure

   The isothermal Compressibility

   $$(\beta_T) = \frac{1}{p}$$

   Valid for ideal gas

2. **Adiabatic bulk modulus ($k_{adiabatic}$) = γp**

   Where $\boldsymbol{\gamma}$ = Ratio of specific heats for air, $\boldsymbol{\gamma} = 1.4$

   $$\beta_{adiabatic} = \frac{1}{\gamma p}$$

   Valid for ideal gas

Note: For real gas to find isothermal bulk modulus we must use thermodynamic relation.

For Incompressible fluid $\beta = 0$

Reason of viscosity(μ) in liquids → Cohesive force (Molecular Bonding)

Reason of viscosity(μ) in gases → Molecular Collision.

- Temperature $\uparrow \Rightarrow \mu_{liquid} \downarrow \Rightarrow \nu_{liquid} \downarrow$

  Where $\nu_{liquid} \Rightarrow$ Kinematic viscosity of liquid.
- Temperature $\uparrow \Rightarrow \mu_{gas} \uparrow \Rightarrow \nu_{gas} \uparrow\uparrow$
- Pressure $\uparrow \Rightarrow \mu_{liquid}$: same $\Rightarrow \nu_{liquid}$: same
- Pressure $\uparrow \Rightarrow \mu_{gas}$: same $\Rightarrow \nu_{gas}$: $\downarrow$

Newton's Law of Viscosity:

$$\tau = \mu \frac{du}{dy} \quad \tau = \text{shear stress}$$

where μ = Viscosity/Dynamic viscosity

Unit of μ → kg/(meter-sec)or pascal-sec

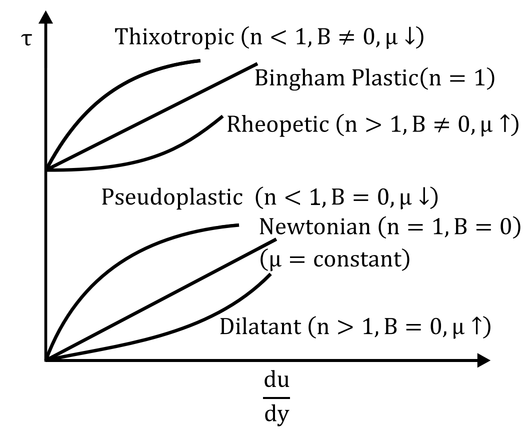
$$\frac{1\text{kg}}{\text{meter-sec}} = 10 \text{ poise}$$

1poise = 0.1pascal-sec

Kinematic viscosity (ν) = meter^2/sec

$$\frac{1\text{m}^2}{\text{sec}} = 10^4 \text{ stokes}$$

Stokes is CGS unit of kinematic viscosity



For Non-Newtonian Fluid:

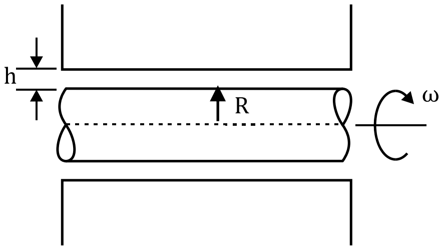
$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

Ideal Fluid:

- Non viscous
- Incompressible

Power Lost in Bearing due to Fluid Friction:

Case 1:



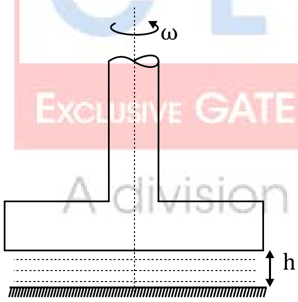
$$\text{Power} = \frac{2\pi\mu\omega^2 L R^3}{h}$$

R = Radius of shaft

h = oil film gap

ω = angular velocity of shaft

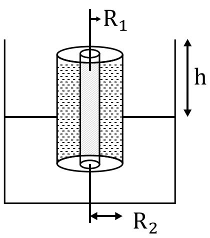
Case 2:



$$\text{Torque} = \frac{\pi\mu\omega R^4}{2h}$$

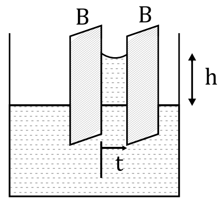
Special Cases

1.



$$h = \frac{2\sigma\cos\theta}{\rho g(R_2 - R_1)}$$

2.



t = distance between plates

$$h = \frac{2\sigma\cos\theta}{\rho g t}$$

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FLUID STATICS

$$p_{absolute} = p_{atm} + p_{gauge}$$

Hydrostatic Law:

$$\frac{dp}{dh} = -\rho g$$

Hydrostatic Forces:

$F = \rho g \bar{h} A$

$\bar{h}$ = distance of centre of gravity of body from free liquid surface.

Location of hydrostatic forces

$$\bar{h}_{cp} = \bar{h} + \frac{I_{Gx}\sin^2\theta}{A\bar{h}}$$

$\bar{h}_{cp}$ = distance of pressure force from free liquid surface.

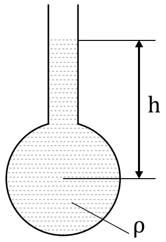
Pascal's Law:

Intensity of pressure at any point in a fluid at rest is same in all directions.

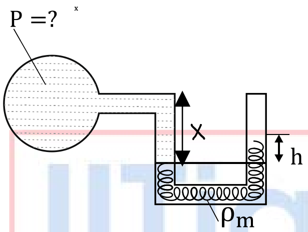
Pressure Measurement Device:

1. **Piezometer:**

   It measures +ve gauge pressure of liquid only.



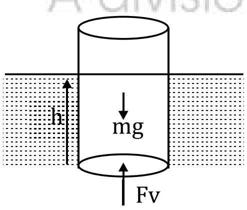
2. **Manometer:**



$p + \rho g x = (\rho_m g h)$

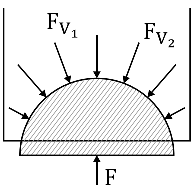
Buoyancy and Flotation:

When a body is immersed either partially or completely then net vertical force exerted by the fluid on the body known as buoyancy force.



$F_v = \rho g V_{displaced}$ $\boxed{mg = F_v}$

Special Case:



$F_B = 0$ (Buoyancy force = 0)

⇒ Stability condition for completely submerged body.

1. **Stable equilibrium:**

   If B above G

   B = centre of Buoyancy

   C = Centre of gravity

2. Unstable equilibrium: B below G
3. Neutral equilibrium: If B coincides G

⇒ Stability condition for partially submerged bodies (Floating bodies)

1. Stable equilibrium: M above G
2. Unstable equilibrium: M below G
3. Neutral equilibrium: M coincide G

   M = Meta centre point.

GM (Meta centric height):

$$GM = \frac{I}{V_{displaced}} - BG$$

I = Moment of inertia of the plane cutted by the surface level of liquid.

$V_{displaced}$ = volume of liquid displaced by the body.

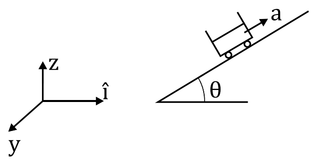
If GM > 0 (stable equilibrium condition)

⇒ Time period of oscillation (T).

$$T = 2\pi\sqrt{\frac{R^2}{g(GM)}}$$

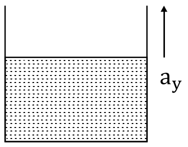
R = Radius of gyration

Accelerated vessel containing liquid.



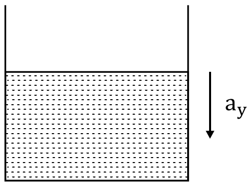
$a = a_x\hat{i} + a_z\hat{k}$ $\boxed{\tan\theta = \dfrac{a_x}{a_z + g}}$

If vessel accelerating in vertical direction:



For vertical upward motion

$$\frac{\partial p}{\partial y} = -\rho(g + a_y)$$



$$\frac{\partial p}{\partial y} = -\rho(g - a_y)$$

Free fall case:

$a_y = g, \qquad a_x = 0$

$$\frac{\partial p}{\partial y} = 0$$

$p = 0(\text{Gauge})$

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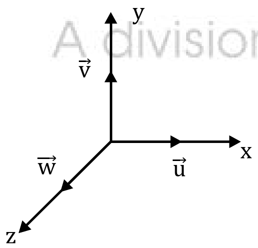
FLUID KINEMATICS

Acceleration:



$$a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$$

These are local component or temporal component of acceleration.

$$\vec{a} = \vec{a}_{convective} + \vec{a}_{temporal}$$

Angular Velocity:

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\omega = \frac{1}{2}(\nabla \times \vec{v})$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Vorticity $(\boldsymbol{\Omega}) = \boldsymbol{\nabla} \times \vec{\mathbf{v}} = 2\vec{\omega}$

Linear strain rate:

$$\varepsilon_{\dot{x}x} = \frac{\partial u}{\partial x}, \varepsilon_{\dot{y}y} = \frac{\partial v}{\partial y}, \varepsilon_{\dot{z}z} = \frac{\partial w}{\partial z}$$

$$\varepsilon_{\dot{v}} = \varepsilon_{\dot{x}x} + \varepsilon_{\dot{y}y} + \varepsilon_{\dot{z}z}$$

$\varepsilon_{\dot{v}}$ = volumetric strain rate

$\varepsilon_{\dot{v}} = 0$ For incompressible flow

Shear Strain Rate :

$$\varepsilon_{\dot{x}y} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\varepsilon_{\dot{x}z} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

$$\varepsilon_{\dot{y}z} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

u = x component of velocity

v = y component of velocity

w = z component of velocity

Shear deformation rate /Angular deformation rate:

$$\gamma_{\dot{x}y} = \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

Circulation (Γ):

$$\Gamma = \int \vec{V} \cdot \vec{dr} \;\; \text{(or)}\; \Omega \times \text{Area} \;\text{(or)} \iint \Omega dA$$

Ω = Vorticity

Velocity Potential Function (ϕ) for 3D flow.

$$u = \frac{\partial \phi}{\partial x}$$

$$v = \frac{\partial \phi}{\partial y}$$

$$w = \frac{\partial \phi}{\partial z}$$

In Polar coordinates:

$$u_r = \frac{\partial \phi}{\partial r}$$

$$V_o = +\frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

- If ϕ exists $\Rightarrow$ Flow is irrotational
- If ϕ satisfies Laplace equation $\Rightarrow$ Flow is possible.
- Slope of equipotential line

$$\frac{dy}{dx} = -\frac{u}{v}$$

Stream Function (ψ) (For 2D Flow):

$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

$$V_\theta = -\frac{\partial \psi}{\partial r}$$

- If ψ exists $\Rightarrow$ Flow is possible
- If ψ satisfied Laplace equation $\Rightarrow$ Flow is irrotational
- Slope of streamline

$$\Rightarrow \frac{dy}{dx} = \frac{v}{u}$$

Laplace Equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$\nabla^2 \phi = 0$$

∇^2 (any variable) = 0 (Laplace Equation)

Discharge Per unit length = $|\psi_1 - \psi_2|$

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FLUID DYNAMICS

$F_{pressure}, F_{gravity}, F_{viscous}$

$\Rightarrow$ Use Navier-strokes equation

$F_{pressure}, F_{gravity}$ $\Rightarrow$ Euler equation

$F_{pressure}, F_{gravity}$ and incompressible

$\Rightarrow$ Bernoulli's equation

Ideal Bernoulli's Equation: (Can be used for rotational flow also but Flow must be along same streamline)

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{Constant}$$

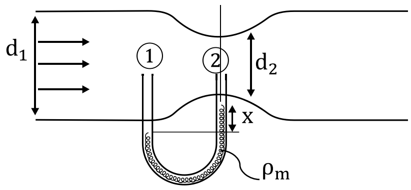
Assumptions:

- Non viscous
- Steady flow
- Incompressible flow of incompressible fluid
- Irrotational flow.

For Practical use (Bernoulli's equation with losses)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{losses}$$

1. **Venturimeter:**



d_1 = Pipe diameter

d_2 = Throat diameter

$$Q_{actual} = \frac{C_d \times A_1 A_2 \times \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$A_1 = \frac{\pi}{4} d_1^2$$

$$A_2 = \frac{\pi}{4} d_2^2$$

C_d = Coefficient of discharge

$C_d = 0.94$ to 0.98

Where h = pressure head difference/Piezometric head difference.

$$h = x\left(\frac{\rho_m}{\rho} - 1\right)$$

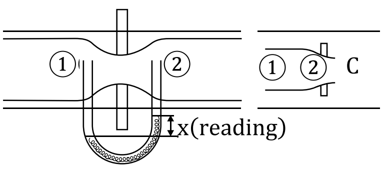
for both horizontal and inclined Venturimeter.

$$h = x\left(1 - \frac{\rho_m}{\rho}\right)$$

for inverted tube manometer where $(\rho_m < \rho)$

x = Manometric deflection

2. **Orifice Meter:**



$C_d = 0.62$ to 0.64

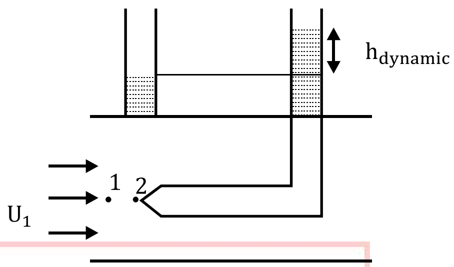
$$C_C = \frac{A_{CC}}{A_2}$$

$$Q_{actual} = \frac{c_d \cdot A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$\boxed{C_d = C_c . C_v}$$

$$C_c = \text{coefficient of contraction} = \frac{A_C}{A_2}$$

Pitot Tube: (Measures stagnation pressure head) (for incompressible flow)



$$u_1 = \sqrt{2g\, h_{dynamic}}$$

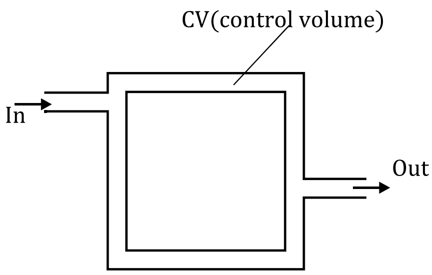
$$h_{dynamic} = x\left(\frac{\rho_m}{\rho} - 1\right)$$

x = Manometric deflection

(or)

$$h = \left(\frac{p_2}{\rho g} + Z_2\right) - \left(\frac{p_1}{\rho g} + Z_1\right)$$

Impulse Momentum Equation:

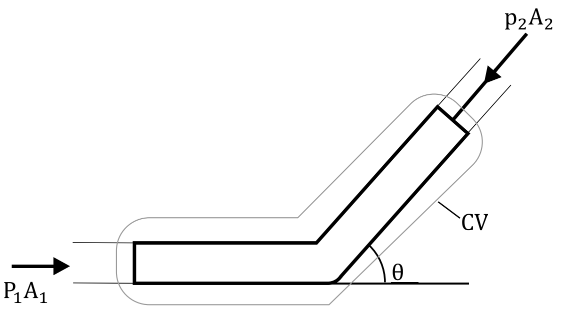


$$\vec{F}_{net} + \sum \dot{m}_i \vec{V}_i = \left(\frac{d(mV)}{dt}\right)_{cv} + \sum \dot{m}_o \vec{V}_o$$

$$f_{net} = \dot{m}_o V_o - \dot{m}_i V_i$$

V_i = inlet velocity of fluid, V_o = exit velocity of fluid

If pressure forces considered, then



$F_x + p_1A_1 - p_2A_2\cos\theta = \dot{m}[V_2\cos\theta - V_1]$

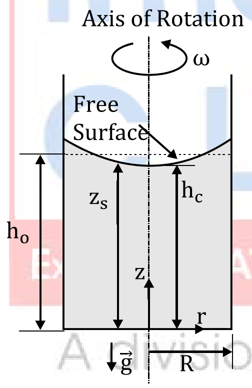
$F_y - p_2A_2\sin\theta = \dot{m}[V_2\sin\theta]$

f_{net} or $F_x, F_y \rightarrow$ Force exerted by pipe bend or structure on fluid element.

Rotational flow in a Cylindrical Container:

Equation of motion for vortex flow

$dp = \rho r\omega^2 dr - \rho g\, dz$

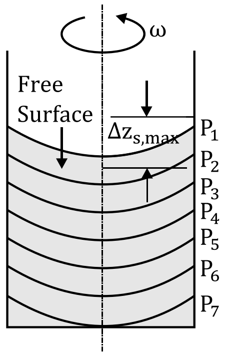


the equation of the free surface

$$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

The maximum vertical height occurs at the edge where r = R, and the maximum height difference between the edge and the center of the free surface is

$$\Delta z_{s,max} = z_s(R) - z_s(0) = \frac{\omega^2}{2g}R^2$$



where h_0 is the original height of the fluid in the container with no rotation

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FLOW THROUGH PIPES

Darcy Weisbech Equation:

Head loss due to friction (h_f)

$h_f = \frac{FLV^2}{2gd}$ (Valid for Laminar and Turbulent)

$F = \frac{64}{Re}$ for Laminar flow

Re = Reynolds Number $= \frac{\rho V D_H}{\mu}$

D_H = Hydraulic diameter $= \frac{4\,A}{P_{wetted}}$

A = cross sectional area

P_{wetted} = wetted parameter

Laminar Flow:

1. **Through Pipe:**

(A) $\tau = -\frac{r}{2}\left(\frac{\partial p}{\partial x}\right)$

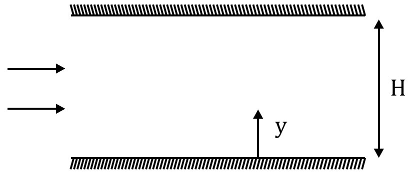
(B) $u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)[R^2 - r^2]$

(C) $u_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2)$

(D) $u_{avg} = \frac{u_{max}}{2}$

2. **Laminar Flow Through Two Fixed Parallel Plates:**

Case A:



$$\tau = -\frac{\partial p}{\partial x}\left(\frac{H}{2} - y\right)$$

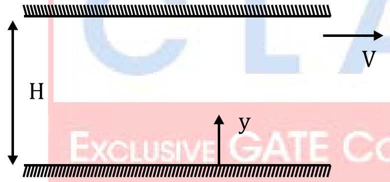
$$\tau_{wall} = -\frac{\partial p}{\partial x}\left(\frac{H}{2}\right)$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(Hy - y^2)$$

$$u_{max} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)(H^2)$$

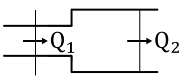
$$u_{avg} = \frac{2\,u_{max}}{3}$$

Case B: (couette flow)



$$u = -\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)[Hy - y^2] + \frac{Vy}{H}$$

Pipe in series



$Q = Q_1 = Q_2$

$h_f = h_{f_1} + h_{f_2}$

h_f = Head loss due to friction

Pipes in Parallel:

$Q = Q_1 + Q_2$

$\boxed{h_{f_1} = h_{f_2}}$

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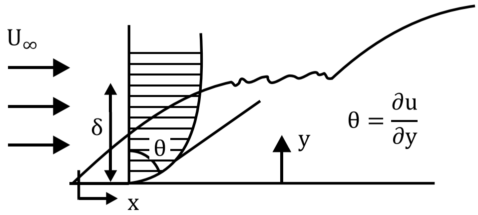
BOUNDARY LAYER THEORY

Boundary layer region is highly viscous and rotational region of flow⇒ so Bernoulli's equation not applicable.



Boundary Condition:

at $x = 0, \delta = 0$

at $y = 0, u = 0$

at $y = \delta, \frac{\partial u}{\partial y} = 0$

at $y = 0, \frac{\partial^2 u}{\partial y^2} = 0$

⇒ **δ^*[Displacement thickness]**

$$= \int_0^{\delta}\left(1 - \frac{U}{U_\infty}\right)dy$$

Loss pf mass flow rate $= (\rho\delta^* U_\infty . b)$

b = Width of plate

Momentum Thickness [θ]

$$\theta = \int_0^{\delta}\frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right)dy$$

$F_{drag} = \rho A U_\infty^2 \Rightarrow \rho(\theta \times b) \times U_\infty^2$

Energy Thickness [δ^{**}]

$$= \int_0^{\infty}\frac{U}{U_\infty}\left(1 - \left(\frac{U}{U_\infty}\right)^2\right)dy$$

Shape Factor $= \frac{\delta^*}{\theta}$

$\delta > \delta^* > \delta^{**} > \theta$ valid for all velocity profile.

Von Karman Momentum Integral Equation.

$$\frac{\tau_o}{\rho U_\infty^2} = \frac{d\theta}{dx}$$ Valid for laminar and turbulent

Used when velocity profile given in the problem.

$$T_o = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Blasius Equation: (Used when velocity profile is not given)

Laminar Flow:

$(Re < 5 \times 10^5)$

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$C_D = \frac{1.328}{\sqrt{Re_L}}$$

$$C_{f,x} = \frac{0.664}{\sqrt{Re_x}}$$

Re = Reynold's Number

C_D = Average drag coefficient.

$C_{f,x}$ = Local Drag Coefficient.

$$\delta^* = \frac{1.72(x)}{\sqrt{Re_x}}$$

$$\theta = \frac{0.664(x)}{\sqrt{Re_x}}$$

Turbulent Flow: (Used when 1/7th law given in problem) $\frac{U}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$

$$\delta = \frac{0.16(x)}{(Re)^{\frac{1}{7}}}$$

$$\delta^* = \frac{0.02(x)}{(Re)^{\frac{1}{7}}}$$

$$\theta = \frac{0.016(x)}{(Re)^{\frac{1}{7}}}$$

Boundary Layer Separation:

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0 \text{ and } \frac{\partial p}{\partial x} > 0$$

*****

DIMENSIONAL ANALYSIS

Dynamic Similarity:

$$\frac{F_{inertia})_m}{F_{inertia})_P} = \frac{F_{viscous})_m}{F_{viscous})_P}$$

$$\left.\frac{F_i}{F_v}\right)_m = \left.\frac{F_i}{F_v}\right)_P$$

$Re)_m = Re)_P$

Reynolds number for model and prototype shall be same.

$F_{inertia} = \rho L^2 V^2$

$F_{viscous} = \mu VL,$

where μ = Viscosity

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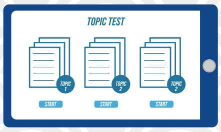

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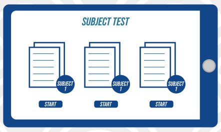

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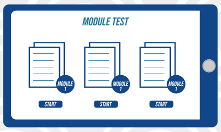

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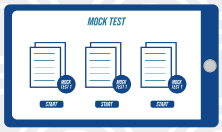

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