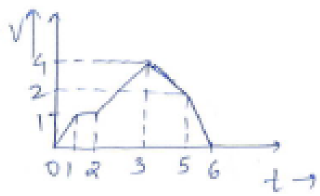
GATE-2017, Mechanics Solution, Set-I.

Q.3) The



distance covered by particle from 0 to 5 sec. ?

Area under the curve from t=0 to t=5 sec

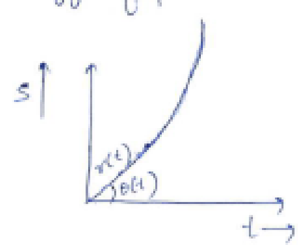
$S = A_1 + A_2 + A_3 + A_4$

$= \frac{1}{2} \times 1 \times 1 + 1 \times 1 + \left(\frac{1+4}{2}\right) \times 1 + \left(\frac{4+2}{2}\right) \times 2$

$= 10$ m (Ans)

Q.8) $r(t) = t^2$, $\theta(t) = t$. The Kinetic Energy of particle at time $t = 2$ is ?

mass = 1 kg.



Ans: Particle is having Linear motion & Angular motion both.

∴ Velocity = Linear velocity in the direction of r. + Angular velocity in some other direction $\hat{t}$.

$\vec{V} = r\omega(\hat{t}) + \frac{dr}{dt}\hat{r}$

$\vec{V} = r\frac{d\theta}{dt}\hat{t} + \frac{dr}{dt}\hat{r} = t^2 \times 1\,\hat{t} + 2t\,\hat{r}$

$|\vec{V}| = \sqrt{(t^2)^2 + (2t)^2}$

$|\vec{V}|_{t=2} = \sqrt{16+16} = 4\sqrt{2}$ m/sec.

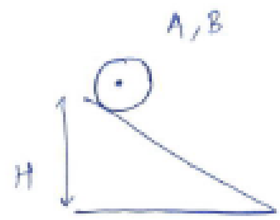
$K.E = \frac{1}{2} m v^2$

$K.E = \frac{1}{2} \times 1 \times 32 = 16$ (Ans)

Q.48.

$I_A = MR^2$

$I_B = \frac{1}{2} MR^2$



Rolling with slipping case.

By energy Conservation,

$$(K.E_T + K.E_R)_A = (K.E_T + K.E_R)_B = P.E.$$

$$\frac{1}{2} M V_A^2 + \frac{1}{2} M (R\omega_A)^2 = \frac{1}{2} M V_B^2 + \frac{1}{2} \frac{MR^2}{2} \omega_B^2 = MgH.$$

$$V_A^2 + V_A^2 = V_B^2 + \frac{V_B^2}{2}$$

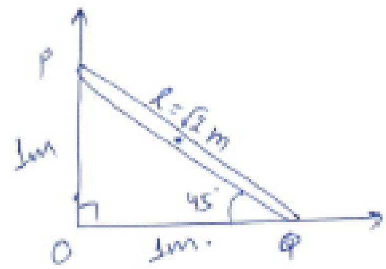
$$\frac{V_A}{V_B} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \text{ (Ans)}$$

GATE-2017, Mechanics Solution, Set-2

Q.30 Rod of Mass = 10 kg.

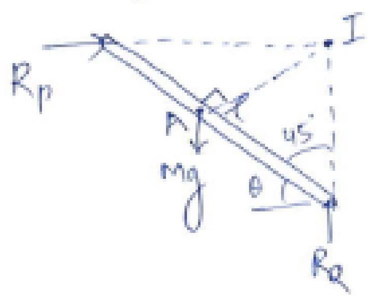
$l = \sqrt{2}$ m.

$g = 10\ m/s^2$



Magnitude of Angular Acceleration, = ?

F.B.D of Rod



By Newton's 2nd law,

Net torque about I $= I\alpha$.

$\tan 45° = \frac{AI}{AQ}$

$\Rightarrow AI = AQ \tan 45° = \frac{l}{2} \times 1 = \frac{1}{\sqrt{2}}$ m.

Moment of Inertia (I) about I

$$I_I = \frac{Ml^2}{12} + M \times (AI)^2 \quad \text{(By Parallel Axis theorem).}$$

$$I_I = 10 \times \frac{2}{12_6} + 10 \times \frac{1}{2} = M\left(\frac{1}{6} + \frac{1}{2}\right) = M \times \frac{2}{3}$$

$\therefore$ Net torque about I, $= Mg \times 0.5 = \frac{2M}{3} \times \alpha$

$$\frac{g}{2} = \frac{2}{3}\alpha.$$

$$\alpha = \frac{3}{4}g = \frac{3}{4} \times 10 = 7.5\ rad/s^2.$$