[ME - SESSION 1][2017],

Q-6 Correct option is B.

As average velocity always remains Constant.

Q-7 $\vec{V} = (5 + a_1 x + b_1 y)\hat{i} + (4 + a_2 x + b_2 y)\hat{j}$

For flow to be incompressible the velocity field has to satisfy the continuity equation for Incompressible flow.

Here they have 2D velocity field so it has to satisfy 2-D Incompressible flow Continuity eqn, and it is given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Given, $u = 5 + a_1 x + b_1 y$, $v = 4 + a_2 x + b_2 y$ after comparing the given velocity vector with $\vec{V} = u\hat{i} + v\hat{j}$

So

$$\frac{\partial}{\partial x}(5+a_1x+b_1y)+\frac{\partial}{\partial y}(4+a_2x+b_2y)=0$$

$$a_1 \quad + \quad b_2 \quad = 0$$

or $\boxed{a_1+b_2=0}$ — Correct option is $\boxed{B}$

Q-8 Given, $\dot{m} = 1$ kg/s

When head loss is h_f than power required to over this head loss is given as

$$P_w = \rho g Q h_f$$

where $\rho Q = \dot{m}$ = mass flow rate $\left[\frac{kg}{m^3}\,\frac{m^3}{s} = \frac{kg}{s}\right]$

and $h_f = \frac{\Delta P}{\rho g} = \frac{\text{pressure drop}}{\rho g}$

So $P_w = \dot{m}\,g\cdot\frac{\Delta P}{\rho g} = 1\cdot\frac{100\times10^3}{1000} = 100$

So required $\boxed{P_w = 100 \text{ Watt}}$ Answer

Q-9 Jet pump is NOT a rotating machine so correct option is C.

Q-30 Displacement thickness

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$\delta^* = \int_0^\delta 1\, dy - \int_0^\delta \frac{u}{U_\infty} dy \quad \text{—— (A)}$$

Given that $\frac{u}{U_\infty} = \sin\left(\frac{\pi y}{2\delta}\right)$

Substituting it in above eq^n (A) and performing the integration

$$\Rightarrow \delta^* = [y]_0^\delta - \int_0^\delta \sin\left(\frac{\pi y}{2\delta}\right) dy$$

$$\Rightarrow \delta^* = \delta - \frac{2\delta}{\pi}\left[-\cos\frac{\pi y}{2\delta}\right]_0^\delta$$

$\because \int \sin x\, dx = -\cos x + C$

In definite integral we donot add constant.

$$\delta^* = \delta - \frac{2\delta}{\pi}\left[-\left(\cos\frac{\pi\delta}{2\delta} - \cos 0\right)\right]$$

$$\delta^* = \delta - \frac{2\delta}{\pi} \quad \text{so} \quad \boxed{\delta^*/\delta = 1 - \frac{2}{\pi}} \text{ — } \boxed{B}$$

Answer

Q-31 The head loss due to friction is given by Darcy-Weisbach equation

$$h_f = \frac{f L V_{avg}^2}{2gD} = \frac{\text{frictional pressure drop}}{\rho g}$$

V_{avg} — average velocity
D — Diameter of pipe
f — friction factor
L — Length of the pipe
ρ — density of the flowing fluid
g — acceleration due to gravity
h_f — head loss due to friction

or $$h_{f1} = \frac{f_1 L_1 V_{1avg}^2}{2g D_1} = \frac{\Delta P_1}{\rho g}$$

or $$\Delta P_1 = \frac{\rho f_1 L_1 V_{1avg}^2}{2 D_1}$$

Similarly

$$\Delta P_2 = \frac{\rho f_2 L_2 V_{2avg}^2}{2 D_2}$$

Now
$$\frac{\Delta P_1}{\Delta P_2} = \frac{\dfrac{\rho f_1 L_1 V_{1avg}^2}{2D_1}}{\dfrac{\rho f_2 L_2 V_{2avg}^2}{2 D_2}}$$

or
$$\frac{\Delta P_1}{\Delta P_2} = \frac{f_1 V_{1avg}^2 \cdot D_2}{f_2 V_{2avg}^2 \cdot D_1} \quad \because \text{ Since } L_1 = L_2$$

Now
$$\frac{f_1}{f_2} = \frac{K\left[\dfrac{\rho V_{1avg} D_1}{\mu}\right]^{-n}}{K\left[\dfrac{\rho V_{2avg} D_2}{\mu}\right]^{-n}} \quad \because \text{ Since } Re = \frac{\rho V D}{\mu}$$

$$\frac{f_1}{f_2} = \frac{V_{1avg}^{-n} D_1^{-n}}{V_{2avg}^{-n} D_2^{-n}}$$

So
$$\frac{\Delta P_1}{\Delta P_2} = \frac{V_{1avg}^{-n} D_1^{-n} \cdot D_2 \cdot V_{1avg}^2}{V_{2avg}^{-n} D_2^{-n} \cdot D_1 \cdot V_{2avg}^2}$$

$$\frac{\Delta P_1}{\Delta P_2} = \frac{V_{1avg}^{2-n} D_2^{n+1}}{V_{2avg}^{2-n} D_1^{n+1}} \quad \text{—— (A)}$$

As we know that volume flow rate will remain constant, since the flow is incompressible and pipes are connected in series.

Volume flow rate is given as

$Q = A_1 V_1 = A_2 V_2 \; (m^3/s)$ Here V_1 and V_2 are average velocities.

or $A_1 V_{1avg} = A_2 V_{2avg}$ $\because A_1 = \frac{\pi}{4} D_1^2$ — Cross Section Area

and $A_2 = \frac{\pi}{4} D_2^2$

So $V_{1avg} = \frac{A_2}{A_1} V_{2avg} = V_{2avg} \frac{D_2^2}{D_1^2}$

Substituting this V_{1avg} in eqn (A)

$$\frac{\Delta P_1}{\Delta P_2} = \frac{V_{1avg}^{2-n} \, D_2^{n+1}}{V_{2avg}^{2-n} \, D_1^{n+1}} = \frac{\left[V_{2avg}\left(\frac{D_2}{D_1}\right)^2\right]^{2-n} D_2^{n+1}}{V_{2avg}^{2-n} \, D_1^{n+1}}$$

$$\frac{\Delta P_1}{\Delta P_2} = \frac{V_{2avg}^{2-n} \, D_2^{4-2n} \, D_2^{n+1}}{V_{2avg}^{2-n} \, D_1^{4-2n} \cdot D_1^{n+1}} = \frac{D_2^{4-2n+n+1}}{D_1^{4-2n+n+1}}$$

$$\frac{\Delta P_1}{\Delta P_2} = \frac{D_2^{5-n}}{D_1^{5-n}} \quad \text{or} \quad \frac{\Delta P_1}{\Delta P_2} = \left(\frac{D_2}{D_1}\right)^{5-n}$$

option A Answer

Q-32 Given, $\vec{V} = (-x^2+3y)\hat{i} + 2xy\hat{j}$

At $(1,-1)$ $|\vec{a}| = ?$

We know $\vec{a} = a_x\hat{i} + a_y\hat{j}$ (2-D flow)

and $|\vec{a}|$ = magnitude of acceleration

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

and a_x — acceleration in x-dirn is given as

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

Similarly in y-dirn acceleration is given as

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

Time derivative $\frac{\partial u}{\partial t}$ and $\frac{\partial v}{\partial t}$ will be zero since it is steady flow.

So $a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$

and $a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$ for the given scenario

and $u = -x^2+3y$, $v = 2xy$ comparing the velocity vector $\vec{V}$ with $\vec{V} = u\hat{i} + v\hat{j}$ (2-D flow)

So $a_x = (-x^2+3y)\frac{\partial}{\partial x}(-x^2+3y) + 2xy\frac{\partial}{\partial y}[-x^2+3y]$

$a_x = (-x^2+3y)[-2x] + 2xy[3]$

$a_x = [-1^2-3][-2] + 2[1][-1][3]$ at $(1,-1)$

$a_x = 8-6 = 2$ unit

Similarly

$a_y = [-x^2+3y]\frac{\partial}{\partial x}[2xy] + [2xy]\frac{\partial}{\partial y}[2xy]$

$a_y = [-x^2+3y][2y] + [2xy][2x]$

$a_y = [-1^2-3][-2] + [2(1)(-1)][2(1)]$ at $(1,-1)$

$a_y = 8-4 = 4$ unit

So acceleration vector

$\vec{a} = 2\hat{i} + 4\hat{j}$

So $|\vec{a}| = \sqrt{2^2+4^2} = \sqrt{20} = \sqrt{4\times5} = 2\sqrt{5}$

So magnitude of the acceleration

$\boxed{|\vec{a}| = 2\sqrt{5}}$ — option C is correct Answer