IITians GATE CLASSES
Exclusive GATE Coaching by IIT/IISc Graduates
A division of PhIE Learning Center

QUICK REVISION

FORMULA SHEET

for

GATE -ME HEAT TRANSFER

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OUR COURSES

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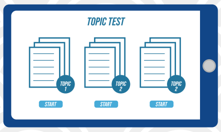

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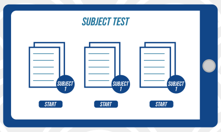

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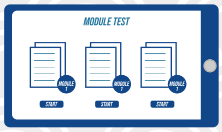

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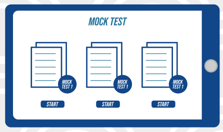

Complete Syllabus Tests

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https://bit.ly/3Wu7lsp

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HEAT TRANSFER

Chapter 1: CONDUCTION

Conduction depends on the medium.

Conduction can happen in solids, liquids and gases.

Solid:

In solid conduction happens due to

1. Free electron
2. Lattice Crystal vibration

Liquid:

In Liquid Conduction happens due to elastic collision.

Gases:

In gases conduction happens due to molecular momentum transfer during elastic collision.

Fourier Law of Heat Conduction:

Heat flux directly proportional to temperature gradient.

$$\dot{q} = -k\left(\frac{\partial T}{\partial x}\right) = \frac{\dot{Q}}{A}$$

Where A = Area normal to heat flow.

k = Thermal conductivity of material.

$k_{gas} < k_{liquid} < k_{solid}$

$$k_{\text{non metallic Crystal (diamond)}} > k_{\text{pure metal}} > k_{\text{alloys}} > k_{\text{non metallic solids}}$$

3D Generalized Conduction Equation:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) + \dot{Q}_g = \rho C \frac{\partial T}{\partial t}$$

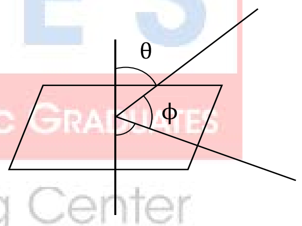
Cylindrical Coordinate:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{Q}_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where α = Thermal diffusivity

$$= \frac{k}{\rho C_p} (m^2/sec)$$

Spherical Coordinate:



θ = zenith angle

ϕ = Azimuthal Angle

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{\dot{Q}_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$\nabla^2 T = 0 \rightarrow$ Laplace equation

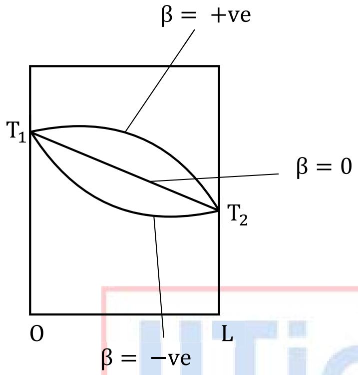
$\nabla^2 T + \frac{\dot{Q}_g}{k} = 0 \rightarrow$ Poission equation

$\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} \rightarrow$ Diffusion equation

$$\dot{Q}_g = \frac{\text{total internal heat generation rate}}{\text{volume}}$$

$$\frac{\partial^2 T}{\partial x^2} = 0 \rightarrow \text{Fourier equation}$$

1D Steady State Conduction Heat Transfer:



Where $k_m = k_o\left(1 \pm \beta\left(\frac{T_1 + T_2}{2}\right)\right)$

$$Q = k_o A\left(1 \pm \beta\left(\frac{T_1 + T_2}{2}\right)\right)\left(\frac{T_1 - T_2}{L}\right)$$

Heat Generation in Slab:

Case 1:

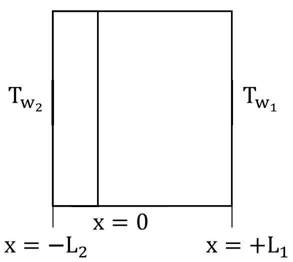
- **Symmetric**

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{Q}_g}{k} = 0$$

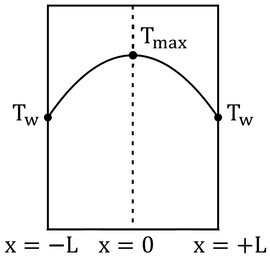
$$T - T_w = \frac{\dot{Q}_g L^2}{2k}\left(1 - \left(\frac{x}{L}\right)^2\right)$$

(Parabolic)

- **Asymmetric**



$$T_{max} - T_w = \frac{\dot{Q}_g L^2}{2k}$$



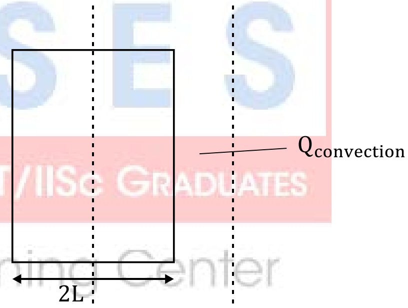
$$Q_{x=L} = \left[\frac{1}{2}\,\dot{Q}_{Total}\right]$$

$$T - T_{w_1} = \frac{\dot{Q}_g}{2k}(L_1^2)\left(1 - \left(\frac{x}{L_1}\right)^2\right)$$

$$T - T_{w_2} = \frac{\dot{Q}_g}{2k}L_2^2\left(1 - \left(\frac{x}{L_2}\right)^2\right)$$

$$T_{max} - T_{w_1} = \frac{\dot{Q}_g L_1^2}{2k}$$

Case 2: Special Case



T_w =Wall Temperature.

$$T_{max} = T_w + \frac{\dot{Q}_g L^2}{2k}$$

$$\text{at}\,\frac{s}{f}, x = L, \frac{1}{2}\dot{Q}_g(A_c \times 2L) = hA_s(T_w - T_\infty)$$

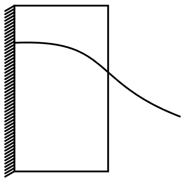
$$T_w = T_\infty + \frac{\dot{Q}_g L}{\bar{h}}$$

$$T_{max} = T_\infty + \frac{\dot{Q}_g L}{\bar{h}} + \frac{\dot{Q}_g L^2}{2k}$$

T_{max} = Temp at centre of wall

$\bar{h}$ = Convective heat transfer coefficient

Case 3:



When one side is insulted

Then

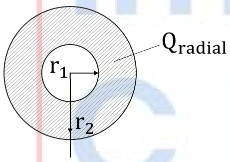
$Q_{convection} = 0$

$q_{conduction} = 0 \Rightarrow \left(\frac{dT}{dx}\right)_{side} = 0$

T = T_{max} at insulated surface

Heat Conduction in Hollow Cylinder:

$$Q_{radial} = \frac{2\pi kL(T_1 - T_2)}{\ln(r_2/r_1)}$$



Replace k with k_m when variable k is given.

Newton's Law of Cooling:

$Q = hA\,(T_w - T_\infty)$

In General

$$Q = \frac{\Delta T}{\sum R_{th}}$$

R = Resistance

Stefan Boltzman Law of Radiation:

$E_b \propto T^4$

$E_b = \sigma T^4$

σ = Stefan Boltzman constant

$= 5.67 \times 10^{-8}\ W/m^2K^4$

(R) Resistance	Slab	Cylinder	Sphere
Conduction	$\frac{L}{kA}$	$\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$	$\frac{r_2 - r_1}{4\pi k\, r_1 r_2}$
Convection	$\frac{1}{hA}$	$\frac{1}{hA}$	$\frac{1}{hA}$

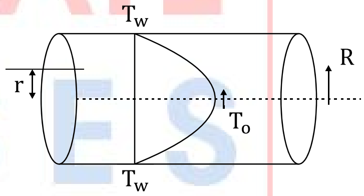
Overall Heat Transfer Coefficient (U)

$$\frac{1}{U} = \sum R_{th} \cdot A$$

$$\frac{1}{U_i} = \sum Rth \times A_i$$

$$\frac{1}{U_o} = \sum R_{th} \times A_o$$

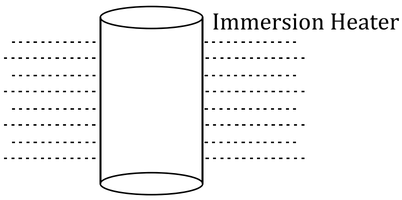
Heat Generation in Cylinder



$$T - T_w = \frac{\dot{Q}_g}{4k} R^2 \left(1 - \frac{r}{R}\right)^2$$

$$T_{max} - T_w = \frac{\dot{Q}_g}{4k} R^2 \rightarrow \text{equation 1}$$

Special Case:



$\dot{Q}_g \times \text{volume} = \bar{h}A_s(T_w - T_\infty)$

$\dot{Q}_g \times \pi R^2 L = \bar{h}\,(2\pi RL)(T_w - T_\infty)$

$$T_w = \frac{\dot{Q}_g}{2\bar{h}} R + T_\infty$$

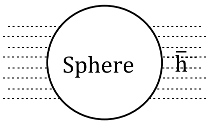
From equation 1

$$T_{max} = T_\infty + \frac{\dot{Q}_g R}{2\bar{h}} + \frac{\dot{Q}_g R^2}{4k}$$

Heat Generation is Sphere:

$$T - T_w = \frac{\dot{Q}_g}{6k} R^2 \left(1 - \left(\frac{r}{R}\right)^2\right)$$

Special Case:



$$\dot{Q}_g \times \frac{4}{3}\pi R^3 = \bar{h} \times 4\pi R^2 (T_w - T_\infty)$$

$$T_w = T_\infty + \frac{\dot{Q}_g R}{3\bar{h}}$$

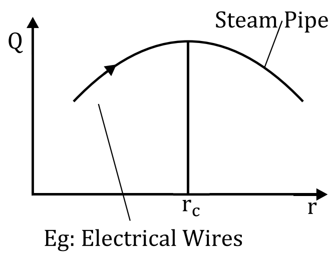
$$T_{max} = T_\infty + \frac{\dot{Q}_g R}{3\bar{h}} + \frac{\dot{Q}_g R^2}{6\, k_{solid}}$$

Critical Radius of insulation

Cylinder $r_c = \frac{k}{h}$

Sphere $r_c = \frac{2k}{h}$

k = Thermal conductivity of insulting material.



Slab (No r_c)

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Chapter 2: FINS

$$\theta = T - T_\infty = C_1 e^{-mx} + C_2 e^{-mx}$$

$$m = \sqrt{\frac{hP}{kA}}$$

P = Perimeter of fin

A = cross-section area

$l_c = l + \frac{t}{2} \rightarrow$ Rectangular Fin

$l_c = l + \frac{d}{4} \rightarrow$ Pin fin

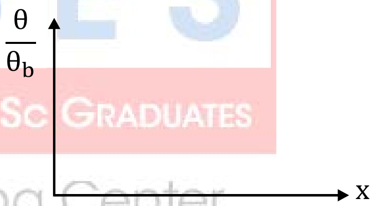
Characteristics

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty}$$

(q fin) at x = 0

$$\eta = \frac{q_{actual}}{q_{max}}$$

$$\epsilon = \frac{q_{with\ fin}}{q_{without\ fin}}$$



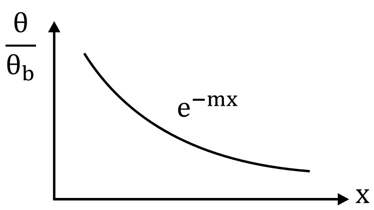
Infinite Long Fin

e^{-mx}

$\sqrt{hPkA}\ \theta_b$

$\frac{1}{ml}$

$\sqrt{\frac{Pk}{hA}}$



B.C

1. $x = 0, \theta = \theta_b$

2. $x = \infty, \theta = 0$

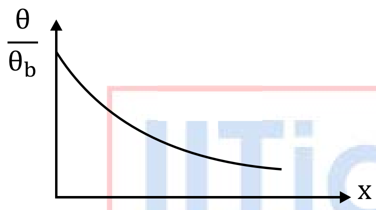
Fin: (Finite length and Insulated tip)

$$\frac{\cos h\left(m\,(l - x)\right)}{\cos h\,(ml)}$$

$$\sqrt{hPkA}(\theta_b)\tan h(ml)$$

$$\frac{\tan hml}{mL}$$

$$\sqrt{\frac{Pk}{hA}} \times \tan h(ml)$$



1. At $x = 0, T = T_b, \theta = \theta_b$
2. At $x = L, -kA\left(\frac{\partial T}{\partial x}\right)_{x=L} = 0$

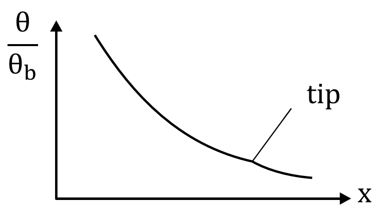
Fin: (Finite length and non Insulated tip)

$$\frac{\cos h\left(m\,(l_c - x)\right)}{\cos h(ml_c)}$$

$$\sqrt{hPkA}\,(\theta_b)\tan h(ml_c)$$

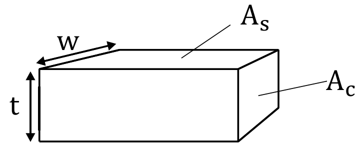
$$\frac{\tan h\,(ml_c)}{(ml_c)}$$

$$\sqrt{\frac{Pk}{hA}} = \tan h(ml_c)$$



If $m_L \leq 2.65 \rightarrow$ Short Fin

$(\epsilon_{min})_{fin} \geq 2\,, \epsilon =$ Effectiveness



$$\epsilon = \frac{\eta_{fin} A_s}{A_c}$$

$A_s = P \times l$

$A_c = w \times t$

Unsteady or Transient Conduction heat Transfer:

$$hA(T - T_\infty) = -mC_p\frac{dT}{dt}$$

$\Rightarrow$ to be used when rate of cooling is asked

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s\tau}{\rho v C_p}}$$

$T_i =$ Initial Temperature

$\tau =$ time

$v =$ volume

$$\frac{\rho v C_p}{hA_s} = t^* \rightarrow \text{Time constant (TC)}$$

Note: Reading on thermocouple is taken at 3(TC).

Lumped Heat Analysis:

This is used when $Bi < 0.1$

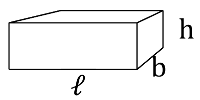
$$\text{Biot No (Bi)} = \frac{hL_c}{k_{solid}} = \frac{\text{Internal conductive resistance}}{\text{External convective resistance}}$$

$$L_c = \frac{\text{Volume}}{\text{Surface Area}}$$

$k_{solid} =$ Thermal conductivity of object.

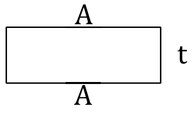
1. **Slab:**

   A.



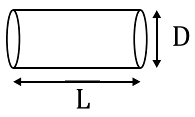
$$L_c = \frac{lbh}{2(\ell b + hb + \ell h)}$$

B.



$$L_c = \frac{At}{2A} = \frac{t}{2}$$

2. **Cylinder**



$$L_c = \frac{\pi R^2 L}{2\pi RL + 2\pi R^2}$$

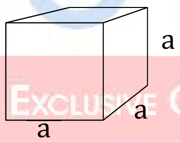
$$L_c = \frac{R}{2}; \text{when } L >> R$$

3. **Sphere**

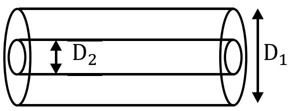
$$L_c = \frac{R}{3}$$

4. **Cube**

$$\frac{V}{SA} = \frac{a}{6}$$



5. **Hollow Cylinder:**

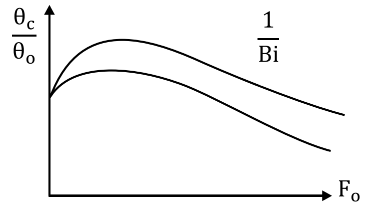


$$L_c = \frac{R_2 - R_1}{2}$$

Semi-infinite solid ($Bi \to \infty$)

When $Bi > 0.1$, then Heisler charts must be used.

Heisler Chart:

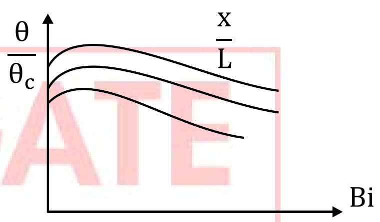


F_o = Fourier number

So $Bi > 0.1$ then

$$\frac{T - T_\infty}{T_o - T_\infty} = \begin{pmatrix}\text{Grober} \\ \text{chart} \\ \text{solution}\end{pmatrix} \times \begin{pmatrix}\text{Heisler} \\ \text{chart} \\ \text{solution}\end{pmatrix}$$

Grober Chart:



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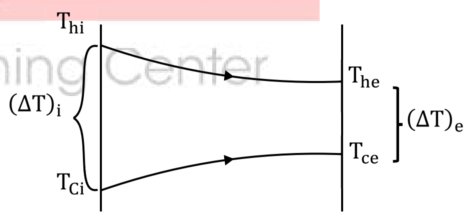
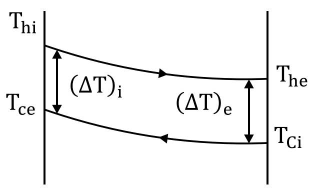
Chapter 3: HEAT EXCHANGERS

Parallel Flow Heat Exchanger



Counter Flow Heat Exchanger



T_{hi} = Hot fluid inlet temperature

T_{he} = Hot fluid exit temperature

T_{ci} = Cold fluid inlet temperature

T_{ce} = Cold fluid exit temperature

$$\text{LMTD} = \Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

Note:

If $T_{ce} > T_{he}$

Then it is (CFHE)

Case 1:

If $Q =$ same $(T_{hi}, T_{he}, T_{ci}, T_{ce}$ =same for both parallel flow and counter flow.

$$(\text{LMTD})_{\text{Counter flow}} > (\text{LMTD})_{\text{Parallel flow}}$$

$(As)_{CF} < (As)_{PF}$

As = Surface area of heat exchanger

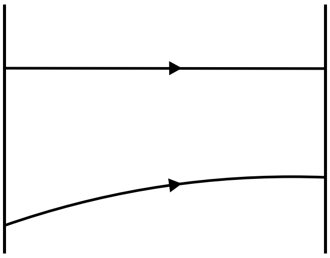
Case 2:

As → same

$Q_{PF} < Q_{CF}$

Case 3:

When one of the fluid is changing its phase.



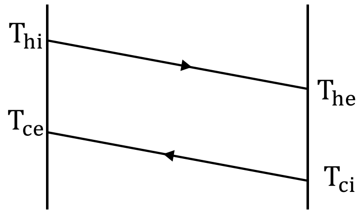
$(\text{LMTD})_{PF} = (\text{LMTD})_{CF}$

Case 4:

When both hot and cold fluid have equal eat capacity $\left(\dot{m}_h C_{ph} = \dot{m}_c C_{pc}\right)$

$(\text{LMTD})_{CF} = \Delta T_i$ or ΔT_e

$$\frac{dT_h}{dx} = \frac{dT_c}{dx} = \text{constant}$$



(Balanced Count Flow Heat Exchanger)

Heat transferred ⇒ $Q = UA(\Delta T_m)$

U = Overall heat transfer coefficient

$$\frac{1}{U} = \frac{1}{h_1} + F_1 + \frac{1}{h_2} + F_2$$

where F_1 and F_2 = Fouling factor

$$F = \frac{1}{U_{dirt}} - \frac{1}{U_{clean}}$$

$$\frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln(r_2/r_1)}{2\pi kL} + \frac{R_{f_o}}{A_o} + \frac{1}{h_o A_o}$$

$Q = U_i A_i \text{LMTD}$ or $U_o A_o$ LMTD

$A = \pi dl \times n \times p$ (p = No. of passes)

n = No. of tubes / pass

$$\dot{m} = \rho \times \frac{\pi}{4} d^2 \times n \times \text{velocity}$$

F ∝ Temperature

$$F \propto \frac{1}{\text{Velocity of flow}}$$

Effectiveness of Heat Exchanger (ϵ)

$$\epsilon = \frac{q_{actual}}{\dot{q}_{max\ possible}}$$

$$\dot{q}_{max} = \left(\dot{m}C_p\right)_{smaller} (\Delta T)_{max}$$

$$\Delta T_{max} = T_{h_i} - T_{c_i}$$

When $\dot{m}_h C_h < \dot{m}_c C_c$

$$\epsilon = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

When $\dot{m}_h C_h > \dot{m}_c C_c$

$$\epsilon = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$

Number of Transfer Units (NTU)

$$NTU = \frac{UA}{(\dot{m}C_p)_{smaller}}$$

Where U is in W/m^2K

A in m^2

$\dot{m}$ in kg/sec

C_p in J/kg K

$$\text{Capacity ratio (C)} = \frac{(\dot{m}C_p)_{smaller}}{(\dot{m}C_p)_{larger}}$$

$0 \le C \le 1$

$\epsilon = f(NTU, C)$

$$\epsilon_{PF} = \frac{1 - e^{-NTU(1+C)}}{1 + c} \Bigg\} \text{Parallel flow}$$

$$\epsilon_{CF} = \frac{1 - e^{-NTU(1-C)}}{1 - ce^{-NTU(1-C)}} \Bigg\} \text{Counter flow}$$

Case 1:

When C = 0

$\epsilon_{PF} = \epsilon_{CF} = \epsilon = 1 - e^{-NTU}$

When LMTD = same

Area same

NTU same

Case 2:

$\dot{m}_h C_h = \dot{m}_c C_c (C = 1)$

$$\epsilon_{PF} = \frac{1 - e^{-2NTU}}{2}$$

$$\epsilon_{CF} = \frac{NTU}{1 + NTU}$$

*****

Chapter 4: RADIATION

E = Total Hemispherical Emissive Power.

$$E = \int_0^\infty E_\lambda d\lambda = \frac{Watt}{m^2}$$

= Area under E_λ and $d\lambda$ graph

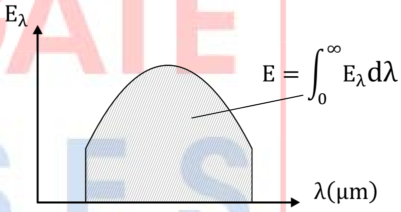
E_λ = Monochromatic/Spectral Hemispherical Emissive Power

E_λ is in $\frac{Watt}{m^2 - \mu m}$

ϵ = Total Emissivity

ϵ_λ = Monochromatic/Spectral Emissivity

$$\epsilon = \frac{E}{E_b}, \epsilon_\lambda = \frac{E_\lambda}{E_{b,\lambda}}$$

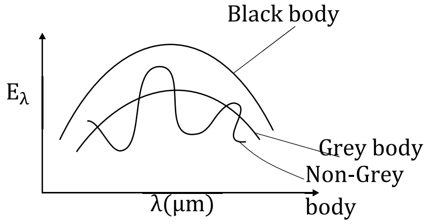


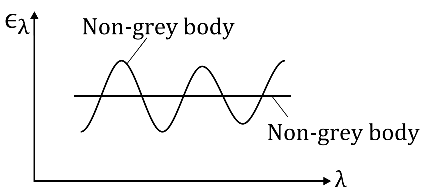
ϵ_{avg} calculation:

$$\epsilon = \frac{E}{E_b} = \frac{\int_0^\infty E_\lambda d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} = \frac{\int_0^\infty (\epsilon_\lambda E_{b\lambda}) d\lambda}{\int_0^\infty E_{b\lambda} d\lambda}$$

Grey body = $\epsilon_\lambda \neq fn(\lambda)$

So for grey body $\epsilon = \epsilon_\lambda$ = constant





$\alpha =$ Absorptivity $\Rightarrow$ Fraction of Incident radiation absorbed.

$\rho =$ Reflectivity $\Rightarrow$ Fraction of incident radiation reflected.

$\tau =$ Transmissivity $\Rightarrow$ Fraction of incident radiation transmitted.

$\alpha + \tau + \rho = 1$

For black body $\alpha_b = 1, \epsilon_b = 1$

Law of Radiation:

1. **Kirchoff's Law:**

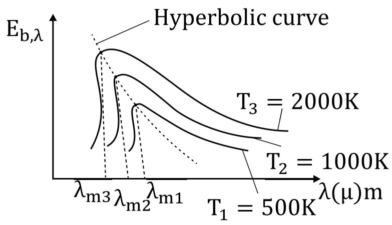
   Whenever a body is in thermal equilibrium with its surrounding then its emissivity is equal to absorptivity $\boxed{\alpha = \epsilon}$

2. **Plank's Law:**

   This law states that monochromatic or spectral emissive power of black body dependent on both absolute temperature of black body and wave length of energy emitted from body.

   $E_{b,\lambda} = f(\lambda, T)$

   $$E_{b,\lambda} = \frac{2\pi C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} \quad \frac{\text{Watt}}{m^2 \mu m}$$



By Increase in temperature, the wavelength of $(E_{b,\lambda})_{max}$ shifted to left

3. **Wien's Displacement Law:**

   $\lambda_{max} \cdot T = 2898$ (in μm K)

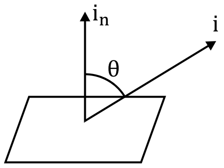
4. **Stefan-Boltzmann Law:**

   $E_b \propto T^4$

   $E_b = \sigma T^4 \ W/m^2$

   Where $\sigma = 5.67 \times 10^{-8} \ W/m^2K^4$

5. **Lambert's Cosine Law:**



$i = i_n \cos\theta$

$\theta =$ Angle with normal of plane

$$i = \frac{\text{Watt}}{m^2 - \text{steradian}}$$

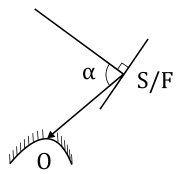
Solid angle(ω) : Unit → Steradian (Sr)

$$dw = \frac{(dA)_N}{r^2} = \sin\theta \ d\theta \ d\phi$$

$$dw = \frac{dA\cos\alpha}{r^2}$$

α is direction normal to the surface and viewing point.

Sphere $\omega = 4\pi$, Hemisphere $(\omega) = 2\pi$



Intensity of Radiation:

$$I_{e(\theta,\phi)} = \frac{dQ_e}{dA\cos\theta d\omega} \quad \frac{\text{Watt}}{m^2 \cdot Sr}$$

$$dE = \frac{dQ_e}{A} = I_{e(\theta,\phi)} \cdot \cos\theta \sin\theta \ d\theta \ d\phi$$

Hemispherical Power:

$$E = \int dE$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\theta=\frac{\pi}{2}} I_{e(\theta,\phi)} \cos\theta \sin\theta \, d\theta d\phi \left(\frac{\text{Watt}}{m^2}\right)$$

Note:

For Diffuse body $I_{e(\theta,\phi)} = \text{constant}$ (Independent of direction)

$E_b = \pi \, i_n \; W/m^2$

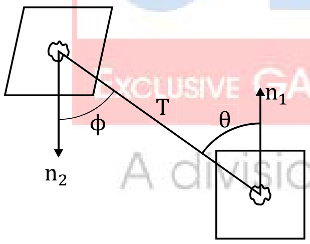
Shape Factor/Configuration/ View Factor:

F_{mn} = Fraction of radiation energy leaving surface (m) that reaches surface (n).

$$\boxed{0 \le F_{mn} \le 1}$$

Reciprocity Relation:

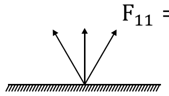
$A_1 F_{12} = A_2 F_{21}$

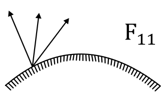


$$F_{12} = \frac{1}{\pi A_1} \int_{A_1} \int_{A_2} \frac{dA_1 dA_2 \cos\theta \cos\phi}{r^2}$$

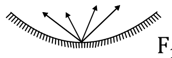
$$F_{21} = \frac{1}{\pi A_2} \iint \frac{dA_1 dA_2 \cos\theta \cos\phi}{r^2}$$

Special Cases:

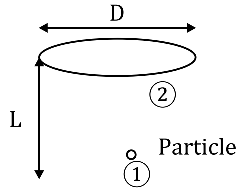




Convex S/F

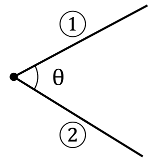


Concave S/F



$A_2 >>> A_1$

$$F_{12} = \frac{D^2}{D^2 + 4L^2}$$



$$F_{12} = F_{21} = 1 - \sin\left(\frac{\theta}{2}\right)$$

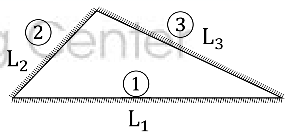
Summation Rule:

$F_{11} + F_{12} + F_{13} + \cdots + F_{1N} = 1$

For N body Enclosure → N^2 Total shape factor

Number of active shape factor

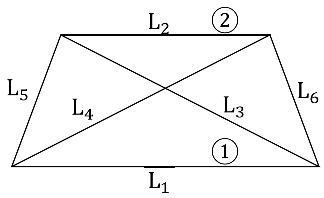
$$= \frac{N(N-1)}{2}$$



$$F_{13} = \frac{L_1 + L_3 - L_2}{2L_1}$$

$w_1 = w_2 = w_3 = w = \text{same (width)}$

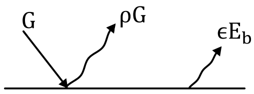
Hottle's Cross String Method (For non-enclosure):



$$F_{12} = \frac{(L_3 + L_4) - (L_5 + L_6)}{2L_1}$$

Irradiation (G):

Total radiation energy incident on surface (W/m^2).



$J = \rho G + \epsilon E_b$

J = Radiosity: total radiation energy leaving the surface.

$$\text{Surface Resistance} = \frac{1-\epsilon}{A\epsilon}$$

$$\text{Space Resistance} = \frac{1}{A_1 F_{12}}$$

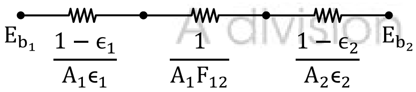
General Radiation Equation

$$(q_{1-2})_{net} = \frac{E_{b_1} - E_{b_2}}{\sum R_{th}} = \frac{\sigma(T_1^4 - T_2^4)}{\sum R_{th}}$$

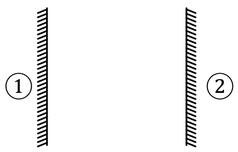
Calculation of R_{th}:

Case 1:

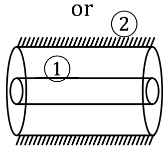
For 2 bodies



$$\sum R_{th} = \frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2}$$

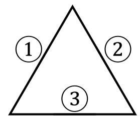


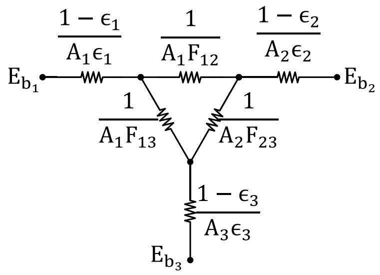
or



Case 2:

For 3 Bodies



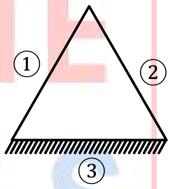


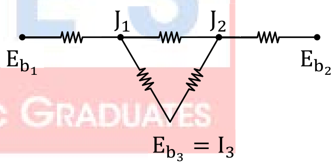
Special Case for Case 2:

A. If one S/F is reradiating S/F (Suppose ③)

$Q_3 = 0$

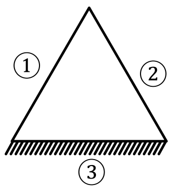
i.e., $E_{b_3} = J_3$





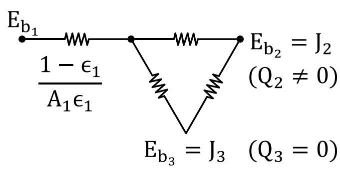
$Q_{1-2} = -Q_{2-1}$

B. If S/F ③ reradiating & S/F ② is large area.

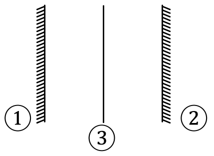


$A_2 \rightarrow \infty$

So $\frac{1-\epsilon_2}{A_2\epsilon_2} \rightarrow 0$



Radiation Shields:



$$\left(\frac{q}{A}\right)_{\text{with shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\sum R_{th}} \quad W/m^2$$

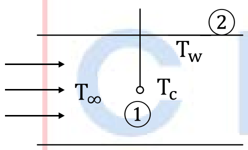
If n No. of shields used:

1. $(2n+2)$ Total surface resistance
2. $(n+1)$ Total space resistance

If $\epsilon_1 = \epsilon_2 \ldots . \epsilon_n = \epsilon$

$$\left(\frac{q}{A}\right) \text{with shield} = \left(\frac{1}{n+1}\right)\left(\frac{q}{A}\right)_{\text{w/o shield}}$$

Estimation of error in temperature measurement:



T_g = True gas temperature

T_c = Reading on thermocouple

T_w = Duct wall temperature

ϵ_c = Emissivity of thermocouple

$$hA\underbrace{(T_g - T_c)}_{\text{error}} = \sigma(T_c^4 - T_w^4) \times A \times \epsilon_c$$

$$\text{Error} = \epsilon_c \sigma\left(\frac{T_c^4 - T_w^4}{h}\right)$$

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Chapter 5: CONVECTION

$$\text{Reylond Number (Re)} = \frac{\rho VD}{\mu}$$

$$= \frac{\text{Inertia force}}{\text{Viscous force}}$$

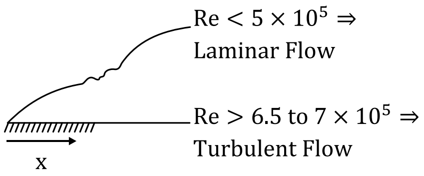
$$\text{Nusselt number (Nu)} = \frac{hD}{k_{fluid}}$$

$$\text{Prandtl Number (Pr)} = \frac{\mu C_p}{k} = \frac{\nu}{\alpha}$$

ν = Kinematic viscosity

$$\text{Stanton Number (St)} = \frac{h}{\rho U_\infty C_p}$$

1. **Forced convection (Flow over flat plate).**



Laminar Flow:

Momentum Equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\tau_w = u\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\rightarrow \delta \; \alpha \; x^{\frac{1}{2}}$$

$$\tau_w \; \alpha \; x^{-\frac{1}{2}}$$

$$h_x \; \alpha \; x^{-\frac{1}{2}}$$

$\delta_h = \delta$ = Boundary layer thickness

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$$

$$= \alpha\frac{\partial^2 T}{\partial y^2}$$

$$-k_f A\left(\frac{\partial T}{\partial y}\right) = h_x . A\,(T_w - T_\infty)$$

$$h_x = \frac{-k_f\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty}$$

For Flat Plate:

Laminar flow

A. **Isothermal**

$$\overline{Nu} = 0.664\,(Re)^{1/2}\,(Pr)^{1/3}$$

B. **Constant Heat Flux**

$$Nu_x = 0.453\, Re_x^{\frac{1}{2}} (Pr)^{\frac{1}{3}}$$

$$\frac{\delta_h}{\delta_t} = \frac{(Pr)^{\frac{1}{3}}}{1.026}$$

δ_h = Hydrodynamic boundary layer thickness

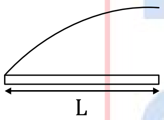
δ_t = Thermal boundary layer thickness

Turbulent Flow:

A. **Isothermal Plate**

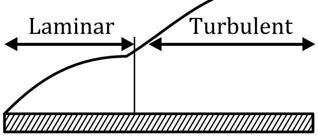
$$(Nu_x)_{local} = 0.028\,(Re)^{0.8}\,(Pr)^{\frac{1}{3}}$$

When whole plate length has turbulent flow.



B.

$$\overline{Nu} = (0.037\,(Re)^{0.8} - 871)(Pr)^{\frac{1}{3}}$$



$$(St)_x * (Pr)^{\frac{2}{3}} = \frac{C_{f,x}}{2}$$

→ Reynolds Analogy

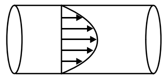
$C_{f,x}$ = Local skin friction coefficient.

C. **Constant Heat Flux:**

$$\overline{Nu} = 1.04\, \overline{Nu}_{T=C}$$

$$\frac{\delta_h}{\delta_t} = 1 \text{ (Approx)}$$

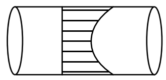
2. **Forced Convection (Through Pipes and Ducts):**



$u = Fn(r)$

$$u_{mean} = \frac{2}{R^2}\int_0^R urdr$$

u_{mean} = mean/Avg velocity



$T = Fn(r)$

$$T_b = \frac{2}{u_m R^2}\int_0^R u\,T\,r\,dr$$

T_b = Bulk mean temperature of fluid

Pipe/Duct

Laminar ($Re_d < 2300$)

A. **Constant temperature**

$$\overline{Nu} = 3.66$$

B. **Constant heat flux**

$$\overline{Nu} = 4.364 \text{ or } \left(\frac{48}{11}\right)$$

Turbulent ($Re_d > 4000$)

$$\overline{Nu} = 0.023\,(Re)^{0.8}(Pr)^n$$

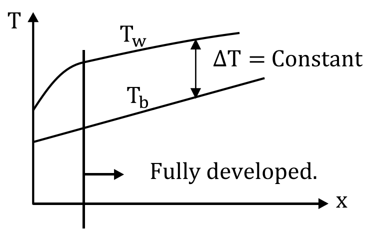
$n = 0.4$ For heating of fluid

$n = 0.3$ For cooling of fluid.

$$St * (Pr)^{2/3} = \frac{F}{8}$$

F = Friction factor

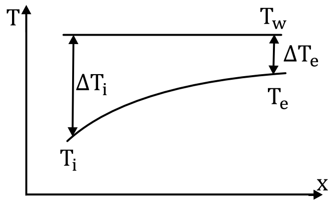
Constant Heat Flux:



$h_x = \text{constant}$

$\dot{m}\,C_p\,(T_e - T_i) = hA_s\Delta T_{const}$

Constant Wall Temperature:



$Q = hA_s\,(\text{LMTD}) = \dot{m}\,C_p\,(T_e - T_i)$

$\text{LMTD} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$

Free convection or natural convection

$Nu = C(GrPr)^m$

$m = \frac{1}{4}$ for laminar flow

$(GrPr < 10^9)$

$m = \frac{1}{3}$ for Turbulent flow

$(GrPr > 10^9)$

Gr = Grashoff number

$Gr = \frac{gB(\Delta T)L_c^3}{\nu^2}$

$\beta = \frac{1}{T_{mean}}$

ν = kinematic viscosity

T_{mean} is in Kelvin

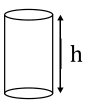
L_c = Characteristics dimension

$\Delta T = T_w - T_\infty$

$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$

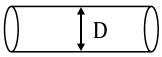
Rayleigh Number (Ra) = (Gr. Pr):

1.



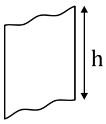
$L_C = h$

2.



$L_C = D$

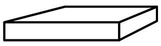
3.



$L_C = h$

Vertical Plate

4.



$L_C = \frac{\text{Surface Area}}{\text{Perimeter}}$

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