IITians GATE CLASSES
Exclusive GATE Coaching by IIT/IISc Graduates
A division of PhIE Learning Center

QUICK REVISION

FORMULA SHEET

for

GATE-AE AERODYNAMICS

Table Of Content

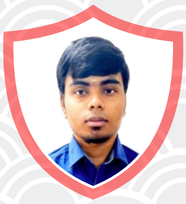
OUR ACHIEVERS

GATE-2023 AE

SRIRAM R
SSN COLLEGE CHENNAI
AIR - 2

Akriti
PEC, CHANDIGARH
AIR - 6

SHREYASHI SARKAR
IIEST, SHIBPUR
AIR - 8



YOKESH K
MIT, CHENNAI
AIR - 11

HRITHIK S PATIL
RVCE, BANGALORE
AIR - 14

And Many More ....

GATE-2022 AE

SUBHROJYOTI BISWAS
IIEST, SHIBPUR
AIR - 4

SANJAY. S
AMRITA UNIV, COIMBATORE
AIR - 7

AKILESH . G
HITS, CHENNAI
AIR - 7

D. MANOJ KUMAR
AMRITA UNIV, COIMBATORE
AIR - 10

DIPAYAN PARBAT
IIEST, SHIBPUR
AIR - 14

And Many More ....

GATE-2021 AE

NILADRI PAHARI
IIEST, SHIBPUR
AIR - 1

VISHAL .M
MIT, CHENNAI
AIR - 2

SHREYAN .C
IIEST, SHIBPUR
AIR - 3

VEDANT GUPTA
RTU, KOTA
AIR - 5

SNEHASIS .C
IIEST, SHIBPUR
AIR - 8

And Many More ....

OUR PSU JOB ACHIEVERS

DRDO & ADA Scientist B

Job Position for Recruitment **(2022-23)** Based on **GATE AE** score

- ***Mr. Abhilash K*** *(Amrita Univ Coimbatore)*
- ***Ms. Ajitha Nishma V*** *(IIST Trivendrum)*
- ***Mr. Dheeraj Sappa*** *(IIEST Shibpur)*
- ***Ms. F Jahangir*** *(MIT Chennai)*
- ***Mr. Goutham*** *(KCG College Chennai)*
- ***Mr. M Kumar*** *(MVJ College Bangalore)*
- ***Mr. Mohit Kudal*** *(RTU Kota)*
- ***Mr. Niladhari Pahari*** *(IIEST Shibpur)*
- ***Mr. Nitesh Singh*** *(Sandip Univ Nashik)*
- ***Mr. Ramanathan A*** *(Amrita Univ Coimbatore)*
- ***Ms. Shruti S Rajpara*** *(IIEST Shibpur)*

HAL DT ENGINEER

Job Position for Recruitment **(2023)**

- ***Mr. Anantha Krishan A.G*** *(Amrita Univ Coimbatore)*
- ***Mr. S.S Sanjay*** *(Amrita Univ Coimbatore)*
- ***Mr. Shashi Kanth M*** *(Sastra Univ Tanjore)*
- ***Mr. Vagicharla Dinesh*** *(Lovely Professional Univ Panjab)*

FATHIMA J
(MIT, CHENNAI)
HAL DT ENGINEER 2022

MOHAN KUMAR .H
(MVJCE, BANGALORE)
HAL DT ENGINEER 2022

ARATHY ANILKUMAR NAIR
(AMRITA UNIV, COIMBATORE)
HAL DT ENGINEER 2021

SADSIVUNI TARUN
(SASTRA TANJORE)
HAL DT ENGINEER 2021

VIGNESHA .M
(MIT, CHENNAI)
MRS E-II CRL BEL

RAM GOPAL SONI
(GVIET, PUNJAB)
CEMILAC LAB, DRDO

AERODYNAMICS

AERODYNAMICS BASICS

1. **Linear Strains of Fluid Element**

$$\dot{\varepsilon}_{xx} = \frac{\partial u}{\partial x}\ ;\ \dot{\varepsilon}_{yy} = \frac{\partial v}{\partial y}\ ;\ \ \dot{\varepsilon}_{zz} = \frac{\partial w}{\partial z}$$

where u, v and w are the velocity components in x, y and z directions respectively.

2. **Rate of angular deformation:**

$$\dot{\gamma}_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad \dot{\gamma}_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

$$\dot{\gamma}_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

3. **Vorticity**

$\xi = \nabla \times \vec{V}$;

IF $\nabla \times \vec{V} = 0 \rightarrow$ Irrotational flow

4. **Angular Velocity**

$$\vec{\omega} = \frac{1}{2}[\nabla \times \vec{V}]$$

5. **Enstrophy**

$$(E) = |\xi|^2$$

$$(E) = \left|\nabla \times \vec{V}\right|^2$$

6. **Continuity Equation for Incompressible Flow:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow\ \dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} + \dot{\varepsilon}_{zz} = 0$$

i.e. No volumetric dilation rate per unit volume

7. **Stream Function:**

$\bar{\psi}(x, y) = c$

For each 'c' value we get a streamline

$$\rho u = \frac{\partial \bar{\psi}}{\partial y}\ ;\ \rho v = -\frac{\partial \bar{\psi}}{\partial x}\ \text{And}$$

$$\rho V_r = \frac{1}{r}\frac{\partial \bar{\psi}}{\partial \theta};\quad \rho V_\theta = -\frac{\partial \psi}{\partial r}$$

For incompressible flow $\psi = \frac{\bar{\psi}}{\rho}$

$$\therefore u = \frac{\partial \psi}{\partial y}\ ;\ v = \frac{-\partial \psi}{\partial x}$$

Note:

a. For compressible flow. $\Delta\bar{\psi} = \bar{\psi}_2 - \bar{\psi}_1$ give the mass flow rate between the two streamlines ψ_1 and ψ_2

b. For incompressible flow, $\Delta\psi$ represents volume flow rate per unit depth.

c. Stream function is valid for 2D flows only.

8. **Velocity Potential Function:**

ϕ = Velocity potential function

So, the flow velocity is defined as

$$\vec{V} = \nabla\phi\ \ \text{i.e.,}\ u = \frac{\partial \phi}{\partial x};\quad v = \frac{\partial \phi}{\partial y};\quad w = \frac{\partial \phi}{\partial z}$$

Polar:

$$V_r = \frac{\partial \phi}{\partial r}; V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}; V_z = \frac{\partial \phi}{\partial z}$$

Spherical:

$$V_r = \frac{\partial \phi}{\partial r}; V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}; V_\Phi = \frac{1}{r\sin\theta}\left[\frac{\partial \phi}{\partial \Phi}\right]$$

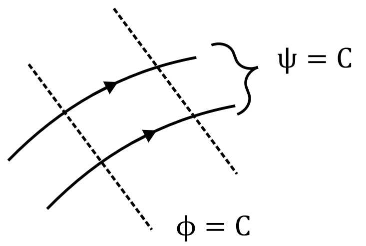
Note:

Velocity potential function is defined for irrotational flow only.

9. Relation b/w stream line and potential lines

$$\left(\frac{dy}{dx}\right)_{\psi=constant} \times \left(\frac{dy}{dx}\right)_{\phi=constant} = -1$$

- Streamlines are isolines of stream function.
- Equipotential lines are isolines of potential function.



- Streamlines and Equipotential lines form an orthogonal flow net.

10. **Reynolds Transport Theorem**

Total derivative of any property

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{C.V} \rho b d\forall + \int_{C.S} \rho b(\vec{V}\cdot\vec{n})dA$$

C.V – Control Volume

C.S. – Control Surface

b- property per unit mass

∀-Volume

V- Velocity

A-Control Surface Area

ρ- Density

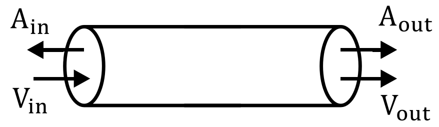
$\vec{n}$ - unit vector normal to control surface

11. **Mass continuity Integral form**

$$\frac{\partial}{\partial t}\int_{C.V} \rho d\forall + \int_{C.S.} \rho(\vec{V}\cdot\vec{n})dA = 0$$

12. **Momentum Conservation**

$$\frac{\partial}{\partial t}\int_{C.V} \vec{V}\rho d\forall + \int_{C.S.} \vec{V}\rho(\vec{V}\cdot\vec{n})dA = \sum F \text{ of C.V}$$



$$\sum F = \vec{F}_{pressure} + \vec{F}_{viscous} + \vec{F}_{body} + \vec{F}_{other}$$

Differential Forms:

13. **Continuity Equation:**

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0$$

$$\Rightarrow \frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Here u, v and w are velocity components.

14. **Cylindrical Coordinate Continuity Equation**

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho V_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) = 0$$

15. **Momentum Conservation**

$$\rho\frac{Du}{Dt} = \frac{-\partial p}{\partial x} + \frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z} + \rho f_x$$

$$\rho\frac{Dv}{Dt} = \frac{-\partial p}{\partial y} + \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\tau_{yy}}{\partial y} + \frac{\partial\tau_{zy}}{\partial z} + \rho f_y$$

$$\rho\frac{Dw}{Dt} = \frac{-\partial p}{\partial z} + \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\tau_{yy}}{\partial y} + \frac{\partial\tau_{zy}}{\partial z} + \rho f_z$$

Here τ_{ij} - Stress tensor components

p- pressure

f_x, f_y and f_z-body forces

16. **Navier-Stokes Equation**

Momentum equation for a Newtonian fluid

Stress Terms:

$$\tau_{xx} = \lambda(\nabla \cdot \vec{V}) + 2\mu\frac{\partial u}{\partial x}$$

$$\tau_{yy} = \lambda(\nabla \cdot \vec{v}) + 2\mu\frac{\partial v}{\partial y}$$

$$\tau_{zz} = \lambda(\nabla \cdot \vec{v}) + 2\mu\frac{\partial w}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

Stokes relation: $\lambda = \frac{-2}{3}\mu$

$\mu \rightarrow$ Molecular viscosity coefficient

$\lambda \rightarrow$ Secondary viscosity coefficient

$\lambda + \frac{2}{3}\mu \rightarrow$ Bulk viscosity

For a Newtonian fluid with constant density and constant viscosity,

$$\rho g_x - \frac{\partial \rho}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] = \rho\frac{Du}{Dt}$$

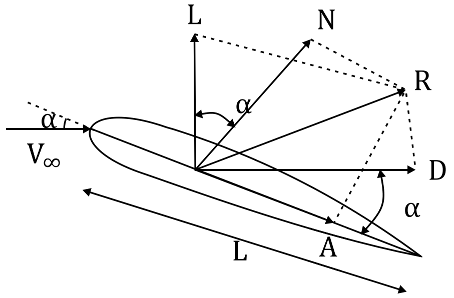
$$\rho g_y - \frac{\partial \rho}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] = \rho\frac{Dv}{Dt}$$

$$\rho g_z - \frac{\partial \rho}{\partial z} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] = \rho\frac{Dw}{Dt}$$

17. **Material Derivative**

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{V} \cdot \nabla)$$

18. **Forces on an airfoil:**



$$L = N\cos\alpha - A\sin\alpha$$
$$D = N\sin\alpha + A\cos\alpha$$

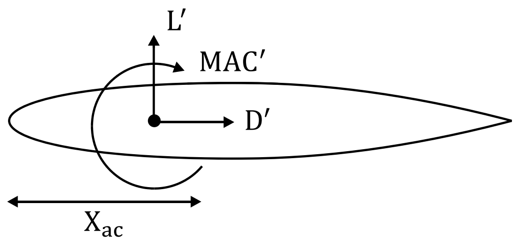
$N \rightarrow$ Normal force; $A \rightarrow$ Axial Force

$L \rightarrow$ Lift force; $D \rightarrow$ Drag Force

$\alpha \rightarrow$ Angle of attack

$V_\infty \rightarrow$ free stream velocity

19. **Aerodynamic Center:**



$$\bar{x}_{ac} = \frac{-m_0}{a_0} + 0.25$$

where, $a_0 = \frac{dC_L}{d\alpha}$ and

$$m_0 = \frac{dC_{m,c/4}}{d\alpha}$$

20. **Aerodynamic Coefficients:**

Lift coefficient,

$$C_L = \frac{L}{q_\infty S} \; ; \; C_N = \frac{N}{q_\infty S}$$

Drag coefficient,

$$C_D = \frac{D}{q_\infty S} \; ; \; C_A = \frac{A}{q_\infty S}$$

Moment Coefficient,

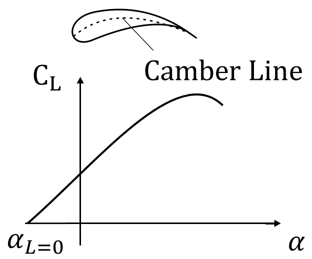
$$C_M = \frac{M}{q_\infty S\bar{c}}$$

Note: $C_L = f_n(\alpha, Re, \text{Mach no.})$

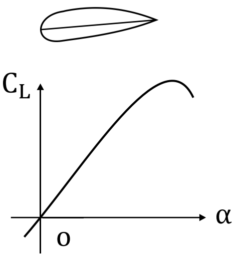
21. **Coefficient of lift v/s angle of attack**

C_L v/s α:

Cambered airfoil



Symmetric airfoil:



22. **NACA Nomenclature:**

Four digits: NACA 2412

First Digit: Maximum camber in hundredths of chord, Ex: Maximum camber 0.02c

Second Digit: Location of maximum camber along the chord from the leading edge in tenths of chord. Ex: 0.4c here

Last 2 digits: Maximum thickness in hundredths of chord. Ex: 0.12c

Five Digit: NACA 23012

First Digit: multiplied by 3/2 gives the design lift coefficient in tenths.

Ex: $\frac{2 \times 3/2}{10} = 0.3$

Second and Third Digit: Divided by 2 gives the location of maximum camber along the chord from the leading edge in hundredths of chord.

Ex: $\frac{30/2}{100} c = 0.15c$

Last 2 Digits: Maximum thickness in hundredths of chord

Ex: $\frac{12}{100} c = 0.12c$

Six-Digit Series: NACA 65-218

First Digit: Identifies series [6]

Second Digit: Location of minimum pressure [0.5c] in tenths of chord from leading edge.

Third Digit: Design lift coefficient in tenths; $C_L = 0.2$

Last 2 Digits: Maximum thickness in hundredths of chord

0.18c or 18% thickness

23. **Pressure Coefficient:**

$$C_P = \frac{p - p_\infty}{q_\infty}$$

q_∞ – Dynamic pressure

[Applicable from incompressible to hypersonic flow]

$$C_P = 1 - \left(\frac{V}{V_\infty}\right)^2$$

[For incompressible flow only]

Note: C_P is non-dimensional quantity and if flow conditions are similar (for kinematic similarity), we will get the same C_p distribution.

POTENTIAL FLUID FLOW

1. **Assumptions:**
   a. Flow is inviscid and irrotational, thereby no vorticity.

$$\vec{\xi} = \nabla \times \vec{V} = 0$$

Note:

a. Potential function was chosen as to satisfy irrotationality condition.

b. Similarly stream function was chosen to satisfy continuity equation.

2. **Potential function and Laplace Equation**

$\nabla \cdot \vec{V} = 0$

By mathematics, $\vec{V}$ should be gradient of scalar function, then,

$\nabla.(\nabla\phi) = 0$ or $\nabla^2\phi = 0$

ϕ satisfies Laplace equation, then only flow is possible as it satisfies continuity equation.

Similarly, $\nabla^2\psi = 0$ for stream function

Note:

1. Any irrotational, incompressible flow has a velocity potential and stream function (for 2D) that both satisfy Laplace equation.
2. Conversely, any solution of Laplace equation represents the velocity potential or stream function (2D) for an irrotational, incompressible flow.

3. **Features of Laplace Equation:**

$\phi = \phi_1 + \phi_2 + \phi_3 + \cdots + \phi_n$

a. Combining several potential flows gives us a new potential flow.

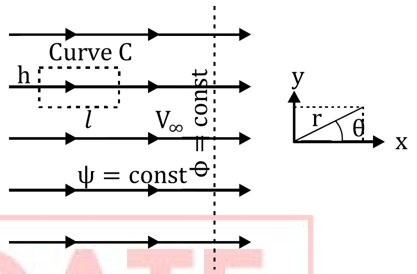
b. Each of the potential flow is called an elementary flow.

c. We can add their derivatives too, i.e., velocities (Observe elementary flows combined).

4. **Circulation:**

$$\Gamma = -\oint V ds = -\iint_A (\nabla \times V) dA$$

5. **Uniform Flow:**



Uniform Flow

In x direction, V_∞ − velocity

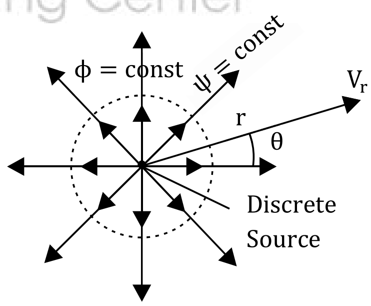
Potential function,

$\phi = V_\infty\, x = V_\infty r\cos\theta$

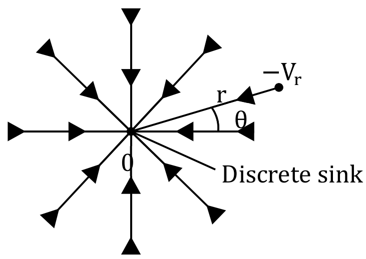
Stream function, $\psi = V_\infty\, y = V_\infty r \sin\theta$

circulation taken in the loop C, $\Gamma = 0$

6. **Source** $(\wedge)$ **and sink** $(-\wedge)$ **Flow:**



Source Flow



Sink Flow

Radial velocity, $V_r = \frac{\wedge}{2\pi r}$,

Tangential $V_\theta = 0$

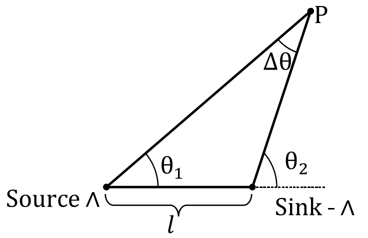
Source strength $\wedge = 2\pi r\, V_r \;\; m^2/s$

Potential and stream function

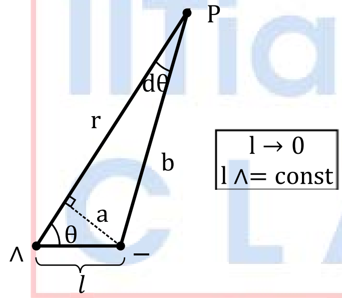
$$\phi = \frac{\wedge}{2\pi}\ln r \;;\; \psi = \frac{\wedge}{2\pi}\,\theta \;;$$

Circulation, $\Gamma = 0$

7. **Doublet Flow:**



(a)Source – sink pair



(b)Limiting case for a doublet

Figure: How a source-sink pair approaches a doublet in the limiting case.

Doublet Strength $k = \wedge\, l$ and $\Gamma = 0$

Potential and stream function

$$\psi = \frac{-k}{2\pi}\frac{\sin\theta}{r} \;\;;\;\; \phi = \frac{k}{2\pi}\frac{\cos\theta}{r}$$

Stream line

$\psi = $ constant

$$= -\frac{k}{2\pi}\frac{\sin\theta}{r} = C$$

$$r = -\frac{k}{2\pi C}\sin\theta$$

$r = d\sin\theta$

d-dia of Lobe

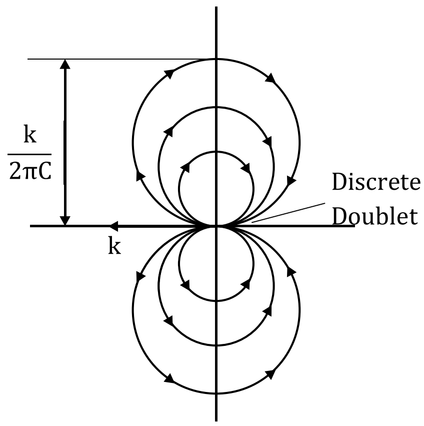


Figure: Doublet Flow with Strength k.

8. **Vortex Flow:**

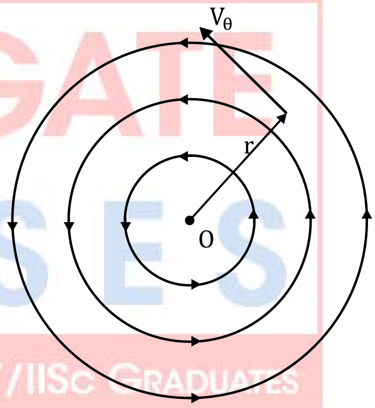


Figure: Vortex Flow.

Vortex Strength is Γ (Circulation)

$$V_\theta = -\frac{\Gamma}{2\pi r} \;;\; V_r = 0$$

$\Gamma = -2\pi r \;\; = $ Constant

Potential and stream function

$$\phi = \frac{-\Gamma}{2\pi}\theta \;;\;\; \psi = \frac{\Gamma}{2\pi}\ln r$$

A vortex of positive strength (Γ) rotates in clockwise direction.

Types of flow	Velocity	ϕ	ψ
Uniform flow in x-direction	$u = V_\infty;\ v = 0$	$V_\infty x$	$V_\infty y$
Source flow	$V_r = \frac{\Lambda}{2\pi r}; V_\theta = 0$	$\frac{\Lambda}{2\pi}\ln r$	$\frac{\Lambda}{2\pi}\theta$
Doublet Flow	$V_r = \frac{-k}{2\pi}\frac{\cos\theta}{r^2}; V_\theta = \frac{-k}{2\pi}\frac{\sin\theta}{r^2}$	$\frac{k}{2\pi}\frac{\cos\theta}{r}$	$\frac{-k}{2\pi}\frac{\sin\theta}{r}$
Vortex Flow	$V_r = 0;\ V_\theta = -\frac{\Gamma}{2\pi r}$	$\frac{-\Gamma}{2\pi}\theta$	$\frac{\Gamma}{2\pi}\ln r$

Source + Uniform Flow:

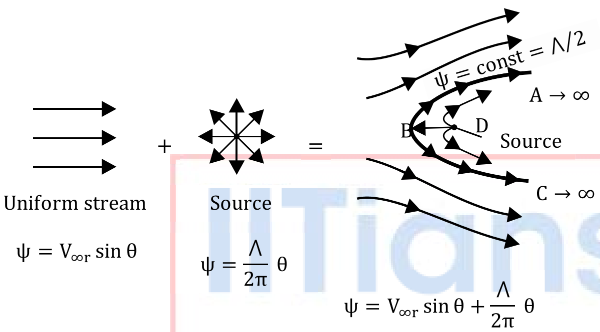


Figure: Superposition of a uniform flow and a source; flow over a semi-infinite body.

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \text{ and } V_\theta = \frac{-\partial\psi}{\partial r}$$

→ Applicable for all polar coordinates

Stagnation points,

$$(r, \theta) = \left(\frac{\Lambda}{2\pi V_\infty}, \pi\right)$$

Stagnation streamline,

$$\psi = \frac{\Lambda}{2} = \text{constant}$$

Width of Rankine half body

$$b = \frac{\Lambda}{V_\infty}$$

9. **Source + Sink + Uniform Flow**

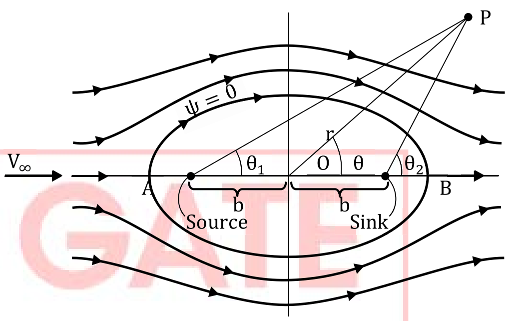


Figure: Superposition of a uniform flow and a source-sink pair; flow over a Rankine oval

$$\psi = V_\infty r \sin\theta + \frac{\Lambda}{2\pi}\theta_1 - \frac{\Lambda}{2\pi}\theta_2$$

Stagnation Points

$$OA = OB = \sqrt{b^2 + \frac{\Lambda b}{\pi V_\infty}}$$

and are at $\theta = 0, \pi$

Stagnation Streamline, $\psi = 0$

10. **Non-Lifting Flow Over Cylinder**

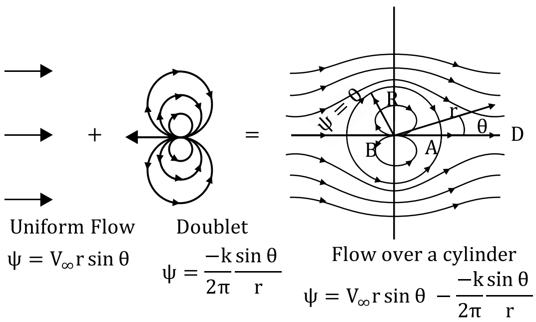


Figure: Superposition of a uniform flow and a doublet; non-lifting flow over a circular cylinder.

$$\psi = V_\infty r \sin\theta\left(1-\frac{R^2}{r^2}\right)$$

$$\text{Where}, R^2 = \frac{k}{2\pi V_\infty}$$

$$\begin{cases} V_r = V_\infty \cos\theta\left(1-\frac{R^2}{r^2}\right) \\ V_\theta = -V_\infty \sin\theta\left(1+\frac{R^2}{r^2}\right) \end{cases}$$

Ther are two stagnation Points, $(R, 0)$ and (R, π)

At stagnation streamline, $\psi = 0$

On cylinder surface, at all points $r = R$

$V_r = 0;\ V_\theta = -2V_\infty \sin\theta$

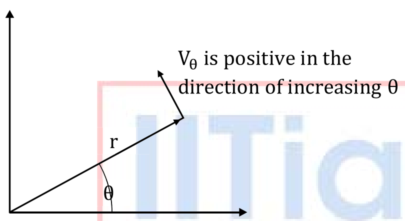
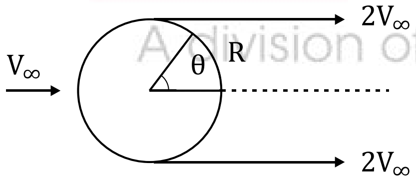


Figure: Sign Convention for V_θ in polar coordinates.

Note:

1. The maximum velocity is $2V_\infty$ at top and bottom of the cylinder.



Maximum acceleration [Flow r →R] occurs at $\theta = 135°, 225°$ and $a_{max} = \frac{2V_{max}^2}{R}$

Maximum deceleration occurs at $\theta = 45°, 315°$

2. **Pressure coefficient**

$$C_P = 1-\left(\frac{V}{V_\infty}\right)^2$$

$$C_P = 1-4\sin^2\theta$$

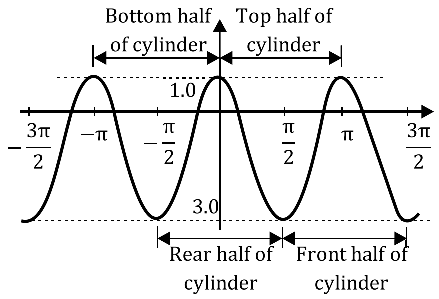


Figure: Pressure coefficient distribution over the surface of a circular cylinder, theoretical results for inviscid, incompressible flow.

11. **Lifting Flow Over Cylinder:**

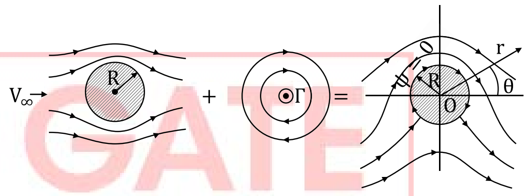


Figure: The synthesis of lifting flow over a circular cylinder.

Non-lifting flow over cylinder + Vortex flow Stream function

$$\psi = (V_\infty r\sin\theta)\left(1-\frac{R^2}{r^2}\right)+\frac{\Gamma}{2\pi}\ln\frac{r}{R}$$

V_r and V_θ can be achieved simply by adding velocities of non-lifting flow and vortex flow.

$$V_r = \left(1-\frac{R^2}{r^2}\right)V_\infty\cos\theta$$

$$V_r = -\left(1+\frac{R^2}{r^2}\right)V_\infty\sin\theta - \Gamma/2\pi r$$

stagnation Point $r = R$

$$\theta = \sin^{-1}\left[\frac{-\Gamma}{4\pi V_\infty R}\right]$$

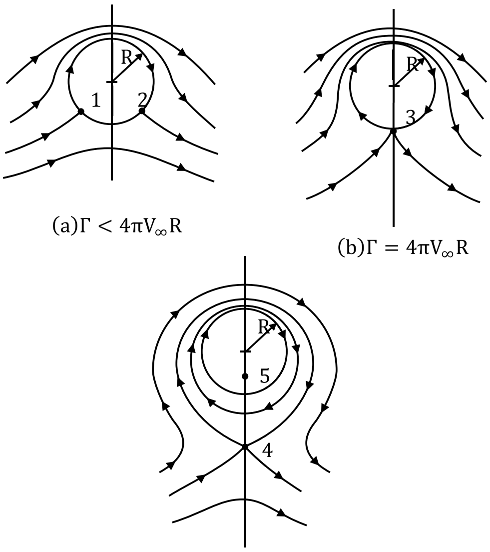


Figure: Stagnation points for the lifting flow over a circular cylinder.

a. $\Gamma < 4\pi V_\infty R \rightarrow$ 2 stagnation points

b. $\Gamma = 4\pi V_\infty R \rightarrow$ 1 stagnation point $\rightarrow$ @ $\left[R, -\frac{\pi}{2}\right]$

c. $\Gamma > 4\pi V_\infty R \rightarrow$ 1 stagnation point inside cylinder and 1 stagnation point lifts off the surface of cylinder.

And this condition is also satisfied by $r = R$ and $\theta = \pi/2$ or $-\pi/2$

So,

$$r = \frac{\Gamma}{4\pi V_\infty} \pm \sqrt{\left(\frac{\Gamma}{4\pi V_\infty}\right)^2 - R^2}$$

Velocity on the surface of cylinder:

$$V = V_\theta = -2V_\infty \sin\theta - \frac{\Gamma}{2\pi R}$$

12. **Kutta-Joukowski Theorem:**

Lift per unit span,

$$L' = \rho_\infty V_\infty \Gamma$$

$$C_L = \frac{\Gamma}{RV_\infty}$$

INCOMPRESSIBLE FLOW OVER AIRFOILS

- Air foil leading edge is usually circular with radius of approx. 0.02c and c is chord length.
- Higher the C_{Lmax}, lower is the stalling speed as $\left(L = \frac{1}{2}\rho_\infty V_\infty^2 SC_L\right)$
- $\alpha_{L=0}$ is negative for positively cambered airfoils and positive for negative cambered airfoils.
- Lift Slope a_0 is not influenced by Re, but C_{Lmax} increases with increase in Re
- The moment coefficient is also insensitive to Re except at large α
- C_d is sensitive to Re and decreases at higher Re for the order of Re~10^6.
- Aerodynamic center, $C_{m,ac}$ = constant

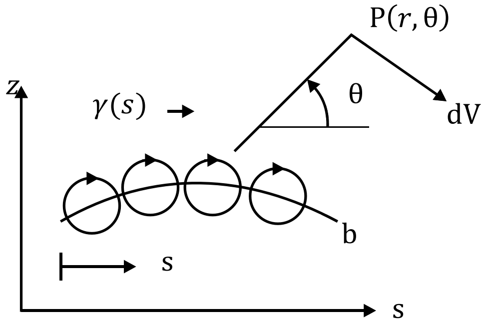
$$\left.\frac{\partial C_m}{\partial \alpha}\right|_{ac} = 0$$

1. **Vortex Sheet:**

$$\phi(s,z) = \frac{-1}{2\pi}\int_a^b \theta\,\gamma\,ds$$

$$\gamma = \int_a^b \gamma(s).\,ds \quad ; \quad dV = \frac{-\gamma ds}{2\pi r}$$

$$\gamma(s) = \frac{d\Gamma}{ds}$$



2. **The local jump in tangential velocity across the vortex sheet is equal to local sheet strength.**

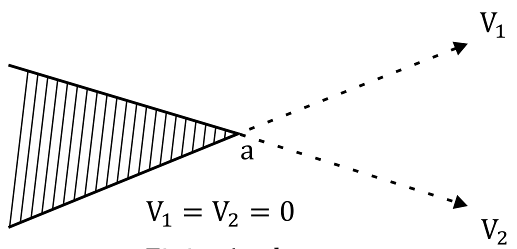
$\gamma = u_1 - u_2$

Note: The strength of vortex sheet $\gamma(s)$ is calculated such that, the camber line become a streamline of the flow.

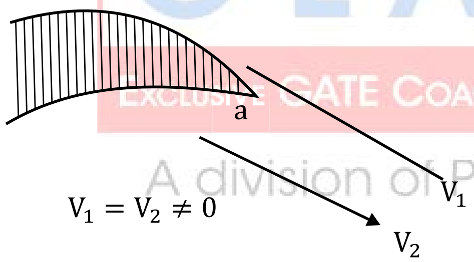
3. **Kutta Condition:**

$$\gamma(\text{T.E}) = 0$$

Trailing edge is a stagnation point for finite angled TE airfoils.

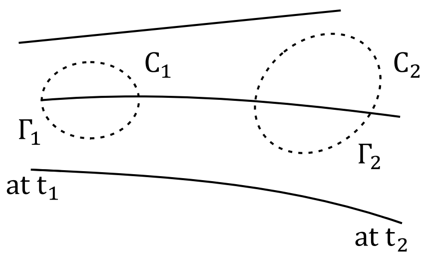


Finite Angle



Cusped

4. **Kelvin's Circulation Theorem:**



$\Gamma_1 = \Gamma_2$

Total derivative $\frac{D\Gamma}{Dt} = 0$

The time rate of change of circulation around a closed curve consisting of same fluid elements is zero.

THIN AIRFOIL THEORY

For the camber line to be a streamline of flow. Net normal velocity must be zero.

$V_{\infty,n} + w(s) = 0$

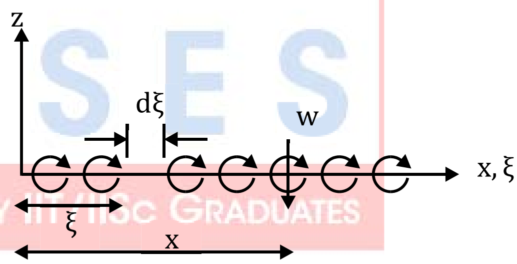
$V_{\infty,n} = V_\infty \sin\left[\alpha + \tan^{-1}\left(\frac{-dz}{dx}\right)\right]$

Approximating, i.e., $V_{\infty,n} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$

and $w(s) \approx w(x)$; $w(x) = -\int_0^c \frac{\gamma(\xi)d\xi}{2\pi(x-\xi)}$

Fundamental equation of thin airfoil theory

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{(x-\xi)} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$



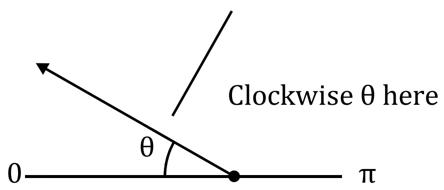
Note:

For symmetric airfoil camber line coincides with chord line,

i.e., $\frac{dz}{dx} = 0$

Symmetric Airfoil

$$\gamma(\theta) = 2\alpha V_\infty \frac{(1+\cos\theta)}{\sin\theta}$$



- $C_L = 2\pi\alpha$

- Lift slope = $\frac{dC_L}{d\alpha} = 2\pi$
- Moment coefficient.

$$C_{m,le} = -\frac{C_L}{4}$$

$$C_{m,\frac{c}{4}} = C_{m,ac} = 0$$

- The aerodynamic center and center of pressure are both at the quarter chord point.

Cambered Airfoil: Circulation

$$\Gamma = cV_\infty\left[\pi A_o + \frac{\pi}{2}A_1\right]$$

$$\gamma(\theta) = 2V_\infty\left[A_o\left(\frac{1+\cos\theta}{\sin\theta}\right) + \sum_{n=1}^{\infty} A_n \sin\theta\right]$$

$$C_L = 2\pi\left[\alpha + \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(\cos\theta_0 - 1)d\theta_0\right]$$

$$C_L = \pi(2A_0 + A_1)$$

$$\alpha_{L=0} = \frac{-1}{\pi}\int_0^\pi \frac{dz}{dx}(\cos\theta_0 - 1)d\theta_0$$

$$[\because C_L = 2\pi(\alpha - \alpha_{L=0})]$$

$$C_{m,le} = -\frac{C_L}{4} + \frac{\pi}{4}(A_1 - A_2)$$

$$C_{m,c/4} = \frac{\pi}{4}(A_2 - A_1)$$

Usuallly, $A_1 > A_2$

$$X_{cp} = \frac{c}{4}\left[1 + \frac{\pi}{C_L}(A_1 - A_2)\right]$$

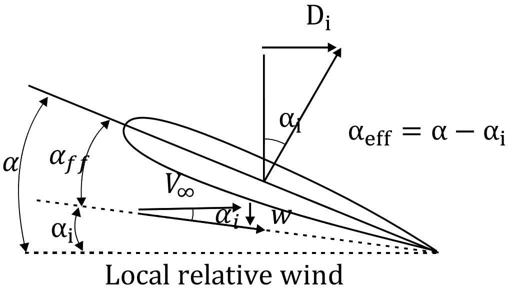
Fourier Coefficients:

$$A_o = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}d\theta_0$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos n\theta_0\, d\theta_0$$

- Lift slope = $\frac{dC_L}{d\alpha} = 2\pi$
- The aerodynamics center is at the quarter-chord point.
- The center of pressure varies with the lift coefficient.

INCOMPRESSIBLE FLOW OVER WING



$\omega \rightarrow$ downwash.

D_i induced drag $[L \sin\alpha_1]$

For airfoil, $C_l = 2\pi\alpha$;

For wing $C_L = a(\alpha - \alpha_i)$

1. **Definitions:**

Geometric twist: α is different at different spanwise locations.

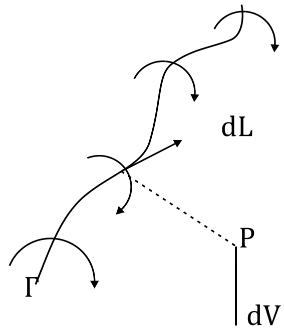
Washout: If the tip is at a higher α than the root

Aerodynamic Twist: Different airfoil sections along the span with different values of $\alpha_{L=0}$

2. **Biot Savart Law:**

The strength of the vortex filament is defined as Γ. Consider a directed segment of the filament dl, as shown in Figure. The radius vector from dl to an arbitrary point P in space is r. The segment dl induces a velocity at P

$$dV = \frac{\Gamma}{4\pi}\frac{dL \times r}{|r|^2}$$



3. **Velocity induced at a point 'P' by an infinite straight vortex filament.**

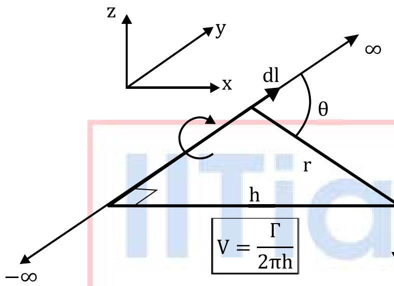


Figure: Velocity induced at point P by an infinite, straight vortex filament.

$$V = \frac{\Gamma}{2\pi h}$$

4. **Semi-infinite Vortex:**

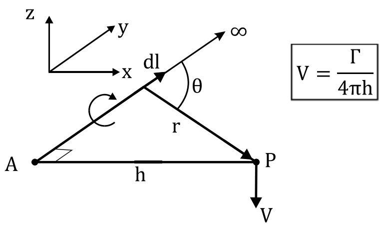
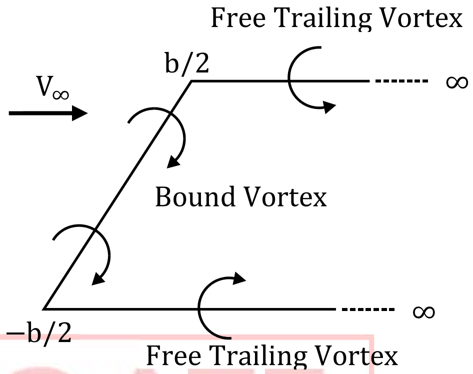


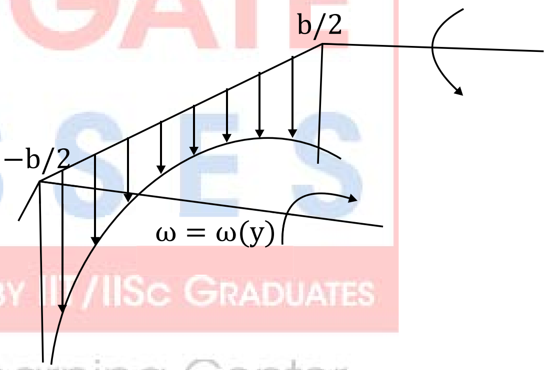
Figure: Velocity induced at point P by a semi-infinite straight vortex filament.

$$V = \frac{\Gamma}{4\pi h}$$

5. **Helmholtz's Vortex Theorems:**
   i. The strength of a vortex filament is constant along its length.
   ii. A vortex filament cannot end in a fluid it must extent to the boundary of the fluid (which can be $\pm\infty$) or form a closed path.

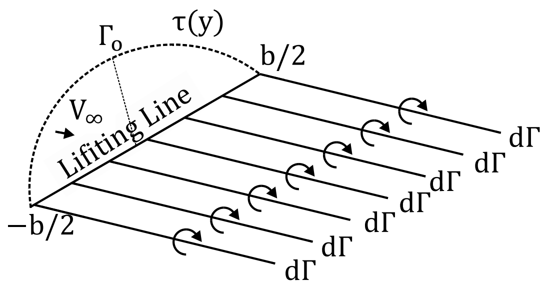
6. **Prandtl's lifting line theory**





$$\omega(y) = \frac{-\Gamma}{4\pi}\frac{b}{\left(\frac{b}{2}\right)^2 - y^2}$$

Note: At trailing edge $y = \pm\frac{b}{2}$, downwash is infinity. So, this model was not accurate.



$$\omega(y_0) = \frac{-1}{4\pi}\int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\left(\frac{d\Gamma}{dy}\right)dy}{y_0 - y}$$

$$\alpha_i(y_0) = \tan^{-1}\left(\frac{-\omega(y_0)}{V_\infty}\right) = \alpha\frac{-\omega(y_0)}{V_\infty}$$

$$\therefore\ \alpha_i(y_0) = \frac{1}{4\pi V_\infty}\int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\left(\frac{d\Gamma}{dy}\right)dy}{y_0 - y}$$

Elliptical Lift Distribution:

$$\Gamma(y) = \Gamma_0\sqrt{1-\left(\frac{2y}{b}\right)^2}$$

∵ Circulation and Lift distribution

$$L'(y) = \rho_\infty V_\infty \Gamma_0\sqrt{1-\left(\frac{2y}{b}\right)^2}$$

Downwash, $\omega(\theta_0) = -\frac{\Gamma_0}{2b}$

→Constant downwash for elliptic distribution

Induced angle of attack,

$$\alpha_i = \frac{\Gamma_0}{2bV_\infty}\ ;\ \left[\alpha_i = -\frac{\omega}{V_\infty}\right]$$

Lift, $L = \rho_\infty V_\infty \Gamma_0 \frac{b}{4}\pi$

Alternative:

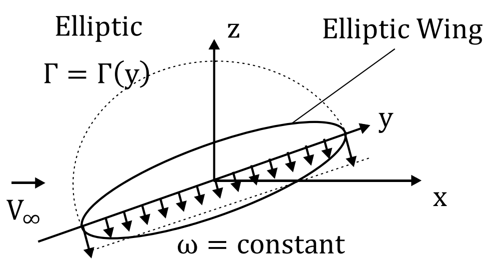
Induced angle of Attack:

$$\alpha_i = \frac{C_L}{\pi AR}$$

Induced drag, $C_{Di} = \frac{C_L^2}{\pi AR}$

AR- Aspect Ratio

Note: For elliptic lift distribution the wing platform is elliptical



General Lift Distribution:

$$C_{Di} = \frac{C_L^2}{\pi AR}(1+\delta) = \frac{C_L^2}{\pi eAR}$$

$$\frac{1}{1+\delta} = e$$

→ Ostwalds Span efficiency factor

Taper ratio $= \frac{c_t}{c_r}$; c_t → tip chord ;

c_r → root chord

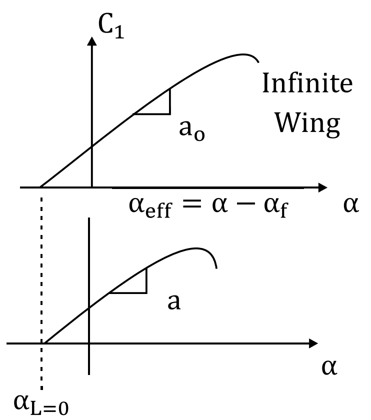
Lift Curve Slope:

For airfoil, $a_o = \frac{dC_l}{d\alpha}$

Finite wing, $a = \frac{dC_L}{d\alpha}$

$a_0 > a$

$$a = \frac{a_o}{1 + \frac{a_o}{\pi eAR}}\ \text{(in radian)}$$



Note:

a. If we use α_{eff} ($\alpha - \alpha_i$) and calculate C_L using slope, a_0 then,

$C_L = a_0(\alpha_{eff} - \alpha_{L=0})$

b. If we use geometric AOA α, then slope used is a

$C_L = a(\alpha - \alpha_{L=0})$

*****

OUR COURSES

GATE Online Coaching

Course Features

- Live Interactive Classes
- E-Study Material
- Recordings of Live Classes
- Online Mock Tests

TARGET GATE COURSE

Course Features

- Recorded Videos Lectures
- Online Doubt Support
- E-Study Materials
- Online Test Series

Distance Learning Program

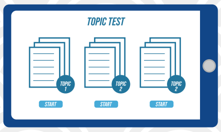
Course Features

- E-Study Material
- Topic Wise Assignments (e-form)
- Online Test Series
- Online Doubt Support
- Previous Year Solved Question Papers

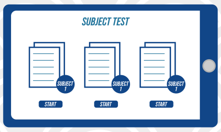
OUR COURSES

Online Test Series

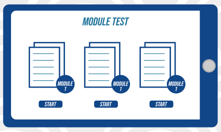
Course Features



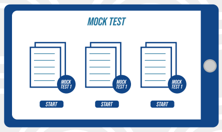
Topic Wise Tests



Subject Wise Tests



Module Wise Tests



Complete Syllabus Tests

More About IGC

SAMPLE STUDY MATERIALS
https://bit.ly/3Wu7lsp

SAMPLE MOCK TESTS
https://bit.ly/3FDyF0f

SAMPLE VIDEO LECTURES
https://bit.ly/3V4xU6j

TEAM IGC GATE
https://bit.ly/3G21khi

OUR TESTIMONIALS
https://bit.ly/3Yp2Hh7

IGC TELEGRAM GROUP
https://bit.ly/3j4koCd

Follow us on:

For more Information Call Us
+91-97405 01604

Visit us
www.iitiansgateclasses.com