IITians GATE CLASSES
Exclusive GATE Coaching by IIT/IISc Graduates
A division of PhIE Learning Center

QUICK REVISION
FORMULA SHEET
for
GATE-AE AIRCRAFT STABILITY

Table Of Content

Flight Stability & Control	05
Longitudinal Stability	05
Stick Free Longitudinal Static Stability	07
Dynamic Longitudinal Stability	08
Directional Stability	09
Contribution of Wing	10
Lateral Stability	13

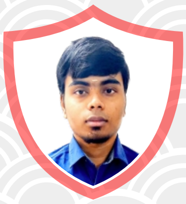
OUR ACHIEVERS

GATE-2023 AE

SRIRAM R
SSN COLLEGE CHENNAI
AIR - 2

Akriti
PEC, CHANDIGARH
AIR - 6

SHREYASHI SARKAR
IIEST, SHIBPUR
AIR - 8



YOKESH K
MIT, CHENNAI
AIR - 11

HRITHIK S PATIL
RVCE, BANGALORE
AIR - 14

And Many More ....

GATE-2022 AE

SUBHROJYOTI BISWAS
IIEST, SHIBPUR
AIR - 4

SANJAY. S
AMRITA UNIV, COIMBATORE
AIR - 7

AKILESH . G
HITS, CHENNAI
AIR - 7

D. MANOJ KUMAR
AMRITA UNIV, COIMBATORE
AIR - 10

DIPAYAN PARBAT
IIEST, SHIBPUR
AIR - 14

And Many More ....

GATE-2021 AE

NILADRI PAHARI
IIEST, SHIBPUR
AIR - 1

VISHAL .M
MIT, CHENNAI
AIR - 2

SHREYAN .C
IIEST, SHIBPUR
AIR - 3

VEDANT GUPTA
RTU, KOTA
AIR - 5

SNEHASIS .C
IIEST, SHIBPUR
AIR - 8

And Many More ....

OUR PSU JOB ACHIEVERS

DRDO & ADA Scientist B

Job Position for Recruitment **(2022-23)** Based on **GATE AE** score

Mr. Abhilash K *(Amrita Univ Coimbatore)*

Ms. Ajitha Nishma V *(IIST Trivendrum)*

Mr. Dheeraj Sappa *(IIEST Shibpur)*

Ms. F Jahangir *(MIT Chennai)*

Mr. Goutham *(KCG College Chennai)*

Mr. M Kumar *(MVJ College Bangalore)*

Mr. Mohit Kudal *(RTU Kota)*

Mr. Niladhari Pahari *(IIEST Shibpur)*

Mr. Nitesh Singh *(Sandip Univ Nashik)*

Mr. Ramanathan A *(Amrita Univ Coimbatore)*

Ms. Shruti S Rajpara *(IIEST Shibpur)*

HAL DT ENGINEER

Job Position for Recruitment **(2023)**

Mr. Anantha Krishan A.G *(Amrita Univ Coimbatore)*

Mr. S.S Sanjay *(Amrita Univ Coimbatore)*

Mr. Shashi Kanth M *(Sastra Univ Tanjore)*

Mr. Vagicharla Dinesh *(Lovely Professional Univ Panjab)*

FATHIMA J
(MIT, CHENNAI)
HAL DT ENGINEER 2022

MOHAN KUMAR .H
(MVJCE, BANGALORE)
HAL DT ENGINEER 2022

ARATHY ANILKUMAR NAIR
(AMRITA UNIV, COIMBATORE)
HAL DT ENGINEER 2021

SADSIVUNI TARUN
(SASTRA TANJORE)
HAL DT ENGINEER 2021

VIGNESHA .M
(MIT, CHENNAI)
MRS E-II CRL BEL

RAM GOPAL SONI
(GVIET, PUNJAB)
CEMILAC LAB, DRDO

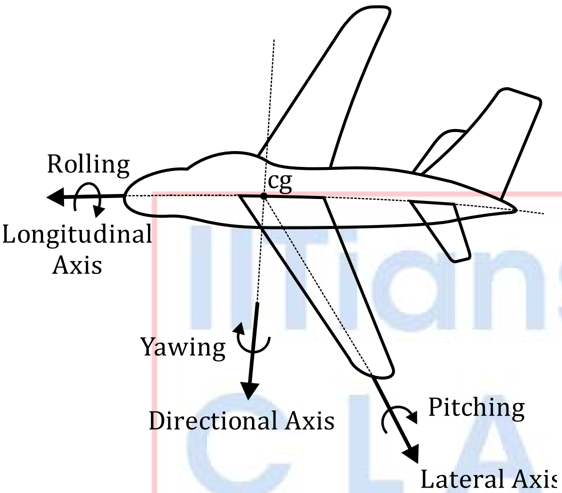
AIRCRAFT STABILITY

FLIGHT STABILITY & CONTROL

Stability:

Static Stability: System response is considered without time.

Dynamic Stability: System response is considered with time



Sign Notation:

For pitching:

Nose up (+ve)

Nose down – (−ve)

Rolling:

Right wing (Starboard) down = (+ve)

Right wing (Starboard) up = (−ve)

Left wing (Port wing) down = (−ve)

Left wing (Port wing) up = (+ve)

Yawing:

Turning towards right wing = (+ve)

Turning towards left wing = (−ve)

LONGITUDINAL STABILITY

Criteria for Longitudinal Stability:

For Statically stable aircraft, trimmed at positive angle of attack

1. $C_{m_o} = (+ve) \rightarrow C_{m_o} > 0$
2. $C_{m_\alpha} < 0$

For trimmed aircraft $C_{m_O} = 0$

For neutrally stable aircraft $C_{m_\alpha} = 0$

For statically unstable aircraft $C_{m_\alpha} > 0$

1. **Wing Contribution:**

   $C_{m_{cg}} = C_{m_{ac}} + C_{L_w}(h - h_{ac})$ .... ①

   $C_{L_w} = C_{L_o} + a_w \alpha_w$

   Where;

   C_{m_o} = Independent on α

   $C_{m_\alpha} = \dfrac{dC_m}{d\alpha} \Rightarrow$ Dependent on α

   $C_{m_o} = C_{m_{ac}} + C_{L_o}(h - h_{ac})$

   $$\frac{\partial C_m}{\partial \alpha} = a_w(h - h_{ac}) > 0$$

   Wing alone configuration is unstable because $\frac{\partial C_m}{\partial \alpha} > 0$.

   For wing alone configuration to be stable, use airfoil with negative camber or reflexed trailing edge.

2. **Contribution of Horizontal Tail:**

   $\alpha_t = (\alpha_w - i_w - \varepsilon) + i_t$

$$\left\{\begin{array}{l}\eta_t = \dfrac{\frac{1}{2}\rho V_t^2}{\frac{1}{2}\rho V_\infty^2} = \dfrac{q_t}{q_\infty} \text{ Tail efficiency factor} \\ V_H = \dfrac{l_t S_t}{S_w C_w} - \text{Tail volume coefficeint}\end{array}\right\}$$

$$\left(C_{m_{cg}}\right)_t = -\eta_t V_H C_{L_t}$$

$$C_{L_t} = a_t \alpha_t$$

$$C_{m_{cg}} = -\eta_t V_H a_t(\alpha_w - i_w - \varepsilon) + i_t$$

$$\left\{\varepsilon = \varepsilon_0 + \frac{\partial \varepsilon}{\partial \alpha}\alpha\right\}$$

$$C_{m_{cg}} = -\eta_t C_H a_t \left(\alpha_w - i_w - \left(\varepsilon_0 + \frac{\partial \varepsilon}{\partial \alpha}\alpha\right) + i_t\right) \ldots ②$$

$$\left\{\begin{array}{l}\text{Empirical relation for } \varepsilon, \varepsilon_o = \dfrac{2C_{Lw}}{\pi e AR} \\ \dfrac{\partial \varepsilon}{\partial \alpha} = \dfrac{2a_w}{\pi e AR}\end{array}\right\}$$

Rewriting ②

$$C_{m_{cg}} = \underbrace{\eta_t V_H a_t (i_t + \varepsilon_0)}_{\text{Independent of } \alpha} - \underbrace{\eta_t V_H a_t \alpha \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)}_{\text{Dependent of } \alpha} \ldots ②$$

$\left(C_{m_{cg}}\right)_t \Rightarrow C_{m_0} > 0$ and $C_{m_\alpha} < 0$

(Satisfied the criteria for longitudinal stability)

Tail alone configuration is statically stable configuration:

$$\left(C_{m_{cg}}\right)_{w,t} = C_{m_{ac}} + C_{L_w}(h - h_{ac}) - \eta_t V_H C_{L_t} + C_{m_{fuselage}}$$

Or

$$\left(C_{m_{cg}}\right)_{w,t} = C_{m_{ac}} + a_w \alpha_w (h - h_{ac}) + C_{L_o}(h - h_{ac}) - \eta_t V_H(\alpha_w - i_w - \varepsilon + i_t) + C_{m_{fuselage}} \ldots ③$$

$$\left.\begin{array}{l} C_{m_o} = \left(C_{m_{ac}}\right) + C_{L_o}(h - h_{ac}) + \eta_t V_H a_t (i_t + \varepsilon_o) \\ C_{m_\alpha} = a_w (h - h_{ac}) - \eta_t V_H \left(1 - \dfrac{\partial \varepsilon}{\partial \alpha}\right) a_t \end{array}\right\} \ldots ④$$

Neutral Point:

A point where $C_{m_{cg}}$ constant for airplane

$$\left(\frac{\partial C_m}{\partial \alpha} \text{ or } \frac{\partial C_m}{\partial C_L} = 0\right)$$

Here $h = h_n$ (Neutral point) equation ③

$$\frac{\partial C_m}{\partial \alpha} = a_w \left[(h - h_{ac}) - \eta_t V_H \frac{a_t}{a_w}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

$$0 = a_w \left[(h_n - h_{ac}) - \eta_t V_H \frac{a_t}{a_w}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

$$h_n = h_{ac} + \eta_t V_H \frac{a_t}{a_w}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) - \frac{1}{a_w}\frac{\partial C_{m_{fuselage}}}{\partial \alpha}$$

Neutral point location

$$\frac{\partial C_m}{\partial \alpha} = a_w (h - h_n)$$

{$(h_n - h)$ – Static margin}

$$\frac{\partial C_m}{\partial \alpha} = -a_w (h_n - h)$$

{Neutral point should behind the CG location}

$$\frac{\partial C_m}{\partial \alpha} = -a_w \times \text{static margin}$$

{For a/c to have static longitudinal stability}

1. $h_n > h$ (Stable configuration)
2. $h_n = h$ (Neutral configuration)
3. $h_n < h$ (Unstable configuration)

Elevator Deflection:

$$C_{L_t} = \frac{\partial C_{L_t}}{\partial \alpha_t}\alpha_t + \frac{\partial C_{L_t}}{\partial \delta_e}\delta_e$$

$$\left\{\frac{\partial C_{L_t}}{\partial \delta_e} - \text{Elevator control effectiveness}\right\}$$

$$\frac{\partial C_{L_t}}{\partial \delta_e} = \alpha_t a_t + \frac{\partial C_{L_t}}{\partial \delta_e}\delta_e$$

$$C_{m_{cg}} = C_{m_{cgw}} + C_{m_{cgt}}$$

$$C_{m_{cg}} = C_{m_{ac}} + C_{L_w}(h - h_{ac}) - \eta_t V_H\left(a_t\alpha_t + \frac{\partial C_{L_t}}{\partial \delta_e}\delta_e\right)$$

$$\frac{\partial C_m}{\partial \delta_e} = -\eta_t V_H \frac{\partial C_{L_t}}{\partial \delta_e} \text{ --- Elevator power}$$

$$\Delta C_{m_{cg}} = -\eta_t V_H \frac{\partial C_{L_t}}{\partial \delta_e}\delta_e$$

Calculate the elevator angle to trim

$$C_{m_{cg}} = C_{m_o} + \frac{\partial C_m}{\partial \alpha}\alpha$$

$$C_{m_{cg}} = C_{m_o} + \frac{\partial C_m}{\partial \alpha} + \Delta C_{m_{cg}}$$

$$C_{m_{cg}} = C_{m_o} + \frac{\partial C_m}{\partial \alpha}\alpha - \eta_t V_H \frac{\partial C_{L_t}}{\partial \delta_e}\delta_e$$

To trim the a/c at $\alpha = \alpha_n$ (new α), $C_{m_{cg}} = 0$

$$\delta_{e_{trim}} = \frac{C_{m_o} + \left(\frac{\partial C_m}{\alpha}\right)\alpha_n}{\eta_t V_H \left(\frac{\partial C_{L_t}}{\partial \delta_e}\right)}$$

Elevator Hinge Moment:

C_{h_e} = Elevator hinge moment coefficient

$$C_{h_e} = f(\alpha_t, \delta_e)$$

$$C_{h_e} = \underbrace{\frac{\partial C_{h_e}}{\partial \alpha_t}}_{\text{Floating Tendency}}\alpha_t + \underbrace{\frac{\partial C_{h_e}}{\partial \delta_e}}_{\text{Restoring Tendency}}\delta_e$$

STICK FREE LONGITUDINAL STATIC STABILITY

If elevator is free to oscillate from position $\delta_{e_{free}}$ can be calculate $H_e = 0$

$$\delta_{e_{free}} = \left(-\frac{\left(\frac{\partial C_{h_e}}{\partial \alpha_t}\right)\alpha_t}{\left(\frac{\partial C_{h_e}}{\partial \delta_e}\right)}\right)$$

Considering tail lift coefficient

$$C_{L_t} = a_t\alpha_t + \left(\frac{\partial C_{L_t}}{\partial \delta_e}\delta_e\right) \quad \{\delta_e = \delta_{free}\}$$

$$C'_{L_t} = a_t\alpha_t + \left(\frac{\partial C_{L_t}}{\partial \delta_e} - \frac{\left(\frac{\partial C_{h_e}}{\partial \alpha_t}\right)\alpha_t}{\left(\frac{\partial C_{h_e}}{\partial \delta_e}\right)}\right)$$

$$C'_{L_t} = a_t\alpha_t - \left(\frac{\partial C_{L_t}}{\partial \delta_e}\left(\frac{\frac{\partial C_{h_e}}{\partial \alpha_t}}{\frac{\partial C_{h_e}}{\partial \delta_e}}\right)\alpha_t\right)$$

$C_{L_t} = a_t\alpha_t F$ (F= Free elevator factor)

$$F = 1 - \frac{1}{a_t}\left(\frac{\partial C_{L_t}}{\partial \delta_e}\right)\left(\frac{\partial C_{h_e}/\partial \alpha_t}{\partial C_{h_e}/\partial \delta_e}\right)$$

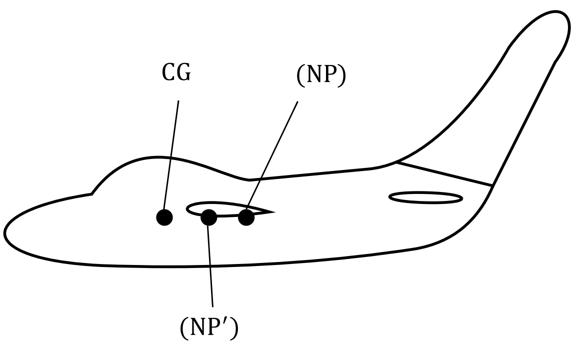
(F is always less than 1)

$$C_{m_\alpha} = a_w(h - h_{ac}) - F\eta_t V_H a_t\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

$$C_{m_\alpha} = a_w\left[(h - h_{ac}) - F\eta_t V_H \frac{a_t}{a_w}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

$$h'_n = h_{ac} + F\eta_t V_H \frac{a_t}{a_w}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

$h'_n < h_n$ because factor F is multifield shift forward and stability gets reduced.



Corresponding C_{m_o}

$$C_{m_o} = C_{m_{ac}} + C_{L_o}(h - h_{ac}) + F\,\eta_t V_H a_t\,(i_t + \varepsilon_o)$$

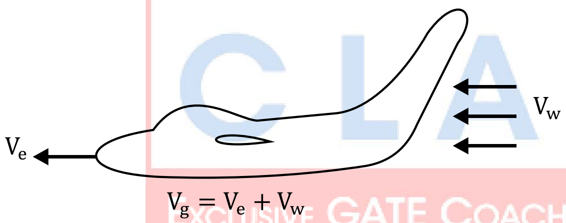
DYNAMIC LONGITUDINAL STABILITY

1. **Short Period Oscillation:**

   Angle of Attack (α) is changed and Velocity kept constant, pitch attitude constant, heavily damped motion

2. **Long Period Oscillation: (Phugoid Mode)**

   AOA kept constant and Velocity is increased or decreased pitch attitude varies, lightly damped motion (V_g – gust wind)



Energy transfer takes place from PE to KE it lose or gain height.

$\Delta PE = mgh \quad kE = \frac{1}{2}\,mV^2$(before wind)

Here the frequency is large time period is less

$$\omega_n = 2\pi f_n \Rightarrow f_n = \frac{\omega_n}{2\pi} \qquad T = \frac{1}{f_n}$$

$$\lambda_{1,2} = -\xi\omega_n \pm i\sqrt{1-\xi^2}\;\omega_n$$

where ξ = damping ratio;

ω_n = Natural frequency

$$\text{Amplitude } \delta = \frac{1}{n}\ln\left(\frac{x_1}{x_{n+1}}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

δ = Logarithmic decrement

$\lambda_{1,2} = -p \pm iq$ complex pair with negative real part.

After Wind Gust:

$$\Delta kE = \frac{1}{2}\,m\,(V+V_w)^2 - \frac{1}{2}\,mV^2$$

$$\Delta kE = \frac{1}{2}m\,(V^2 + V_w^2 + 2VV_w) - \frac{1}{2}mV^2$$

$$\Delta kE = \frac{1}{2}\,(mV_w^2) + (mVV_w)$$

The aircraft velocity V_∞ is greater than wind velocity.

$\left\{\left(\frac{1}{2}mV_w^2\right) \simeq 0\right\}$ Neglected

$V >> V_W \quad \Delta PE = \Delta KE$

$mgh = mVV_W$

$gh = VV_w$

Before wind, $L = W = \frac{1}{2}\rho V^2\,SC_L$

After wind, $L = \frac{1}{2}\,\rho(V+V_w)^2 SC_L$

$$\frac{L}{W} = \frac{\frac{1}{2}\,\rho(V+V_w)^2 SC_L}{\frac{1}{2}\,\rho V^2 SC_L} = \frac{(V+V_w)^2}{V^2}$$

$$\frac{L}{W} = \frac{V^2 + V_w^2 + 2VV_w}{V^2}$$

(V_w is very small neglected)

$$\frac{L}{W} = 1 + \frac{2VV_w}{V^2} = \left(1 + \frac{2V_w}{V}\right)$$

$(L - W)$net change in lift

$$L = W\left(1 + \frac{2V_w}{V}\right)$$

$$L = W + \frac{2V_w}{V}W \Rightarrow \left(L - W = \frac{2V_w}{V}W\right)$$

$$\omega_n = \frac{\sqrt{2}g}{V}$$

$$f_n = \frac{g}{\sqrt{2}\pi V},$$

$$T = \frac{\sqrt{2}\pi V}{g}$$

For Dynamic Longitudinal Stability

The characteristic equation given as

$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$

To check whether it is the characteristic equation for Longitudinal mode

1. All coefficients A, B, C, D and E are positive
2. B and C $>$ D and E
3. A is almost equal to 1

Routh's Criteria:

$$R = BCD - D^2A - B^2E$$

Case 1: R > 0 the aircraft is stable

$(-p_1 \pm iq)$and $(-p_2 \pm iq)$

One of the complex pair represents (phugoid mode) and other complex pair represents the short period oscillation.

Case 2: R = 0 then aircraft is dynamically neutral

Case 3: R < 0 then aircraft is dynamically unstable.

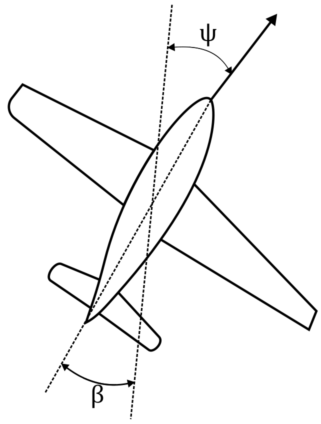
Case 4: If E = 0 then one of the λ value is zero which shows that mode is dynamically unstable.

If other mode is negative then it shows pure convergence.

If other mode is positive then it shows pure divergence.

Case 5: If any coefficient in the characteristic's equation is negative then mode will give pure divergence or dynamically unstable condition.

DIRECTIONAL STABILITY (WEATHER COCK STABILITY)



β = Sideslip Angle

ψ = Yawing Angle

N = Yawing Moment

$\beta = -\psi$

Criteria for Yaw stability

$\frac{\partial N}{\partial \psi} < 0 \quad \frac{\partial N}{\partial \beta} > 0$

$C_N = \frac{N}{\frac{1}{2}\rho V^2 S b}$

$\frac{\partial C_N}{\partial \psi} < 0$ and $\frac{\partial C_N}{\partial \beta} > 0$

Case 1: If the yawing moment trying to restore the original equilibrium position i.e., zero yaw condition then airplane is statically directionally stable.

$\frac{\partial C_N}{\partial \psi} < 0$ (or) $\frac{\partial C_N}{\partial \beta} > 0$ (stable)

Case 2: If the yawing moment tends to take the airplane further away from equilibrium position, then the airplane is statically directionally unstable.

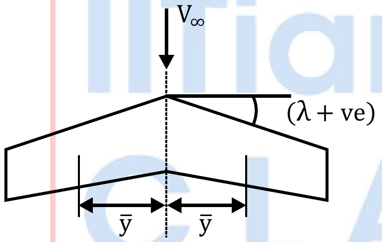
$$\frac{\partial N}{\partial \psi} > 0 \text{ (or) } \frac{\partial N}{\partial \beta} < 0 \text{ (stable)}$$

Case 3: If the yawing moment created the airplane neither return back to original equilibrium position it takes away from its original equilibrium position then the airplane is statically directionally neutral.

$$\frac{\partial N}{\partial \psi} = 0 \text{ (or) } \frac{\partial N}{\partial \beta} = 0$$

CONTRIBUTION OF WING

1. **Sweep Back Wing:**



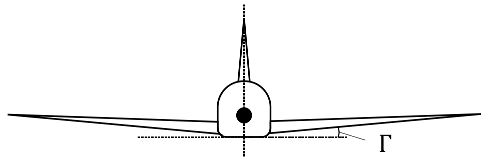
- Drag Component gives yawing moment
- Lift component gives rolling moment.

$$\frac{\partial C_N}{\partial \psi} = -C_D \frac{\bar{y}}{b} \frac{\sin 2\lambda}{57.3} \text{(per radian)}$$

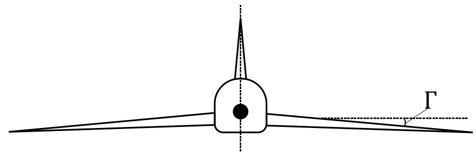
- Sweep back angle contributes positive directional stability.
- If $\frac{\partial C_N}{\partial \psi} = 0$ where $(\lambda = 0)$ Neutrally stable. (For straight wing)
- For forward sweep

$$(\lambda = -ve) \; \frac{\partial C_N}{\partial \psi} > 0 \text{ (Unstable).}$$

2. **Dihedral Wing:**



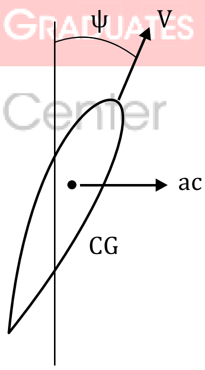
(Dihedral Wing)
$(\Gamma = +ve)$



(Anhedral Wing)
$(\Gamma = -ve)$

- Due to dihedral or anhedral (cathedral) wing the lift vector on each wing (Span wise) is getting tilted there by it creates minor destabilizing effect.
- Dihedral and anhedral wing results negative directional stability.

Contribution of Fuselage:



Sideways force towards starboard side is (+ve).

Sideways force acts on ac of fuselage.

- (CG) well below ac is creating yawing moment towards starboard side (+ve).

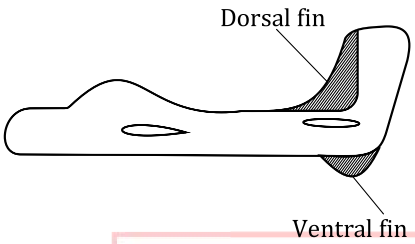
$$\frac{\partial N}{\partial \psi} = (+ve)$$

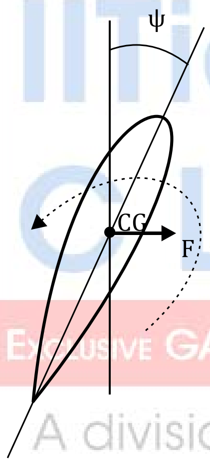
Yawing moment towards starboard is (+ve)

Yawing angle towards starboard is (+ve)

- Fuselage alone contribution unstable for directional stability.

Contribution of Dorsal and Ventral Fin:





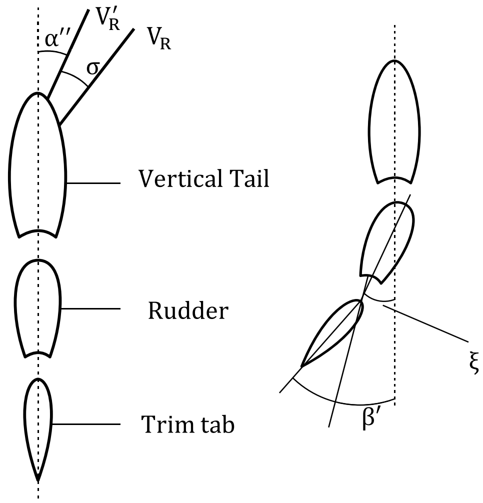
It creates yawing moment towards port side (−ve).

{ F = sideway force acts on ac of Dorsal and Ventral fin}.

$$\frac{\partial N}{\partial \psi} = \frac{-ve}{+ve} = -ve \quad \frac{\partial N}{\partial \psi} < 0$$

Dorsal and Ventral fin provides positive directional stability.

Contribution of Vertical Tail:



$\alpha'' + \sigma = \psi$

α'' = Effective incidence of fin

V'_R = Resultant velocity of vertical tail.

V_R = resultant velocity of a/c.

σ = Sidewash angle.

ξ = Rudder deflection angle

β' = Trim tub deflection angle

l_f = Fin lift arm.

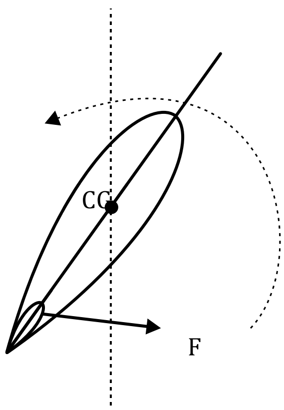
$N_{VT} = -L' l_f$ (Net moment acting due to vertical tail)

L' = Lift due to vertical tail

(−ve) because vertical tail is behind the CG it creates a restoring moment towards port side.

$C_{L_{VT}} = a'_1 \alpha'' + a'_2 \xi + a'_3 \beta'$

Since trim tab deflection D very small ($\beta' \simeq 0$)



(Anticlockwise moment)

$C_{LVT} = a_1'\alpha'' + a_2'\,\xi$

$a_1' = \dfrac{\partial C_{LVT}}{\partial \alpha''}$

$a_2' = \dfrac{\partial C_{LVT}}{\partial \xi}$

$L' = \dfrac{1}{2}\,\rho\,(V_R')^2\,S_{VT}\,C_{LVT}$

Now since, $N_{VT} = -L'l_f \qquad \{l_f = l_{VT}\}$

Now since,

$N_{VT} = -L'l_f$

$= -C_L'\left(\dfrac{1}{2}\,\rho(V_R')^2\,S_{VT}\right)l_{VT}$

$= -(a_1'\alpha'' + a_2'\xi)\left(\dfrac{1}{2}\,\rho(V_R')^2 S_{VT}\right)l_{VT}$

$N_{VT}@CG = -\dfrac{1}{2}\rho(V_R')^2 S_{VT} l_{VT}(a_1'(\psi-\sigma) + a_2'\xi)$

Non-Dimensional zing: $\left\{\div \dfrac{1}{2}\rho V_R^2 S_w b_w\right\}$

$C_{N_{VT}} = -\dfrac{\frac{1}{2}\,\rho(V_R')^2}{\frac{1}{2}\,\rho\,(V_R)^2}\dfrac{S_{VT}\,l_{VT}}{S_w b_w}\;(a_1'(\psi-\sigma) + a_2'\,\xi)$

$C_{N_{VT}} = -\eta_{VT}\,\bar{V}_{VT}(a_1'\,(\psi-\sigma) + a_2'\,\xi)$

$\dfrac{\partial C_{N_{VT}}}{\partial \psi} = -\eta_{VT}\,\bar{V}_{VT}\left(a_1'\left(1-\dfrac{\partial\sigma}{\partial\psi}\right) + a_2'\dfrac{\partial\xi}{\partial\psi}\right)$

If rudder is fixed (Pedal fixed condition) then

$\left(\dfrac{\partial\xi}{\partial\psi} = 0\right)$

$\dfrac{\partial C_{N_{VT}}}{\partial \psi} = -\eta_{VT}\bar{V}_{VT}\left(a_1'\left(1-\dfrac{\partial\sigma}{\partial\psi}\right)\right) \rightarrow \left(\text{Pedal fixed Condition}\right)$

$\left(\dfrac{\partial C_N}{\partial \psi} < 0\right)$

Vertical tail provides positively for directional stability (Main contributing member)

Pedal Free Directional Stability:

Rudder is free to oscillates until hinge moment tends to zero ($H_e = 0$)

$H' = \dfrac{1}{2}\,\rho\,(V_R')^2 S_R C_H'\,\bar{C}_R$

(When rudder is deflected hinge moment exerted).

S_R = Surface area of rudder

$\bar{C}_R$ = Rudder chord

C_H' = Hinge moment coefficient

$C_H' = \dfrac{H'}{\frac{1}{2}\,\rho(V_R')^2 S_R\bar{C}_R}$

$C_H' = b_1'\alpha'' + b_2\xi + b_3'\beta'$

($\beta' \simeq 0$ trim tab deflection)

$C_H' = b_1'\alpha'' + b_2'\xi$

$b_1' = \dfrac{\partial C_H'}{\partial\alpha''} \quad b_2' = \dfrac{\partial C_H'}{\partial\xi}$

Rudder is free to oscillate until (He) hinge moment is zero

$C_H' = 0 = b_1\alpha'' + b_2'\xi + b_3'\beta'(\beta' \simeq 0)$already

$0 = b_1'\,\alpha'' + b_2'\,\xi$

$\xi = -\dfrac{b_1'\,\alpha''}{b_2''} \quad \xi = -\dfrac{b_1'(\psi-\sigma)}{b_2'}$

$\dfrac{b\xi}{\partial\psi} = -\dfrac{b_1'}{b_2'}\left(1-\dfrac{\partial\sigma}{\partial\psi}\right)$

$C_{N_{cg}} = C_{N_{fuselage}} + C_{N_{wing}} - \eta_{NT}\bar{V}_{VT}\;[a_o(\psi-\sigma) + a_2'\xi]$

$\dfrac{\partial C_{N_{cg}}}{\partial\psi} = \dfrac{\partial C_{N_{fuselage}}}{\partial\psi} + \dfrac{C_{N_{wing}}}{\partial\psi} - \eta_{NT}\bar{V}_{VT}\;\left[a_1'\left(1-\dfrac{\partial\sigma}{\partial\psi} + a_2'\dfrac{\partial\xi}{\partial\psi}\right)\right]$

When rudder is free to oscillate

$\dfrac{\partial\xi}{\partial\psi} = -\dfrac{b_1'}{b_2'}\left(1-\dfrac{\partial\sigma}{\partial\psi}\right)$

$$\frac{\partial C_{N_{cg}}}{\partial \psi} = \left(\frac{\partial C_N}{\partial \psi}\right)_f + \left(\frac{\partial C_N}{\partial \psi}\right)_{wing} - \eta_f V_f a_1' \left(1 - \frac{\partial \sigma}{\partial \psi}\right)\left(1 - \frac{a_2'}{a_1'} \cdot \frac{b_1'}{b_2'}\right)$$

$$\frac{\partial C_{N_{cg}}}{\partial \psi} = \left(\frac{\partial C_N}{\partial \psi}\right)_f + \left(\frac{\partial C_N}{\partial \psi}\right)_{wing} - \eta_f V_f a_1' \left(1 - \frac{\partial \sigma}{\partial \psi}\right) F$$

F = free rudder factor

$$a_1' = \frac{\partial C_{L_{VT}}}{\partial \alpha''} \qquad a_2' = \frac{\partial C_{L_{VT}}}{\partial \xi}$$

$$b_1' = \frac{\partial C_H}{\partial \alpha''} \qquad b_2' = \frac{\partial C_H'}{\partial \xi}$$

Pure Yawing Motion (Dynamics):

Directional

Yaw motion is a function of $\underbrace{\psi \;\xi\; r}_{\text{State variables}}$

characteristics equation $\lambda^2 - N_r\lambda + N_\beta = 0$

$$\omega_n = \sqrt{N_\beta} \qquad 2\xi\omega_n = -N_r$$

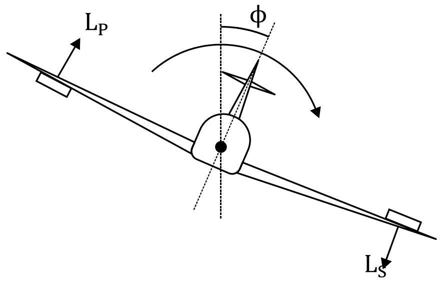
$$\xi = -\frac{N_r}{2\omega_n} \Rightarrow -\frac{N_r}{2\sqrt{N_\beta}}$$

$$N_r = \frac{\partial N/\partial r}{I_z} \qquad N_\beta = \frac{\partial N/\partial \dot{r}}{I_z}$$

$$N_\beta = \frac{\partial N}{\partial \beta} > 0$$

LATERAL STABILITY

Sign Convention: Rolling moment towards starboard is positive; rolling moment towards port side is negative & rolling angle towards starboard is positive ; rolling angle towards port is negative



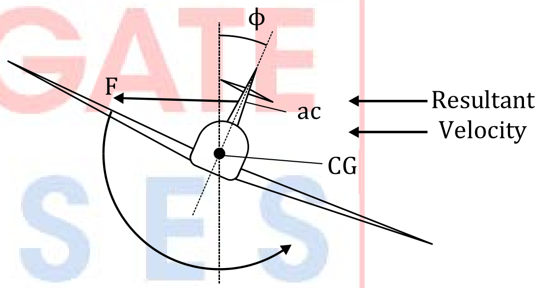
Criteria for Lateral Stability:

$$\frac{\partial C_L'}{\partial \phi} = 0$$

L' = Rolling moment

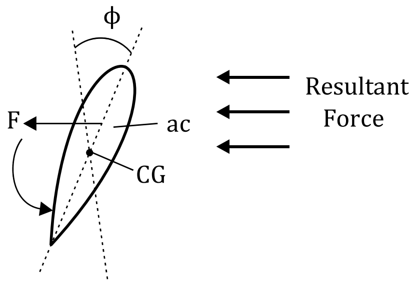
ϕ = Rolling angle

1. **Vertical tail contribution for lateral stability:**



Vertical tail stabilizes for lateral stability.

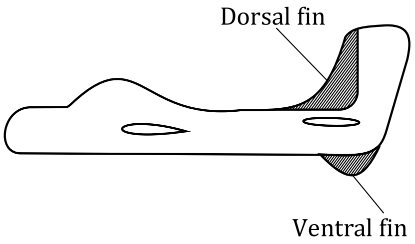
2. **Contribution of Fuselage:**

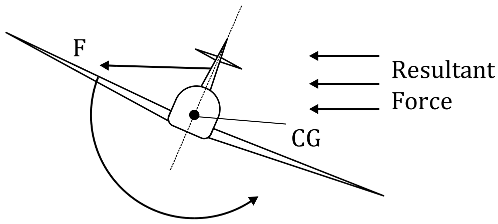


$$\frac{\partial C_L'}{\partial \phi} = \frac{-ve}{+ve} < 0 \text{ (stable)}$$

Fuselage contribution for lateral stability is $\left(\frac{\partial C_L'}{\partial \phi} < 0\right)$ which stabilize in lateral mode.

3. **Contribution of Dorsal and Ventral Fin:**



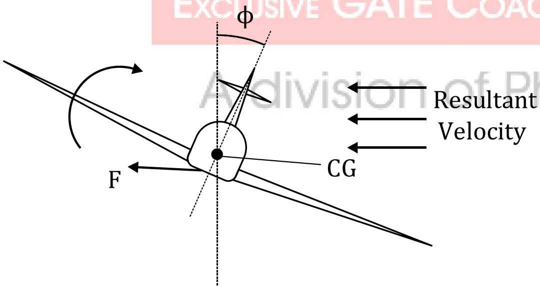


For Dorsal Fin

$$\frac{\partial C_L'}{\partial \phi} = \frac{-ve}{+ve} = (-ve) < 0 \text{ (stable)}$$

Dorsal fin contributes positively for lateral stability.

4. **Ventral Fin:**



$$\frac{\partial C_L'}{\partial \phi} = \frac{+ve}{-ve} > 0$$

Ventral fin contributes negatively for lateral stability.

5. **Wing Contribution:**

- **Dihedral Wing:**

  Dihedral angle (Γ)

  Dihedral angle (β)

$$\Delta\alpha = \beta\Gamma(\text{rad}) \Rightarrow \Delta\alpha = \frac{\beta\Gamma}{57.3}\ (\text{deg})$$

$$\Delta C_L = \Delta_\alpha \frac{dC_L}{d\alpha} \Rightarrow \Delta C_L = \frac{\beta\Gamma}{57.3}\frac{dC_L}{d\alpha}$$

Net rolling moment acting on airplane

$L' = -2\Delta L\bar{y}$ ($-$ve sign for anticlockwise moment)

ΔL = Change in lift.

Net change in both the wing so ($\times$ by 2)

$$\Delta L = \frac{1}{2}\rho V^2 S_\Gamma \Delta C_L$$

S_Γ = Dihedral wing area of one wing.

$$\Delta L = qS_\Gamma \Delta C_L$$

$$\Delta L = qS_\Gamma \frac{\beta\Gamma}{57.3}\frac{dC_L}{dC_\alpha}$$

Net change in rolling moment

$L' = 2\Delta L\bar{y}$.

$$L' = -2q\, S_\Gamma \frac{\beta\Gamma}{57.3}\frac{dC_L}{d\alpha}\bar{y}$$

$$C'_{L_{roll}} = -2\frac{\beta\Gamma}{57.3}\frac{dC_L}{d\alpha}\frac{\bar{y}}{b}\frac{S_\Gamma}{S} \leftarrow \frac{dC'_{L_{roll}}}{\partial\beta} < 0$$

$$C'_{L_\beta} = -\frac{2\Gamma}{57.3}\frac{dC_L}{d\alpha}\frac{\bar{y}}{b}\frac{S_\Gamma}{S}$$

$\rightarrow$ Dihedral wing contributes stabilizing effect for lateral stability.

- **Sweep Back Wing:**

  Net rolling moment,

$$L' = -(L_s - L_p)\bar{y}$$

L_s = Starboard Lift L_p = port lift.

$$L_s = \frac{1}{4}\rho V^2 SC_L(\cos(\lambda - \beta))^2$$

$$L_s = \frac{1}{4}\rho SC_L(V\cos(\lambda - \beta))^2$$

$$\frac{\partial C_L'}{\partial \beta} = -\frac{C_L}{57.3}\frac{\bar{y}}{b}\sin 2\lambda$$

Sweep back wing provides stabilizing effect for lateral stability

Lateral Directional (Routh's Criteria):

$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$

To check the above equation is the characteristic equation for lateral direction

1. D and E > B and C
2. $(A \simeq 1)$
3. One of the coefficients is negative.

$R = BCD - D^2A - B^2E > 0 \ (R > 0)$

If one root is (+ve); it is corresponding to directional divergence.

If roots are (−ve); it is corresponding to damping in roll.

If roots are (−ve) but very small value i.e., in the order of 10^-3 then it is corresponding to spiral motion

If roots are complex pair with -ve real part; it is corresponding to Dutch-roll mode $-p \pm iq$.

Pure Pitching Motion (Dynamics): (Longitudinal)

θ = Pitch angle; q = Pitch axis

Governing DE for pitching moment at CG of a/c

$\sum M_{cg} = I_y\ddot{\theta}$

$\Delta M_{cg} = I_y\,\Delta\ddot{\theta}$

Characteristics Equation:

$\lambda^2 - (M_q - M_{\dot{\alpha}})\lambda - M_\alpha = 0$

$\omega_n = \sqrt{-M_\alpha}$

$2\xi\omega_n = -(M_q = M_{\dot{\alpha}})$

$\Rightarrow \xi = -\frac{(M_q - M_{\dot{\alpha}})}{2\omega_n}$

$\xi = -\frac{(M_q - M_{\dot{\alpha}})}{2\sqrt{-M_\alpha}}$

Ignoring compressibility,

$\xi = \frac{1}{(L/D)\sqrt{2}}$

$\omega = \frac{\sqrt{2}\,g}{V}$ and $T = \frac{\sqrt{2}\,V\pi}{g}$

where T is the time period of phugoid oscillation

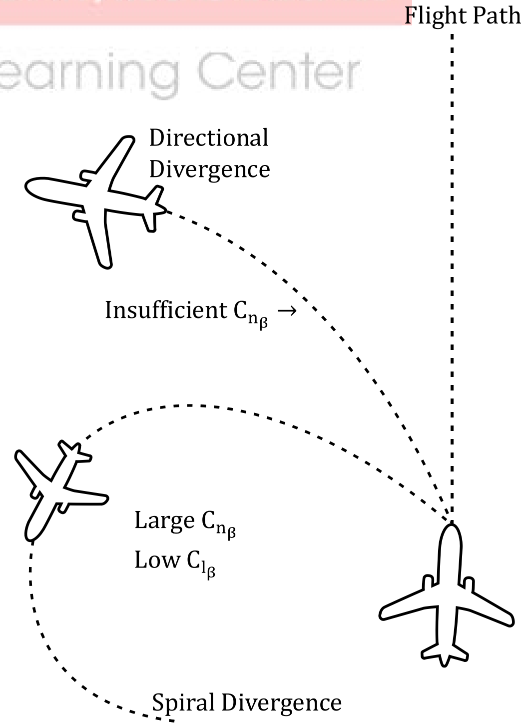
Pure Rolling Motion (Dynamics): (Lateral)

$\sum$ Rolling motion $= I_x\,\ddot{\phi}$

$\frac{\partial L}{\partial \delta_a}\,\Delta\,\delta_a$

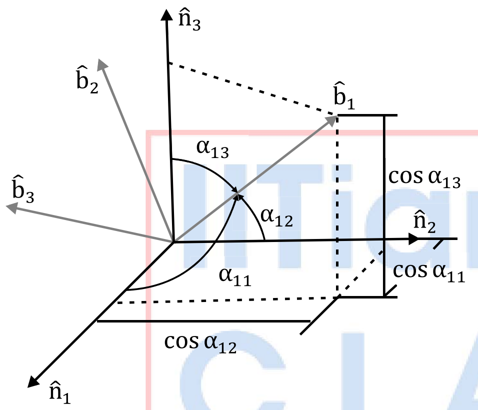
Roll Moment due to aileron deflection

$\frac{\partial L}{\partial P}\,\Delta P$ Roll damping moment



Direction Cosines And Euler Angles

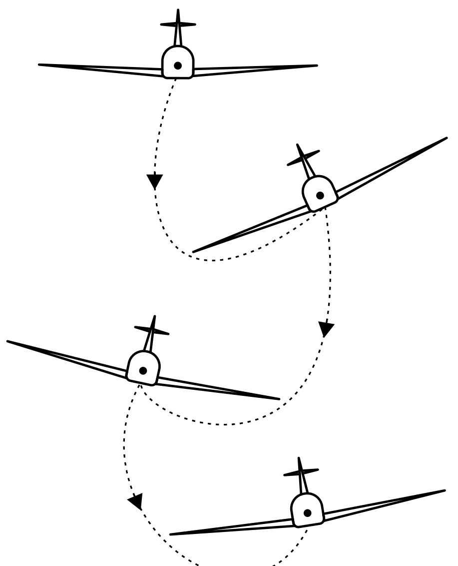
Coordinates that completely describes the orientation of a rigid body relative to some reference frame. **Three** parameters are necessary and sufficient for orientation description (minimal set). A minimal set will contain at least one orientation where the kinematic differential equations are singular Singularity is avoided by using redundant sets of more than three parameters.



$$\begin{bmatrix} \cos\alpha_{11} & \cos\alpha_{12} & \cos\alpha_{13} \\ \cos\alpha_{21} & \cos\alpha_{22} & \cos\alpha_{23} \\ \cos\alpha_{31} & \cos\alpha_{32} & \cos\alpha_{33} \end{bmatrix}$$

$[C_I^R] = [C_B^R][C_I^B]$ projects N frame to R frame

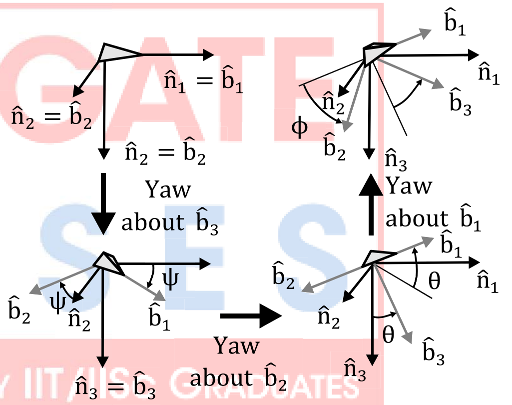
$([\dot{C}] + [\tilde{\omega}][C])\{\hat{n}\} = 0$



Dutch Roll Mode

Euler Angles

Most commonly used sets of attitude parameters, Describe attitude of reference frame B relative to the frame I through three successive rotation angles about axes $\{\hat{b}\}$. Standard set of Euler angles used in aircraft and missile is (3(Yaw)-2(Pitch)-1(Roll)) i.e., (ψ, θ, ϕ). This sequence is called **asymmetric** set. Other is (3-1-3) set of Euler angles, to define the orientation of orbital planes of satellites relative to the equatorial plane of planet. It is called **symmetric** set.



The final DCM is obtained by three cascading rotations

$$[M(\psi)] = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[M(\theta)] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$[M(\phi)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

The direction cosine matrix in terms of the (3-2-1) Euler angles is

$C = [M(\phi)][M(\theta)][M(\psi)]$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{32} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{bmatrix}$$

$\theta = \sin^{-1}(-C_{13})$:

Not unique, $-\frac{\pi}{2} \le \sin^{-1}(-C_{13}) \le \frac{\pi}{2}$,

$\psi = \tan^{-1}\left(\frac{C_{12}}{C_{11}}\right)$, $\phi = \tan^{-1}\left(\frac{C_{23}}{C_{33}}\right)$

*****

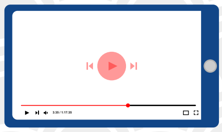
OUR COURSES

GATE Online Coaching

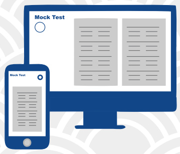
Course Features

Live Interactive Classes

E-Study Material



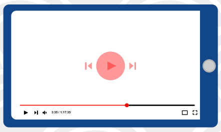
Recordings of Live Classes



Online Mock Tests

TARGET GATE COURSE

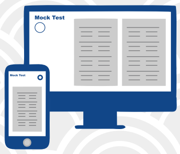
Course Features



Recorded Videos Lectures

Online Doubt Support

E-Study Materials



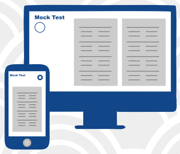
Online Test Series

Distance Learning Program

Course Features

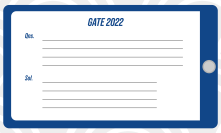
E-Study Material

Topic Wise Assignments (e-form)



Online Test Series

Online Doubt Support

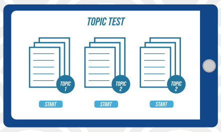


Previous Year Solved Question Papers

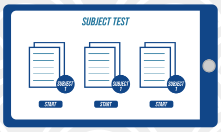
OUR COURSES

Online Test Series

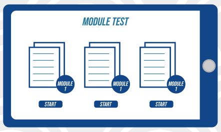
Course Features



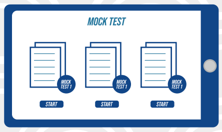
Topic Wise Tests



Subject Wise Tests



Module Wise Tests



Complete Syllabus Tests

More About IGC

SAMPLE STUDY MATERIALS

https://bit.ly/3Wu7lsp

SAMPLE MOCK TESTS

https://bit.ly/3FDyF0f

SAMPLE VIDEO LECTURES

https://bit.ly/3V4xU6j

TEAM IGC GATE

https://bit.ly/3G21khi

OUR TESTIMONIALS

https://bit.ly/3Yp2Hh7

IGC TELEGRAM GROUP

https://bit.ly/3j4koCd

IITians GATE CLASSES

Exclusive GATE Coaching by IIT/IISc Graduates

A division of PhIE Learning Center

Admission Open for

GATE 2025/26

Live Interactive Classes

AEROSPACE ENGINEERING

ENROLL NOW

For more Information Call Us

+91-97405 01604

Visit us **www.iitiansgateclasses.com**