[AE][2017], Q-32 [Correct: 2 Marks, Wrong: 0]

Consider an Incompressible flow over a flat plate with the following approximation to the velocity profile:

$$\frac{u(y)}{U} = \begin{cases} y/\delta & \text{for } y \le \delta \\ 1 & \text{for } y > \delta \end{cases}$$

where δ is the boundary layer thickness and U the free-stream speed. The normalized momentum thickness (θ/δ) for this profile is ______ (in three decimal places)

Solution → For Incompressible flow the momentum thickness, θ, is given as

$$\theta = \int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy \approx \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

and the Integrand essentially will be zero for $y > \delta$ [for this particular problem]

* Although for $y \ge \delta$, Integrand will be zero.

But for this case they have mentioned

$\frac{u(y)}{U} = \frac{y}{\delta}$ for $y \leq \delta$, So Integrand will not be zero at $y = \delta$.

As $\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$

$$\Rightarrow \theta = \int_0^{\delta} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy$$

$$\Rightarrow \theta = \int_0^{\delta} \left[\frac{y}{\delta} - \frac{y^2}{\delta^2}\right] dy$$

$$\Rightarrow \theta = \int_0^{\delta} \frac{y}{\delta} dy - \int_0^{\delta} \frac{y^2}{\delta^2} dy$$

$$\Rightarrow \theta = \left[\frac{y^2}{2\delta}\right]_0^{\delta} - \left[\frac{y^3}{3\delta^2}\right]_0^{\delta}$$

$$\Rightarrow \theta = \left[\frac{\delta^2}{2\delta}\right] - \left[\frac{\delta^3}{3\delta^2}\right]$$

$$\Rightarrow \theta = \frac{\delta}{2} - \frac{\delta}{3}$$

So $\frac{\theta}{\delta} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

or $\frac{\theta}{\delta} = 0.16667 = 0.167$ (three decimal places)

End

[AE][2017], Q-33, [Correct: 2 Marks, Wrong: 0]

An Idealized velocity field is given by $\vec{V} = 4tx\hat{i} - 2t^2y\hat{j} + 4xz\hat{k}$. At point (−1, 1, 0) and at $t = 1$, the magnitude of the material acceleration vector of the fluid element is ____

Solution Material acceleration is given by Substantial derivative of velocity vector or total derivative of velocity vector. It is given as

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

or

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

where

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

where u, v and w are velocity component is x, y and z direction respectively

Now $a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$

as $\vec{V} = 4tx\hat{i} - 2t^2y\hat{j} + 4xz\hat{k}$ —— (A)

moreover $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$ —— (B)

comparing eqn (A) and (B) we get

$u = 4tx$

$v = -2t^2y$

$w = 4xz$

So $a_x = \frac{Du}{Dt} = \frac{\partial}{\partial t}(4tx) + 4tx\frac{\partial}{\partial x}(4tx) + [-2t^2y]\frac{\partial}{\partial y}[4tx]$

$+ 4xz\frac{\partial}{\partial z}(4tx)$

$a_x = \frac{Du}{Dt} = 4x + 16t^2x + 0 + 0 = 4x + 16t^2x$

$a_x = \frac{Du}{Dt} = 4(-1) + 16(1)^2(-1)$, at $(-1, 1, 0)$ and $t=1$

So $a_x = -20$ unit

Similarly

$$a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_y = \frac{Dv}{Dt} = \frac{\partial}{\partial t}[-2t^2y] + 4tx\frac{\partial}{\partial x}[-2t^2y] + [-2t^2y]\frac{\partial}{\partial y}[-2t^2y] + 4xz\frac{\partial}{\partial z}[-2t^2y]$$

$$a_y = \frac{Dv}{Dt} = -4ty + 0 + 4t^4y$$

$a_y = -4 + 4 = 0$, at $(-1, 1, 0)$ and $t = 1$.

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial}{\partial t}[4xz] + [4tx]\frac{\partial}{\partial x}[4xz] + [-2t^2y]\frac{\partial}{\partial y}[4xz] + [4xz]\frac{\partial}{\partial z}[4xz]$$

$$a_z = \frac{Dw}{Dt} = 0 + 16xzt + 0 + 16x^2z$$

$$a_z = 16(-1)(0)(1) + 16(-1)^2(0) = 0$$

at $(-1, 1, 0)$ and $t = 1$.

So $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} = -20\hat{i} + 0\hat{j} + 0\hat{k}$

$$\boxed{\vec{a} = -20\hat{i}}$$

magnitude of $\vec{a} \Rightarrow |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$|\vec{a}| = \sqrt{(-20)^2} = 20$ units

End