IITians GATE CLASSES

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QUICK REVISION

FORMULA SHEET

for

GATE-AE AIRCRAFT PERFORMANCE

Table Of Content

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AIR - 2

Akriti
PEC, CHANDIGARH
AIR - 6

SHREYASHI SARKAR
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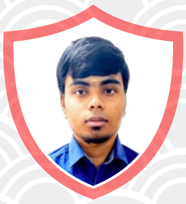
AIR - 8



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AIR - 14

And Many More ....

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OUR PSU JOB ACHIEVERS

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Job Position for Recruitment **(2022-23)** Based on **GATE AE** score

- ***Mr. Abhilash K*** *(Amrita Univ Coimbatore)*
- ***Ms. Ajitha Nishma V*** *(IIST Trivendrum)*
- ***Mr. Dheeraj Sappa*** *(IIEST Shibpur)*
- ***Ms. F Jahangir*** *(MIT Chennai)*
- ***Mr. Goutham*** *(KCG College Chennai)*
- ***Mr. M Kumar*** *(MVJ College Bangalore)*
- ***Mr. Mohit Kudal*** *(RTU Kota)*
- ***Mr. Niladhari Pahari*** *(IIEST Shibpur)*
- ***Mr. Nitesh Singh*** *(Sandip Univ Nashik)*
- ***Mr. Ramanathan A*** *(Amrita Univ Coimbatore)*
- ***Ms. Shruti S Rajpara*** *(IIEST Shibpur)*

HAL DT ENGINEER

Job Position for Recruitment **(2023)**

- ***Mr. Anantha Krishan A.G*** *(Amrita Univ Coimbatore)*
- ***Mr. S.S Sanjay*** *(Amrita Univ Coimbatore)*
- ***Mr. Shashi Kanth M*** *(Sastra Univ Tanjore)*
- ***Mr. Vagicharla Dinesh*** *(Lovely Professional Univ Panjab)*

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SADSIVUNI TARUN
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VIGNESHA .M
(MIT, CHENNAI)
MRS E-II CRL BEL

RAM GOPAL SONI
(GVIET, PUNJAB)
CEMILAC LAB, DRDO

AIRCRAFT PERFORMANCE

BASICS OF ATMOSPHERE

- **Density (ρ):**

  $$\text{Density } (\rho) = \frac{\text{Mass}}{\text{Volume}} \left(\frac{\text{kg}}{\text{m}^3}\right)$$

  Density at sea level for

  Water $= 1000 \text{ kg/m}^3$

  Air $= 1.2256 \text{ kg/m}^3$

- **Specific Weight:**

  $$\text{Specific weight} = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

  Weight $\{w = mg\}$

  $$\text{Specific weight (w)} = \frac{mg}{V} = \rho g$$

- **Specific volume (v):**

  $$\text{Specific volume (v)} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}}$$

  $$= \frac{V}{m} = \frac{1}{\rho}$$

- **Specific Gravity/Relative Density:**

  1. For Liquid $= \left(\dfrac{\text{Weight density of fluid}}{\text{Weght density of water}}\right)$
  2. For Gas $= \left(\dfrac{\text{Weight density of gas}}{\text{Weght density of air}}\right)$

- **Viscosity:**

  $$\tau = \mu \frac{du}{dy}$$

  $\frac{du}{dy}$ velocity gradient $\left(\frac{1}{s}\right)$

  τ = Shear stress (N/m^2)

  μ = Coefficient of VIscosity (kg/m-s)

- **Maxwell Law of Viscosity:**

  $$\frac{F}{A} = \mu \frac{V}{h}$$

  μ = coefficient of viscosity

  At sea level for air,

  $\mu_\infty = 1.7894 \times 10^{-5}$ kg/m-s (or) N-s/m^2

  $$1 \text{ poise} = 1 \frac{\text{dyne} - \text{sec}}{\text{cm}^2} \text{ in CGS}$$

  1 dyne $= 10^{-5}$N

  $\{1 \text{ Ns/m}^2 \text{ or kg/m-s} = 10\text{poise}\}$

  For Liquids:

  $$\mu = \mu_o \left(\frac{1}{1 + \alpha t + \beta t^2}\right)$$

  (μ and μ_o in poise)

  μ_o - Viscosity of liquid at °C

  μ - Viscosity of liquid any (t)

  t - Temperature (°C)

  α and β are constant

  eg: Water

  $\mu_o = 1.79 \times 10^{-3}$ poise

  $\alpha = 0.03368 \quad \beta = 0.00221$

  For Gasses: (Air)

  $\mu = \mu_o + \alpha t - \beta t^2$

  $\mu_o = 1.7 \times 10^{-5}$poise

  $\alpha = 5.6 \times 10^{-8} \quad \beta = 0.1189 \times 10^{-9}$

- **Rayleigh Law of Viscosity:**

  $$\frac{\mu_2}{\mu_1} = \left(\frac{T_2}{T_1}\right)^{3/4} \text{(or)} \frac{\mu}{\mu_\infty} = \left(\frac{T}{T_\infty}\right)^{3/4}$$

  At sea level

  $T_\infty = 288.16$ K (Air)

  $\mu_\infty = 1.789 \times 10^{-5}$ kg/m-s

Kinematic Viscosity (ν)

$$= \frac{\text{Dynamic viscosity}}{\text{Density of air}}$$

$$= \frac{\mu}{\rho}, unit - \left(\frac{m^2}{s}\right)$$

At sea level

$$\nu = \frac{\mu}{\rho} = \frac{1.7894 \times 10^{-5}}{1.2256}$$

$$= 1.46 \times 10^{-5}\ m^2/s$$

- **Bulk Modulus (k):**

$$(k) = \frac{\text{Increase in pressure}}{\text{Volumetric Strain}}$$

$$k = \frac{\text{Compressive stress}}{\text{VOlumetric Strain}} = -\frac{dp}{dV/V}$$

$$\left\{\frac{1}{k}\ \text{compressibility}\right\}$$

- **Pressure:**

At sea level

$P_\infty = 101325$ bar $\{1\ \text{bar} = 10^5\ N/m^2]$

$P_\infty = 760$ mm of Hg (Mercury)

$P_\infty = 10.3$ m of water column

1 stroke $= 1\ cm^2/sec$ in CGS unit

$\{1\ \text{stroke} = 10^{-4}\ m^2/s\}$

1 ata $= 1\ kg/cm^2$

$$1\ \text{ata} = \frac{9.81}{10^{-4}} N/m^2 = 0.98100\ \text{bar}$$

GAS CONSTANT

For air

$R = 287.026$ J/kgK

- **Density (ρ):**

$R = C_p - C_V$

$$\gamma = \frac{C_p}{C_v} = 1.4\ \text{(for air)}$$

Where

C_p of air $= 1005$ J/kgK

C_v of air $= 718$ J/kgK

For monoatomic gases = 1.67

For diatomic gases = 1.33

- **Gravitational Acceleration:**

$g = 9.80665\ m/s^2$

ATMOSPHERE

- Refer Altitude vs Temperature graph

Basic composition of atmosphere

Nitrogen 78.09%

Carbon dioxide 0.03%

Oxygen 20.95%

Ozone 0.000003%

Water vapor 0 to 3% (Mostly 0.93%)

- **Ideal Gas Equation:**

$$PV = mRT \Rightarrow P = \frac{m}{V}RT \Rightarrow P = \rho RT$$

m = number of moles × Molecular weight

$pV = mRT \Rightarrow pV = nMRT\ \{M \times R = \bar{R}_u\}$

$pV = n\bar{R}_u T\ \ (\bar{R}_u = 8314$ J/kg mole-K)

R = specific gas constant

$\bar{R}_u$ = Universal gas constant

- **Layers of Atmosphere:**

1. Troposphere (0-11)km → (Aircraft Application).
   Troposphere (11-25)km → (Lower Stratosphere).
2. Stratosphere (25-50 km) → weather balloon (Upper Stratosphere).
3. Mesosphere (50-80 km) -Meteors

4. Thermosphere (Ionosphere) (80-500 km)
   Ionosphere Aurora (Northern Hemisphere)
   Ionosphere Borolis (Southern Hemisphere)
5. Exosphere (500-10000 km) Rarified gases.
   - **Troposphere (0-11 km)**

Lapse rate $\lambda = -\dfrac{dT}{dh}$,

$$\lambda = -\frac{T - T_\infty}{h - h_\infty}$$

$h_\infty = 0$ at sea level altitude

$$\lambda = -\frac{T - T_\infty}{h}$$

$\Rightarrow T = T_\infty - \lambda h$

h is upto 11 kms

T = Temperature

Using Hydrostatic Equation:

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{g/\lambda R}$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{g}{2\lambda}-1}$$

For Troposphere

$\lambda = 0.0065$ k/m

$g = 9.80665\ \text{m/s}^2$ (3/4 = 0.75)

$R = 287.026\ \text{J/kg K}$

$$\frac{\mu_2}{\mu_1} = \left(\frac{T_2}{T_1}\right)^{0.75}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{5.256}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{4.256}$$

   - **Lower Stratosphere (11-25km)** (Isothermal Region)

$$\lambda = \frac{dT}{dh} = 0 \quad (T = \text{constant})$$

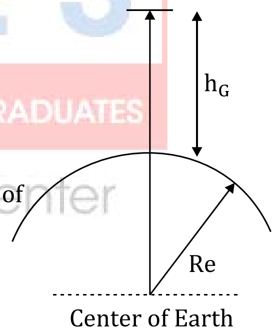
$T_s = 216.66$ K

$$\frac{P_2}{P_1} = e^{-g/RT_s(h_2-h_1)}$$

$$\frac{\rho_2}{\rho_1} = e^{-g/RT_s(h_2-h_1)}$$

$$\frac{\mu_{25}}{\mu_{11}} = 1 \ (T = C)$$

- **Various Kind of Altitude:**
  1. **Geometric altitude (h_G):**
     Used for airplane altitude.
  2. **Absolute altitude (h_a):**
     Height above centre of earth.
     $h_a = h_G + R_E$; R_E =Latitude of 45°
     $R_E = 6356.766$ km
     Approx $R_E = 6400$ km



$$g \propto \frac{1}{(h_a)^2} \Rightarrow g = g_o\left(\frac{R_E}{h_a}\right)^2$$

$$g = g_o\left(\frac{R_e}{h_G + R_E}\right)^2$$

  3. **Geopotential Altitude (H):** $g = g_o$

$$h = \left(\frac{R_E}{R_E + h_a}\right)h_a \Rightarrow h = \left(\frac{h_G}{h_a}\right)R_E$$

  4. **Pressure Altitude:**

$$h = 19220\log_{10}\left(\frac{101.325}{P}\right)$$

P is in kPa Emperical relation

5. **Temperature Altitude:**

   $T = T_\infty - \lambda h$

   λ – lapse rate

   here h is in meter

6. **Density Altitude:**

   $$\frac{\rho}{\rho_\infty} = \frac{20 - h}{20 + h} \text{ Emperical relation}$$

   (h is in km)

   Valid upto 12-30 kms

   $$\frac{\rho}{\rho_\infty} = \left(\frac{T}{T_\infty}\right)^{4.256} \quad \text{for trospopheric}$$

   $$\frac{\rho_2}{\rho_1} = e^{-(g/RT_s)(h_2 - h_1)} \text{ For Stratosphere}$$

   Note: For pressure altitude refer previous page.

DIFFERENT KIND OF AIRSPEEDS

1. Indicated air speed (IAS)
2. Calibrated air speed (CAS)
3. Equivalent air speed (EAS)
4. True air speed (TAS)

For Incompressible Flow (M < 0.3):

$$P_o = P + \frac{1}{2}\rho v^2$$

$$V_{TAS} = \sqrt{\frac{2(P_o - P)}{\rho}}, V_{EAS} = \sqrt{\frac{2(P_o - P)}{\rho_{SL}}}$$

ρ = Density at altitude

ρ_{SL} = Density at sea level

$$\frac{V_{EAS}}{V_{TAS}} = \sqrt{\frac{\rho}{\rho_{SL}}} = \sqrt{\sigma} \quad (\sigma = \text{Relative density})$$

At sea level ($\sigma = 1$) $P_o = P + \frac{1}{2}\rho V^2$

$$\frac{P_o - P}{\frac{1}{2}\rho V^2} = 1$$

$V_{EAS} = V_{TAS}$

For Compressible flow ($M > 0.3$)

$$V_{TAS}^2 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{P_o - P}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$V_{EAS}^2 = \frac{2a_{SL}^2}{\gamma - 1}\left[\left(\frac{P_o - P}{P_{SL}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

For incompressible flow,

$$C_P = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \left(1 - \left(\frac{V}{V_\infty}\right)^2\right)$$

$$\rightarrow P_o - P = \frac{1}{2}\rho v^2\left[1 + \frac{M^2}{4} + \frac{M^4}{40} + \cdots\right]$$

At stagnation point ($V = 0$) so, $C_p = 1$ (maximum +ve value)

C_p = Pressure coefficient.

AERODYNAMICS FORCES

Lift (L) $= \frac{1}{2}\rho V^2 S C_L$

Drag (D) $= \frac{1}{2}\rho V^2 S C_D$

Drag Polar

$C_D = C_{D_o} + KC_L^2 + C_{DW}$

$$C_M = \frac{M}{\frac{1}{2}\rho V^2 S\bar{c}}$$

C_{D_o} = Parasite darg

S = Surface area

ρ = density

C_L = coefficient of lift

C_D = Coefficient of drag

C_{D_i} = Induced drag

C_{DW} = Wave drag (Mach drag)

C_M = Moment coefficient

$\bar{c}$ = chord

for subsonic ($C_{DW} = 0$) $\Rightarrow C_D = C_{D_o} + kC_L^2$

$C_{D_i} = kC_L^2$ where, $\quad k = \dfrac{1}{\pi eAR}$

e = span efficiency factor;

(for elliptic wing $e = 1$)

(for rectangular & tapered wing $e < 1$)

C_{D_o} (Linear curve)

C_L^2 (Parabolic curve) (k – Slope)

AERODYNAMIC CENTRE

The point about which pitching moment coefficient remains constant

$$\frac{dC_m}{d\alpha} \text{ or } \frac{dC_m}{dC_L} = 0$$

$$\frac{x_{ac}}{\bar{C}} = \frac{a}{c} - \frac{dC_m}{dC_L}$$

$\frac{a}{c}$ = Spindle ratio

For Subsonic airfoil x_{ac} lies at 23 % to 25% $\bar{C}$. where $\alpha < 10°$

For supersonic airfoil x_{ac} lies at half of chord 50% of $\bar{C}$

Category of Aircrafts based on Mach Numbers:

1. Subsonic aircraft ($M < 1$)
2. Supersonic aircraft ($M > 1$)
3. Transonic aircraft ($0.8 > M < 1.2$)
4. Hypersonic aircraft ($M > 5$)

   ($M = 1$) Transonic aircraft

   $$M = \frac{V}{\sqrt{\gamma RT}}$$

   {V = Velocity, T = Temperature}

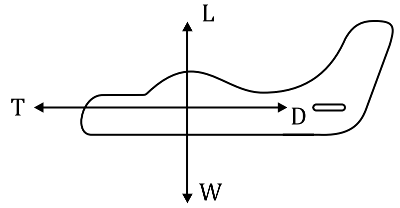
FLIGHT PERFORMANCE

Level Flight: (Steady and unaccelerated flight)

$L = W$

$T = D$

$n = 1$



where W = weight; T = Thrust; L = Lift; D = Drag

Climbing Flight

$L = W\cos\gamma \Rightarrow \frac{L}{W} = \cos\gamma \quad \ldots (1)$

$n = \frac{L}{W}$ load factor for flight ($n < 1$)

as $\cos\gamma < 1$

$T - W\sin\gamma - D = 0$

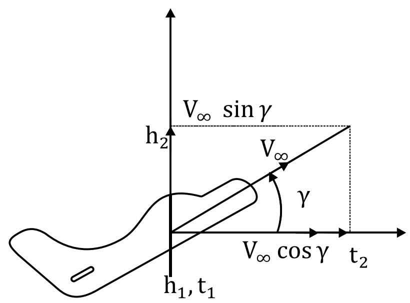
$\Rightarrow \frac{T-D}{W}$

$= \sin\gamma$ (Multiplying both sides with V_∞)

Rate of climb$=\frac{R}{C}$

$$V_\infty\left(\frac{T-D}{W}\right) = V_\infty \sin\gamma \ldots (2)$$

(V_∞) Free – stream velocity



P_{av} = Power available $= T.V_\infty$

P_R = Power required $= D.V_\infty$

$$\frac{V_\infty T - V_\infty D}{W} = V_\infty \sin\gamma$$

$$\frac{P_{av}-P_R}{W} = V_\infty \sin\gamma = \frac{R}{C}$$

$P_{av} - P_R$ – Excess power

V_c = Climbing velocity

$$\frac{dh}{dt} = V_\infty \sin\gamma = \frac{h}{\text{Endurance}}$$

For piston engine aircraft $(V_C)_{max}$ occurs at $(P_R)_{min}$ condition $(V_C)_{max} = \frac{(P_{av})-(P_R)_{min}}{W}$

For Jet engine

$$\frac{R}{C} = \left(\frac{T}{W}C_L^{-\frac{1}{2}} - (C_{D_o} + kC_L^2)C_L^{-\frac{3}{2}}\right)\sqrt{\frac{2W}{P_\infty S}}$$

Condition for maximum (ROC) (R/C):

$$\frac{T}{W}C_L - 3CD_o + kC_L^2 = 0$$

Calculate C_L from above relation and substitute in $\frac{R}{C}$ equation

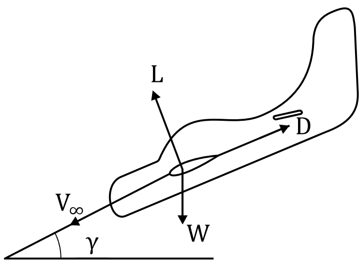
$$\left(\frac{R}{C}\right)_{max} = \left(\frac{T}{W} - \frac{(C_{D_o} + kC_L^2)}{C_L}\right)\sqrt{\frac{2W}{\rho S C_L}}$$

GLIDING FLIGHT:

$W\sin\gamma = D$ ... ① (Parallel force)

$L = W\cos\gamma$ .... ② (Perpendicular force)

$$n = \frac{L}{W} = \cos\gamma \ (n < 1)$$



1. $\Rightarrow D = W\sin\gamma$ {÷ ① with ②}
2. $\Rightarrow L = W\cos\gamma$

$$\frac{D}{L} = \frac{W\sin\gamma}{W\cos\gamma} \Rightarrow \frac{D}{L} = \tan\gamma$$

$$\tan\gamma = \frac{1}{\frac{L}{D}}$$

$$\Rightarrow \gamma_{min} = \tan^{-1}\left(\frac{1}{\left(\frac{L}{D}\right)_{max}}\right) .. ③$$

Referring figure, $\tan\gamma = \frac{h}{\text{Range}}$

$$\tan\gamma = \frac{1}{\left(\frac{L}{D}\right)_{max}} = \frac{h}{(\text{Range})_{max}}$$

Conditions for range and Endurance:

a. **Minimum Drag Condition:**

It is the condition for maximum range of piston engine a/c. and Condition for maximum endurance of jet engine a/c.

For level flight

$L = W$; $T = D$

$$\frac{L}{W} = 1 \quad \frac{D}{T} = 1$$

{T_R – Thrust required}

$$D_{min} = (T_R)_{min} = \frac{W}{\left(\frac{L}{D}\right)_{max}}$$

For minimum drag, $\frac{L}{D}$ should be maximum

$T_R = D$

$$= \underbrace{\left(C_{D_o} \times \frac{1}{2}\rho S\right)}_{K_1} V_\infty^2 + \underbrace{\left(\frac{kW^2}{\frac{1}{2}\rho s}\right)}_{K_2} \frac{1}{V_\infty^2} .. ①$$

$$T_R = D = K_1 V_\infty^2 + \frac{K_2}{V_\infty^2} .. ②$$

For minimum drag condition

$$\frac{dT_R}{dV_\infty} = 0$$

$$V_{md} = \sqrt[4]{\frac{k}{C_{D_o}}} \cdot \sqrt{\frac{2W}{\rho s}}$$

V_{md} is minimum drag velocity

For minimum drag condition

$$C_{D_o} = kC_L^2$$

$$\Rightarrow C_L = \sqrt{\frac{C_{D_o}}{k}}$$

$$C_{D_{md}} = C_{D_o} + kC_L^2$$

$$C_{D_{md}} = C_{D_o} + C_{D_o}$$

$$\Rightarrow C_{D_{md}} = 2C_{D_o}$$

$$\left(\frac{L}{D}\right)_{md} = \left(\frac{L}{D}\right)_{max} = \left(\frac{C_L}{C_D}\right)_{md}$$

$$= \left(\frac{\sqrt{C_{D_o}/k}}{2\,C_{D_o}}\right)$$

$$\left(\frac{L}{D}\right)_{md} = \left(\frac{C_L}{C_D}\right)_{md} = \frac{1}{2\sqrt{kC_{D_o}}}$$

$$\left(\frac{L}{D}\right)_{max} \Rightarrow C_{D_o} = C_{D_i}$$

Minimum drag is independent of altitude

$$D_{min} = 2W\sqrt{kC_{D_o}}$$

$$C_D = 2C_{D_o}$$

b. **Minimum Power required conditions:**

It is the condition for max endurance for piston engine aircraft.

$$P_R = T_R V_\infty$$

$P_R =$ Power required

$$P_R = K_1 V_\infty^3 + \frac{K_2}{V_\infty}$$

For minimum P_R required condition

$$\frac{dP_R}{dV_\infty} = 0$$

$$V_{mp} = \sqrt[4]{\frac{k}{3C_{D_o}}} \sqrt{\frac{2W}{\rho s}}$$

$$\frac{V_{mp}}{V_{md}} = \sqrt[4]{\frac{1}{3}} = 0.7598$$

$$V_{mp} = 0.7598\, V_{md}$$

$$V_{mp} < V_{md}$$

For minimum power required condition

$$3C_{D_o} = kC_{L_{mp}}^2$$

$$C_{D_{mp}} = C_{D_o} + 3C_{D_o}$$

$$C_{L_{mp}} = \sqrt{\frac{3C_{D_o}}{k}}$$

$$C_{D_{mp}} = 4C_{D_o}$$

$$\Rightarrow C_{D_{mp}} = 2\, C_{D_{md}}$$

$$\left(\frac{L}{D}\right)_{mp} = \frac{\sqrt{3}\, C_{L_{md}}}{2C_{D_{md}}}$$

$$= 0.866 \left(\frac{C_L}{C_D}\right)_{md}$$

$$\left(\frac{L}{D}\right)_{mp} = 0.866 \left(\frac{L}{D}\right)_{md}$$

$P_{R_{min}}$ occurs at $\left(\frac{C_L^{\frac{3}{2}}}{C_D}\right)$ i.e. $C_{D_o} = \frac{1}{3} C_{D_i}$

$$C_D = 4C_{D_o}$$

c. **Minimum drag to velocity ratio**

$$\left(\frac{\mathbf{D}}{\mathbf{V}}\right)_{\mathbf{min}}$$

It is the condition for maximum range of jet engine aircraft

$$\left(\frac{D}{V_\infty}\right) = K_1 V_\infty + \frac{K_2}{V_\infty^3}$$

for $\left(\frac{D}{V_\infty}\right)_{min}$

$$\frac{d\left(\frac{D}{V_\infty}\right)}{dV_\infty} = 0$$

$$V_{m(D/V)} = (\sqrt[4]{3}).\sqrt[4]{\frac{k}{C_{D_o}}}\cdot\sqrt{\frac{2W}{\rho s}}$$

$$V_{m(D/V)} = 1.316\ V_{md}$$

$$V_{m(D/V)} > V_{md} > V_{mp}$$

$$C_{D_{min(D/V)}} = C_{D_o} + \frac{1}{3}C_{D_o}$$

$$C_{D_{min(D/V)}} = \frac{4}{3}C_{D_o}$$

$$C_{D_{min(D/V)}} = \frac{2}{3}C_{D_{md}} = 4C_{D_i}$$

$$C_{D_o} = kC_L^2$$

Find C_L

$$C_L = \sqrt{C_{D_o}/k}$$

$$\left(C_{L_{(D/V)}}\right) = \sqrt{\frac{1}{3}}\,(C_{L_{md}})$$

$$\left(C_{L_{(D/V)}}\right) = 0.577(C_{L_{md}})$$

Breguet Range equation for piston engine

$P_A = \eta_p \times P_s$

$P_R = D\cdot V_\infty$

P_s = shaft power

η_p = efficiency of propeller

$$P_s = \frac{D\cdot V_\infty}{\eta_p}$$

{W_o = Gross weight of an aircraft}

{W_1 = weight of aircraft when fuel load is empty}

$$R = \frac{\eta_p}{SFC}\times\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_o}{W_1}\right) \ldots\ldots (1)$$

SFC = Specific fuel consumption

$$SFC = \frac{(N)\text{of fuel}}{\sec J/s}$$

Unit of SFC

$$\frac{N}{s\times\frac{Nm}{s}} = SFC = \frac{1}{m}$$

Breguet range equation for jet engine aircraft

TSFC = Thrust Specific fuel consumption

$$TSFC = \frac{(N)\text{of fuel}}{\sec\times N}$$

Unit of TSFC

$$TSFC = \frac{N}{Ns} = \frac{1}{s}$$

$$R = \frac{2}{TSFC}\times\sqrt{\frac{2}{\rho s}}\times\left(\frac{C_L^{\frac{1}{2}}}{C_D}\right)(\sqrt{W_o}-\sqrt{W})..(2)$$

Breguet Endurance equation for piston engine a/c.

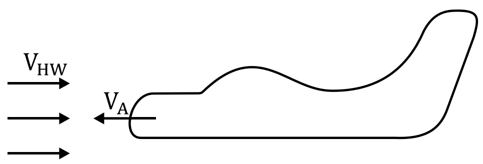
$$E = \frac{\eta_p}{SFC}\times\left(\frac{C_L^{\frac{3}{2}}}{C_D}\right)\times\sqrt{2\rho s}\times\left(\frac{1}{\sqrt{W_1}}-\frac{1}{\sqrt{W_o}}\right)..(3)$$

Breguet Endurance equation for Jet engine a/c.

$$E = \frac{1}{TSFC}\times\left(\frac{C_L}{C_D}\right)\ln\frac{W_o}{W_1}\ldots(4)$$

Effect of wind on range:

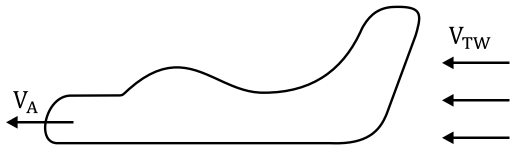
1. **Head wind (HW):**



$V_g = V_\infty - V_{HW}$

V_g – ground speed decreases thereby range decreases

2. **Tail Wind (TW):**



$V_g = V_\infty + V_{TW}$ (Range increases)

$V_{TW} < V < V_{HW}$

$V_g -$ ground speed increases thereby range increases

TURING FLIGHT PERFORMANCE

1. **Level Turn:**

$\psi =$ turn angle

$\phi =$ Banking angle

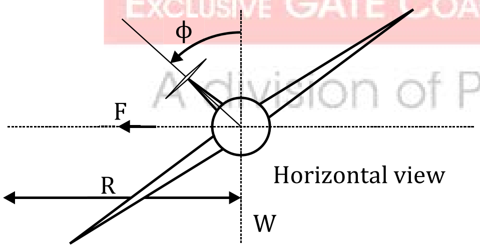
Altitude remains constant

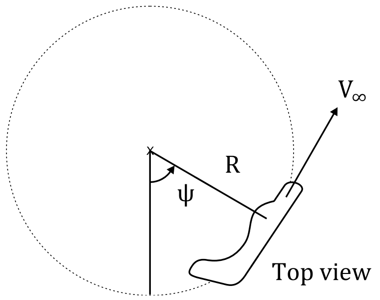
$L\cos\phi = W \Rightarrow \frac{L}{W} = \sec\phi$

$n =$ Load factor $= \sec\phi > 1$

$$R = \frac{V^2}{g\sqrt{n^2 - 1}}$$

$$\text{Also, } R = \frac{V^2}{g\tan\phi}$$





Turn Rate:

$$\omega = \frac{d\psi}{dt} = \frac{V_\infty}{R}$$

$$\omega = \frac{g\sqrt{n^2 - 1}}{V_\infty} \text{ so, } \omega = \frac{g\tan\phi}{V_\infty}$$

Time required to complete 1 cycle

$$= \frac{2\pi R}{V_\infty}$$

Time taken to turn through angle ψ

$$= \frac{2\pi R}{V_\infty} \times \frac{\psi}{2\pi} (\psi - \text{radians})$$

$$\psi = \frac{R\psi}{V_\infty} \Rightarrow \frac{V_\infty^2}{g\tan\phi} \times \frac{\psi}{V_\infty}$$

$$T = \frac{V}{g\tan\phi} \times \psi$$

Comparing (Level turn flight with straight and level flight).

For level flight: $L_o, D_o T_o; V_o, P_o(P_{av})$

$L_o = W;\ T_o = D_o$

For turn flight $= L, D, T, V, P_{av}$

$$n = \text{Load factor} = \frac{L}{W} = \frac{L}{L_o} = \frac{D}{D_o}$$

$$T_o = D_o \Rightarrow \frac{D_o}{T_o} = 1 \quad \frac{L_o}{W} = 1$$

$$n = \frac{L}{W} = \frac{D}{T} \text{ equating } \frac{D_o}{T_o} = \frac{L_o}{W}$$

$$\frac{L}{L_o} = n = \frac{D}{D_o} = \frac{T}{T_o} = \sec\phi$$

For turning flight, $L = nW = nL_o$

$$C_L \times \frac{1}{L}\rho V^2 S = nC_L \times \frac{1}{2}\rho V_o^2 S$$

$$V^2 = n V_o^2 \Rightarrow \frac{V^2}{V_o^2} = n \Rightarrow \frac{V}{V_o} = \sqrt{n}$$

$$\text{Also, } \frac{V}{V_o} = \sqrt{\sec\phi}$$

$$P_{av} = TV = nT_o \times \sqrt{n}V_o$$

$$P_{av} = n^{3/2}P_o$$

$$\frac{P}{P_o} = n^{3/2} = (\sec\phi)^{3/2}$$

$\rightarrow \eta_{max} = \sec\phi_{max}$

(For level turn L = nW; T=D)

$$\rightarrow L = nW = C_L \times \frac{1}{2}\rho V^2 S$$

$$C_L = \frac{2nW}{\rho V^2 S}$$

$$\rightarrow D = \frac{1}{2}\rho V^2 S\, C_D$$

$$\frac{1}{2}\rho V^2 S\left(C_{D_o} + kC_L^2\right) = D$$

$$T = D = \frac{1}{2}\rho V^2 S\left(C_{D_o} + kC_L^2\right)$$

Substituting the C_L i.e.,

$$C_L = \frac{2nW}{\rho V^2 S} \text{ and solving for (n)}$$

$$n = \frac{\rho V^2}{2k\left(\frac{W}{s}\right)}\left[\frac{T}{W} - \frac{\rho V^2 C_{D_o}}{2\left(\frac{W}{S}\right)}\right]$$

$$V_{stall} = \sqrt{\frac{2nW}{\rho S C_{L_{max}}}}$$

Stalling velocity is the minimum velocity

Minimum turn radius:

$$R = \frac{V_\infty^2}{g\sqrt{n^2 - 1}}$$

$$q_\infty = \frac{1}{2}\rho V_\infty^2$$

$$V_\infty^2 = \frac{2q_\infty}{\rho_\infty}$$

$$R = \frac{2q_\infty}{g\rho_\infty\sqrt{n^2 - 1}} \quad \ldots.\#$$

Referring the above equation of n we get

$$n^2 = \frac{q_\infty}{k\left(\frac{W}{S}\right)}\left[\frac{T}{W} - \frac{q_\infty C_{D_o}}{\left(\frac{W}{S}\right)}\right] \ldots..(A)$$

Condition for minimum turn radius

$$\frac{q_\infty (T/W)}{k\left(\frac{W}{S}\right)}\left(\frac{1}{2}\right) = 1$$

$$q_\infty = \frac{2k\,(W/S)}{(T/W)}$$

Required velocity for the turn rate to be minimum

$$V_\infty = \left(\frac{4k(W/S)}{\rho_\infty(T/W)}\right)^{1/2}$$

Required load factor for turn rate to be minimum.

Referring (A) substituting the value of q_∞

i.e., $\frac{2k(W/S)}{(T/W)}$ and solving

$$n = \sqrt{2 - \frac{4kC_{D_o}}{T/W}}$$

$$R_{min} = \frac{V_\infty^2}{g\sqrt{n^2 - 1}}$$

$$R_{min} = \frac{4k(W/S)}{\rho_\infty (T/W) g\sqrt{1 - \frac{4kC_{D_o}}{T/W}}}$$

Above equation is minimum turn radius possible

Maximum Turn Rate:

$$W = \frac{g\sqrt{n^2 - 1}}{V_\infty}$$

$$V_\infty = \sqrt{\frac{2q_\infty}{\rho_\infty}}$$

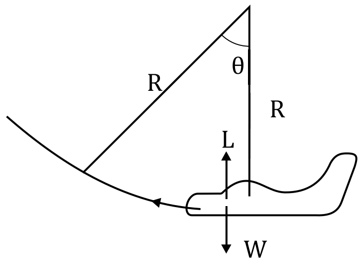
$$V_{\omega_{max}} = \left(\frac{2(W/S)}{\rho_\infty}\right)^{\frac{1}{2}}\left(\frac{k}{C_{D_o}}\right)^{\frac{1}{4}}$$

This is also corresponding to minimum drag velocity i.e. ω_{max} occurs at min drag

$$n_{\omega_{max}} = \left[\frac{T/W}{\sqrt{k/C_{D_o}}} - 1\right]^{1/2}$$

$$\omega_{max} = g\sqrt{\frac{\rho_\infty}{W/S}\left[\frac{T/W}{2k} - \left(\frac{C_{D_o}}{k}\right)^{1/2}\right]}$$

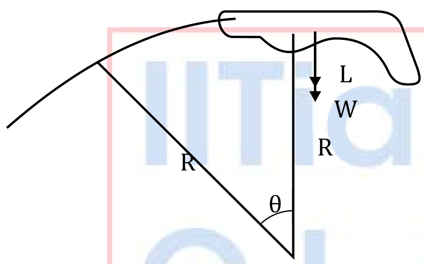
2. **Pull-up Maneuver**



$$R = \frac{V_\infty^2}{g(n-1)}$$

$$\omega = \frac{g(n-1)}{V_\infty}$$

3. **Pull-down Maneuver**



$$R = \frac{V_\infty^2}{g(n+1)}$$

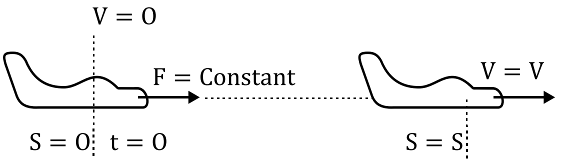
$$\omega = \frac{g(n+1)}{V_\infty}$$

TAKE-OFF PERFORMANCE

1. Ground roll (S_g)
2. Airborne Phase (S_a)

$S_{T_o} = S_g + S_a$

By rectilinear motion of the aircraft



$F = ma$ $(2^{nd}$ law of motion$)$

$$F = m\frac{dV}{dt} \Rightarrow \int_0^V dV = \int_0^t \frac{F}{m}\, dt$$

$$V = \frac{F}{m}t$$

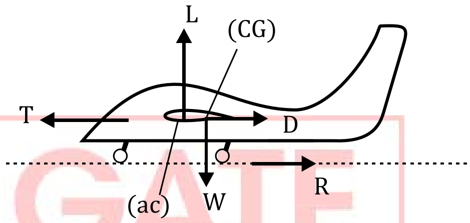
$$\Rightarrow t = \frac{Vm}{F}$$

$$\text{Also, } V = \frac{ds}{dt} \qquad ds = Vdt$$

$$\Rightarrow \int_0^s ds = \int_o^t Vdt \Rightarrow S = \int_o^t \frac{F}{m} tdt$$

$$\Rightarrow S = \frac{F}{m}\frac{t^2}{2}$$

$$S = \frac{F}{2m}\left(\frac{Vm}{F}\right)^2 \Rightarrow S = \frac{V^2 m}{2F}$$



$R = (L - W)\mu_R$ (μ_R – Coefficient of friction)

Total force, $F = T - D - R$

$$F = T - D - \mu_r(W - L)$$

$$F = T - D - \mu_r(W - L) = \frac{mdV}{dt}$$

ΔC_{D_o} Extended parasite drag due to landing gear

ϕ = Ground effect.

$$C_D = C_{D_o} + \Delta C_{D_o} + \phi\, kC_L^2$$

$$\phi = G = \frac{C_{D_i}\text{(in Ground effect)}}{C_{D_i}\text{ (without ground effect)}}$$

$$\phi = \frac{(16\ h/b)^2}{1 + (16\,h/b)^2}$$

h = Height of wing above the ground

b = Wing span

$$\Delta C_{D_O} = \frac{W}{S}\ K_{UC}\, m^{-0.25}$$

Where m is mass of airplane

$K_{UC} = 5.81 \times 10^{-5}$ (zero flap deflection)

$K_{UC} = 3.16 \times 10^{-5}$ (Maximum flap deflection)

$$S = \frac{V^2 m}{2F} \qquad S_{L_o} = \frac{V_{L_o}^2\, m}{2F}$$

$$S_{L_o} = S_g = \frac{V_{L_o}^2 (W/g)}{2\{T - D - \mu_r (W - L)\}_{0.7\, V_{L_o}}}$$

$$V_{L_o} = 1.2\, V_{stall}$$

$$S_{L_o}$$

$$= \frac{1.44 \times W^2}{g\, \rho s C_{L_{max}} \{T - (D + \mu_r (W - L))\}_{0.7\, V_{L_o}}} \quad (1)$$

S_a, (Pull up manoeuver)

$$V_\infty = 1.15\ V_{stall}$$

$$C_L = 0.9\, C_{Lmax}$$

$$\sin\theta_{obs} = \frac{S_a}{R}$$

$$S_a = R\sin\theta_{obs} \quad \ldots (2)$$

$$n = \frac{L}{W} = \frac{\frac{1}{2}\rho_w (1.15\, V_{stall}) \times S \times 0.9\, C_{L_{max}}}{\frac{1}{2}\rho_\infty V_{stall}^2 \times S \times C_{L_{max}}}$$

$$n = 1.15^2 \times 0.9\,, \qquad n = 1.19$$

$$R = \frac{((1.15)V_{stall})^2}{g(n-1)}$$

$$\Rightarrow R = \frac{1.3225\, V_{stall}^2}{g(1.19 - 1)}$$

$$R = \frac{6.96}{g} V_{stall}^2$$

LANDING PERFORMANCE

V_a = Approach velocity

$V_a = 1.3\, V_{stall}$ (Commerical a/c)

$V_a = 1.2\, V_{stall}$ (Military a/c)

V_{TD} = Touch down velocity

$V_{TD} = 1.15\, V_{stall}$ (Commercial a/c)

$V_{TD} = 1.1\, V_{stall}$ (Military a/c)

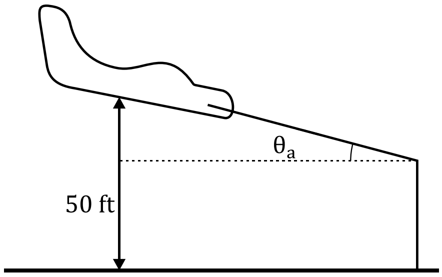
$$S_{L_o} = S_a + S_f + S_g \quad \ldots (1)$$

S_a = Approach distance

S_f = Flare distance

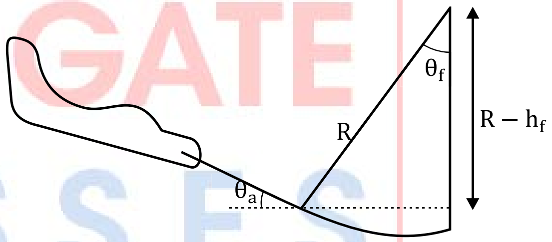
s_g = Ground roll

For s_a



$$s_a = \frac{50 - h_f}{\tan\theta_a}\ (\text{ft})$$

$$s_a = \frac{15.24 - h_f}{\tan\theta_a}\ (\text{m}) \quad \ldots (2)$$



$$\theta_a = \sin^{-1}\left(\frac{1}{L/D} - \frac{T}{W}\right)$$

$$s_f = R\sin\theta_a \ \ldots . (3)$$

$$R = \frac{V^2}{g(n-1)}$$

n during flare is 1.2 (Assumed)

$$R = \frac{V_f^2}{g(0.2)}$$

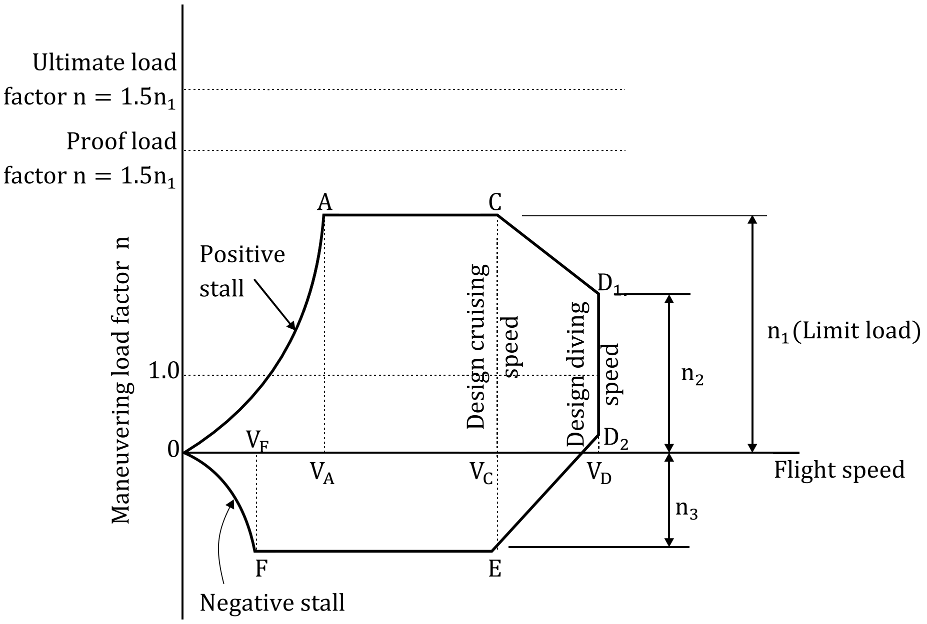
$V_f = 1.23\, V_{stall}$ (Commerical)

$V_f = 1.15\, V_{stall}$ (Military)

$$S_g = NV_{TD} + \frac{WV_{TD}^2}{2g}\left[\frac{1}{T_{rev} + D + \mu_r (W - L)}\right]_{0.7 V_{TD}} . (4)$$

FLIGHT ENVELOPE

Flight Envelope



$$V^* = V_a = \sqrt{\frac{2nW}{\rho S C_{L_{max}}}} \text{ (Corner velocity)}$$

n = 1 for straight and level flight

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L_{max}}}}$$

Load Factor	Normal Flight	Semi Aerobatic (a/c) (Military a/c)	Aerobatic (a/c)
n_1	$2.1 + \frac{24000}{W + 10000}$	4.5	6
n_2	$0.75\, n_1 < 2.0$	3.1	4.5
n_3	-1	-1.8	-3.0

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