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GATE 2017 AE

Topic: Aerodynamics

Question 6 :

Which one of the following statements is NOT true

- (A) The pitching moment of any airfoil at any angle of attack is always zero at the center of pressure
- (B) The pitching moment of any airfoil at any angle of attack is always zero at the aerodynamic center
- (C) The center of pressure and aerodynamic center coincide for a symmetric airfoil
- (D) The pitching moment about the aerodynamic center, for any airfoil, does not vary with angle of attack

ANS : [B]

Question 7 :

Which one of the following statements is NOT true

- (A) Compared to a laminar boundary layer, a turbulent boundary layer is more desirable on a wing operating at large angle of attack
- (B) The skin friction drag for a turbulent boundary layer is larger than that for a laminar boundary layer
- (C) The location of transition from laminar to turbulent boundary layer depends only on the operating Reynolds number
- (D) A separated flow does not necessarily lead to a turbulent boundary layer

ANS : [C]

Question 8 :

A De Laval nozzle is to be designed for an exit Mach number of 1.5. The reservoir conditions are given as $P_0=1$ atm (gage), $T_0=20^\circ\text{C}$, $\gamma=1.4$. Assuming shock free flow in the nozzle, the exit absolute pressure (in atm) is _____ (in three decimal places)

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Solution: (8)

Design Mach No = 1.5

$P_0 = 1 \text{ atm (g)}$

$T_0 = 20^\circ \text{C}$

$\gamma = 1.4$

$\rightarrow M_0 = 1.5$

for shock free flow

De-Laval Nozzle

exit pressure (P_e) is given by

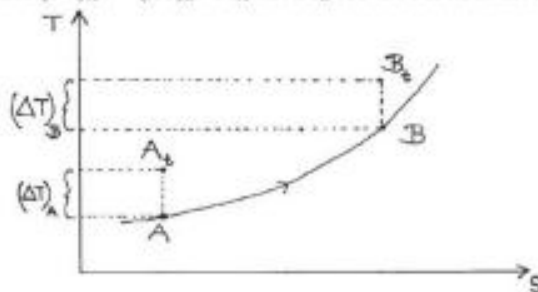
$$\frac{P_e}{P_0} = \left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{\frac{\gamma}{\gamma - 1}} \quad \left[\begin{array}{l} P_0 = 1 \text{ atm (gag)} \\ = 1 + 1 = 2 \text{ atm (absolute)} \end{array} \right]$$

$$\frac{2}{P_e} = \left(1 + \frac{1.4 - 1}{2} (1.5)^2\right)^{\frac{1.4}{0.4}}$$

$$P_e = 0.5448 \text{ atm (absolute)} \quad \underline{\text{Ans}}$$

Question 9:

Consider a steady one dimensional flow of a perfect gas with heat transfer in a duct. The T-s diagram (shown below) shows both the static and the stagnation conditions at two locations, A and B, in the duct. A_s and B_s denote stagnation conditions for states A and B, respectively. It is known that $(\Delta T)_A = (\Delta T)_B$. M_A and M_B are the Mach numbers of the flow at locations A and B.



Which of the following statements is true about the flow.

- (A) Flow is subsonic and $M_A < M_B$.
- (B) Flow is supersonic and $M_A > M_B$.
- (C) Flow is subsonic and $M_A > M_B$.
- (D) Flow is supersonic and $M_A < M_B$.

ANS : [B]

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Question 31:

Consider a straight wing with rectangular planform of aspect ratio 10 and with a NACA 0012 airfoil. The span effectiveness factor for this wing is 0.95. Assume the flow to be incompressible and governed by thin airfoil theory. The lift coefficient of this wing, at an angle of attack of 6 deg, is _____ (in three decimal places).

ANS:

Solution (31)

→ NACA 0012

→ AR = 10

→ e (span effectiveness factor) = 0.95

→ flow is incompressible and governed by thin airfoil theory

→ $C_L = ?$ @ $\alpha = 6$ deg.

for finite wing

$$C_L = a \times (\alpha - \alpha_{L=0}) \quad (\text{for symmetric airfoil } \alpha_{L=0} = 0)$$

$$a = \frac{a_0}{1 + \frac{a_0}{\pi \cdot AR}} \cdot (1 + \delta) \quad \text{where } e = \frac{1}{1 + \delta} \quad (e = \text{span effectiveness factor})$$

$$a_0 = \frac{dC_L}{d\alpha} = 2\pi \quad (\text{from thin airfoil theory})$$

$$e = \frac{1}{1 + \delta} \Rightarrow 1 + \delta = 1.052$$

$$a = \frac{2\pi}{1 + \frac{2\pi}{\pi \cdot 10} (1.052)} = 5.19$$

$$C_L = 5.19 \times \left(\frac{2\pi \times 6}{360} \right) = 0.5434$$

$$C_L (@ \alpha = 6^\circ) = 0.5434 \quad \text{Ans}$$

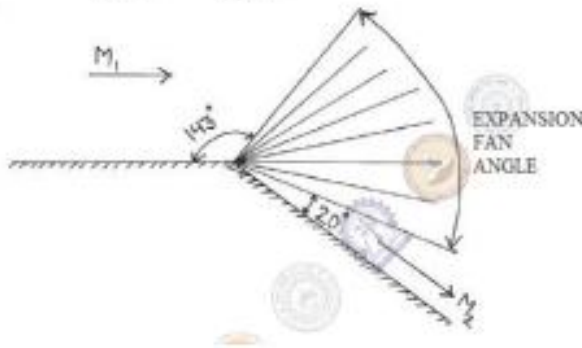
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Question 34:

A trace from the schlieren photograph of the flow around a corner reveals the edges of the expansion fan as shown below. The leading and trailing edges of the expansion fan make the angles as shown. Assuming $\gamma = 1.4$, the angle of the expansion fan (in degrees) is _____ (in two decimal places)

Prandtl Meyer function is given by

$$v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1}$$



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Solution (34)

$$\mu_1 = 180^\circ - 143^\circ = 37^\circ$$

$$\mu_2 = 20^\circ$$

$$\mu_1 = \sin^{-1} \left(\frac{1}{M_1} \right) \Rightarrow M_1 = 1.66$$

$$\mu_2 = \sin^{-1} \left(\frac{1}{M_2} \right) \Rightarrow M_2 = 2.923$$

$$v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1}$$

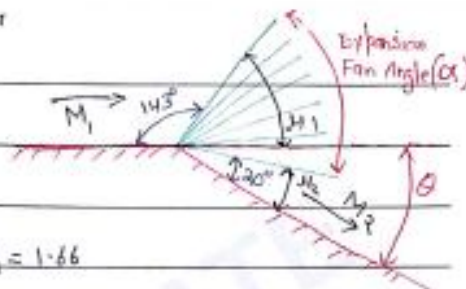
$$v(M_1) = 16.56^\circ$$

$$v(M_2) = 48.10^\circ$$

$$\theta = v(M_2) - v(M_1) = 31.54^\circ$$

$$\alpha = \mu_1 + \theta - \mu_2 = 37 + 31.54 - 20 = 48.54^\circ$$

Expansion fan angle $(\alpha) = 48.54^\circ$ Ans



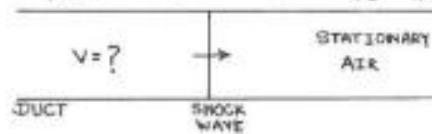
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Question 35:

A strong normal shock wave, with a pressure ratio of 29 across it, is travelling into stationary air ($\gamma=1.4$) at $T=280$ K in a straight duct (see figure). The magnitude of the velocity of the air induced behind the shock wave is _____ m/s. (round to nearest integer)

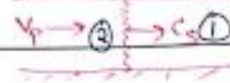
(Gas constant = 287 J/kg.K; Shock wave relations:

Pressure ratio: $\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M^2 - 1)$; Density ratio: $\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2+2}$



Q.35- Solution

$$\frac{P_2}{P_1} = 29$$



$$T_1 = 280 \text{ K}$$

$$\gamma = 1.4$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 280} \approx 335.41 \text{ m/s}$$

Velocity of air induced behind the shock wave (V_p)

$$V_p = \frac{a_1}{\gamma} \left(\frac{P_2}{P_1} - 1 \right) \left(\frac{\frac{2\gamma}{\gamma+1}}{\frac{P_2}{P_1} + \frac{\gamma-1}{\gamma+1}} \right)^{1/2}$$

For derivation of this formula Ref.
 ⇒ Gas dynamics, 2nd edition
 E. Rathakrishnan
 Page No. 112

$$V_p = \frac{335.41}{1.4} (29 - 1) \left(\frac{2 \times 1.4}{1.4 + 1} \right)^{1/2} \left(\frac{1}{29 + \frac{1.4 - 1}{1.4 + 1}} \right)^{1/2}$$

$$V_p = 1341.64 \text{ m/s} \quad \underline{\underline{\text{Ans}}}$$