IITians GATE CLASSES
Exclusive GATE Coaching by IIT/IISc Graduates
A division of PhIE Learning Center

QUICK REVISION

FORMULA SHEET

for

GATE-ME ENGINEERING MECHANICS

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Engineering Mechanics

Chapter 1: STATICS (REST)

Newton 1st law (NFL)

$\vec{a} =$ Acceleration vector

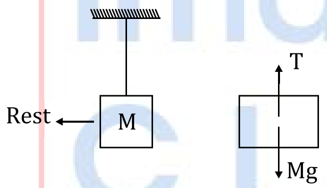
For a Particle:

$\sum\vec{F} = 0$, then $\vec{a} = 0 \rightarrow$ Rest; Uniform velocity

For a Rigid Body:

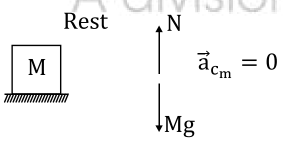
$\sum\vec{F}_{ext} = 0$ then $\vec{a}_{cm} = 0$

Case 1:

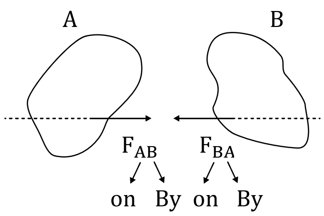


$\vec{a}_{c_m} = 0;\ \sum\vec{F}_{ext} = 0;\ \ T - mg = 0\ , T = mg$

Case 2:

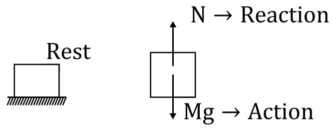


Newton's Third Law (NTL):



$\vec{F}_{AB} = -\vec{F}_{BA}$

Case:



Equilibrium:

Rest or uniform linear velocity

i.e. $\sum\vec{F} = 0$

For Particle $\sum\vec{F} = 0$

For a rigid body $\sum\vec{F} = 0 \Rightarrow \sum\vec{F}_x = 0, \sum F_y = 0$

$\sum\vec{F}_z = 0$

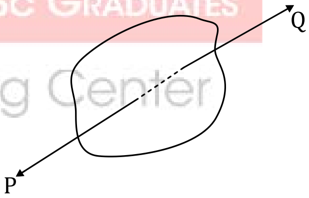
$\sum M = 0$(about any point or line in plane)

System of Equilibrium:

1. **Two force system**

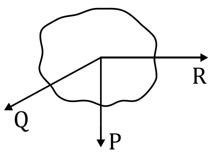
   $\vec{P} + \vec{Q} = 0, \vec{P} = -\vec{Q}$

   $\vec{M} = 0 \rightarrow$ Collinear



2. **Three Force System**

   For equilibrium three forces must be coplanar and con-current.

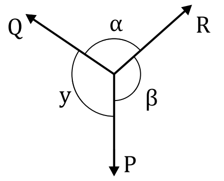


$\vec{P} + \vec{Q} + \vec{R} = 0$

$\sum\vec{M} = 0$ (Concurrent)

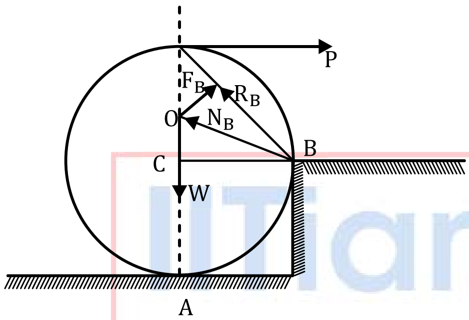
Lami's Theorem:

For a three-force system



$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin y}$$

Important concepts:



a. As roller is about to move out of curb, normal reaction at A becomes zero, as it loses its contact at point A and cylinder or roller will be under equilibrium at verge of motion.

b. Here comes a contact force (Resultant of normal reaction and friction at B (i.e., R_B)

c. So, only three forces are P, W, R_B which maintain equilibrium, so these must be coplanar and concurrent.

d. If the horizontal force 'P' is applied at centre, no friction will be there i.e., reaction force at B will acts as normal reaction.

For P to be minimum its line of action should be $\perp^r$ to R_B

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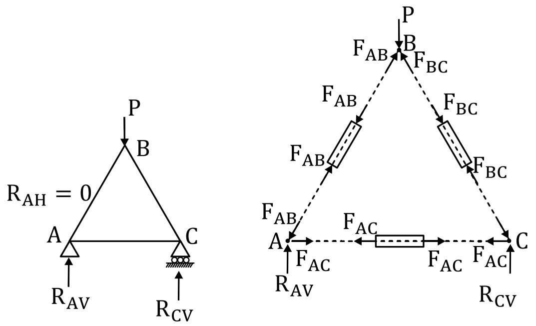
Chapter 2: PLANE TRUSS

Numbers of members = m

Number of Joints = J



J = No. of Joints (Whether it is binary or ternary joint)

For perfect truss: $m = 2j - 3$ (DOF = 0)

$m < 2j - 3$ Unstable truss (DOF > 0)

$m > 2j - 3$ Redundant, stable and (DOF < 0) indeterminate truss

Method of Joint:

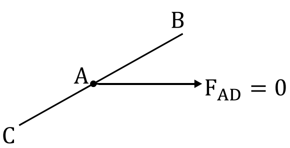
Equilibrium of joint is considered in method of joint to find loading in the member.

Procedure:

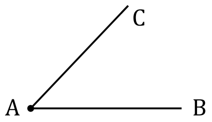
1. Find support reactions if required
2. Consider equilibrium of joint where only two members are meeting and use $\sum f_x = 0, \sum f_y = 0$.

Note:

1. At a joint if 3 members are meeting and 2 members are collinear the forces in 3rd member = 0



2. If at a joint two members are meeting and they are non-collinear then forces in both members will be zero.



$F_{AB} = F_{AC} = 0$

Method of Section:

Equilibrium of a section of truss is considered.

Procedure:

1. Find reactions at support if required
2. Cut the member under consideration by section 1 .... 1 and consider the equilibrium of either LHS at RHS of section 1 .... 1 and use $\sum f_x = 0, \sum f_y = 0, \sum M = 0$ to find unknowns.

Note: Don't cut more than 3 members.

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Chapter 3: Principle of Virtual Work (POVW)

$WD(\text{Work done}) = \vec{F} \cdot \vec{d}_s$

$\vec{d}_s$ = Displacement vector of a point where $\vec{F}$ is acting.

Principle of virtual work stats that if the system is in equilibrium then sum of virtual work by all the forces = 0

Virtual work = $F \cdot ds$

F = Actual force

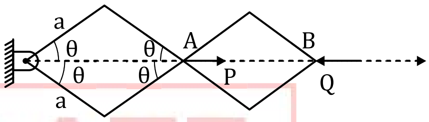
ds = virtual displacement.

Procedure:

1. Take any fixed point is problem as origin. fix coordinate axes and find coordinates of all joint where forces are acting.
2. Find virtual displacements.
3. Use POVW to find unknowns

Note: In POVW, we don't consider reaction at supports, because their work done is zero.

Example:



Find P: Q =?

Solution:

$x_A = 2a\cos\theta, \partial x_A = -2a\sin\theta\ \partial\theta$

$x_B = +4a\cos\theta, \partial x_B = -4a\sin\theta\ \partial\theta$

$(VW)_P + (VW)_Q = 0$ for equilibrium

$P\cdot\partial x_A - Q\,\partial x_B = 0$

$P[-2a\sin\theta]\,\partial\theta - Q[-4a\sin\partial\theta] = 0$

$-2P + 4Q = 0$

$$\frac{P}{Q} = 2$$

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Chapter 4: TRANSLATORY MOTION

Kinematics:

For a particle with $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{V} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{V}}{dt}$$

Acceleration ($\vec{a}$):

- **Uniform**

  $v = u + at$

  $S = ut + \frac{1}{2}at^2$

  $V^2 = u^2 + 2as$

- **Non-uniform**

  $a = \frac{dV}{dt}$

  $a = V \cdot \frac{dV}{ds}$

Newton's Second Law (NSL):

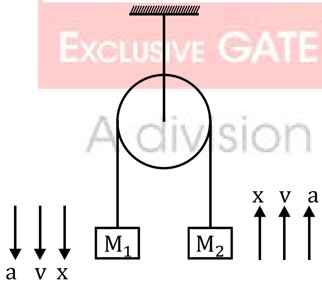
For a particle:

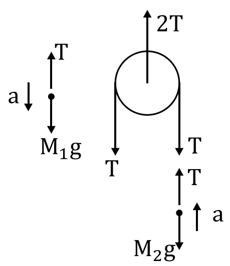
$\Sigma\vec{F}_{ext} \neq 0$ then $\vec{a} \neq 0$

$\boxed{\vec{a} = \frac{\Sigma\vec{F}_{ext}}{m}}$ NSL

Dynamics:

Case A:

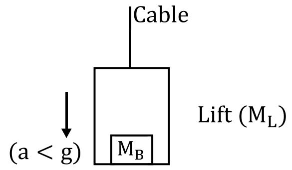


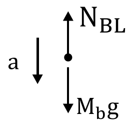


$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \qquad \begin{cases} m_1g - T = m_1a \rightarrow (1) \\ T - m_2g = m_2a \rightarrow (2) \end{cases}$

$T = \left(\frac{2m_1m_2}{m_1+m_2}\right)g$

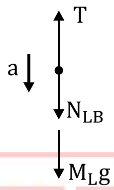
Case B :





$\boxed{M_B\,g - N_{BL} = M_B\,a}$ NSL

For Lift



$\vec{N}_{BL} = -\vec{N}_{LB}$

$\boxed{M_L\,g + N_{LB} - T = M_L\,a}$ NSL

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Chapter 5: CIRCULAR MOTION

Curvilinear Motion

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$

$\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath} + V_z\hat{k}$

$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$

Projectile Motion:

Total time $(T) = \frac{2U\sin\theta}{g}$

Max height $(H_{max}) = \frac{U^2\sin^2\theta}{g}$

Horizontal Range $(R) = U_x.T$

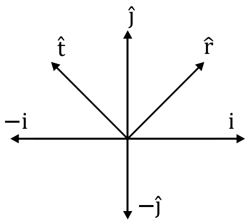
$\boxed{R = \frac{U^2\sin 2\theta}{g}}$ R_{max} at $\theta = 45°$

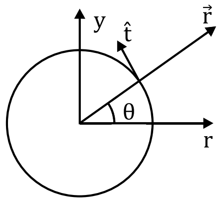
Equation of Trajectory:

$$y = x\tan\theta - \frac{gx^2}{2\,U^2\cos^2\theta}$$

or $\quad y = x\tan\theta\left(1 - \frac{x}{R}\right)$

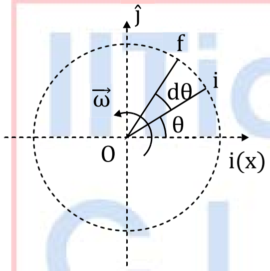
Circulation Motion:





$\vec{r} = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath}$

$\hat{t} = \sin\theta\,(-\hat{\imath}) + \cos\theta(\hat{\jmath})$



Angular velocity

$$\omega = \frac{d\theta}{dt} \rightarrow (\text{rad/sec})$$

Angular acceleration

$$\alpha = \frac{d\omega}{dt} \rightarrow \text{rad/sec}^2$$

Direction of 'ω' and 'α' → Right hand thumb rule.

Angular acceleration:

- **Uniform**

  $\omega = \omega_o + \alpha t$

  $d\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

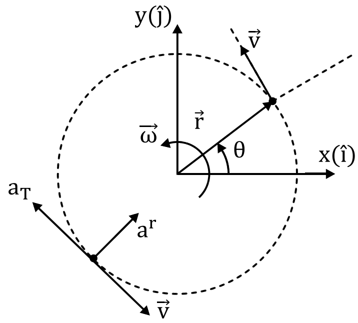
  $\omega^2 = \omega_0^2 + 2\alpha(d\theta)$

- **Non-uniform**

  $\alpha = \frac{d\theta}{dt}$

  $\alpha = \omega\frac{d\omega}{d\theta}$

Vector Analysis of Circular Motion:



$\vec{r} = |r|\cdot\hat{r}$

$\vec{r} = r[\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}]$

$\vec{V} = \frac{d\vec{r}}{dt} = r[-\sin\theta\hat{\imath} + \cos\theta\,\hat{\jmath}]\frac{d\theta}{dt}$

$\vec{V} = r\omega(\hat{t})$

$$\vec{a} = \frac{d\vec{V}}{dt} = r\left(\omega\left(-\cos\theta\frac{d\theta}{dt}\,\hat{\imath} - \sin\frac{d\theta}{dt}\hat{\jmath}\right)\right)(-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath})\frac{d\omega}{dt}$$

$\vec{a} = r\omega^2(-\cos\theta\,\hat{\imath} + (-\sin\theta)\hat{\jmath}) + r\alpha(-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath})$

$\vec{a} = r\omega^2(-\hat{r}) + r\alpha(\hat{t})$

$\vec{a} = \vec{a}_r + \vec{a}_T$

$\vec{a}_r$ = radial/centripetal acceleration

$\vec{a}_T$ = tangential acceleration

$a_r = r\omega^2 = \frac{V^2}{r}$

For uniform circular motion

$V = r\cdot\omega$ = constant

ω = constant

$\alpha = 0, \qquad \vec{a}_T = 0$

$\vec{a} = \vec{a}_r$

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Chapter 6: FRICTION

Dry Friction/Coulomb Friction:

- **Static friction**

  Rest + Verge of motion

  $0 < f_s < f_{(s)max}$

  $\boxed{(f_s)_{max} = \mu_s N}$

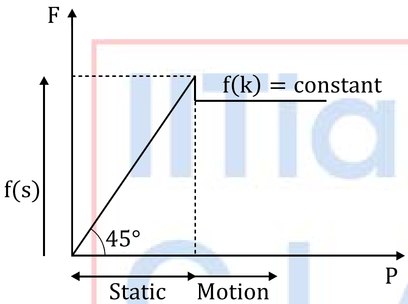
  $\mu_s =$ Coefficeint of static friction

  $N =$ Normal reaction

  If applied force i.e:

  $P < (f_s)_{max}$

  then $\left(f_{(s)} = P\right)$



Note:

$f_{(s)max} > F_k$

$\mu_s > \mu_k$

Then $f_s = \mu_k N$

$f_s = F_k$

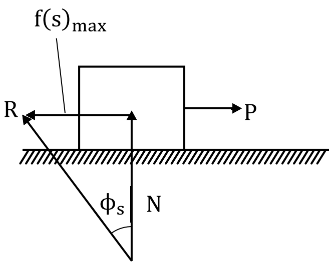
- **Kinetic friction**

  Motion

  $\boxed{F_K = \mu_k N}$ constant

Angle of Static Friction (ϕ_s):

Angle made by normal reaction with contact force.

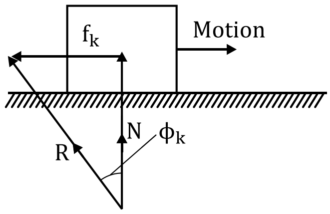


$$\tan\phi_s = \frac{(f_s)_{max}}{N}$$

$$\tan\phi_s = \frac{\mu_s N}{N} = \mu_s$$

Angle of Kinetic Friction (ϕ_k):

During motion of body, the angle made by contact force with normal reaction is called as ϕ_k.



$$\tan(\phi_k) = \frac{f_k}{N} = \frac{\mu_k N}{N} = \mu_k$$

Note:

If μ_s and μ_k not given separately

1. $\mu_s = \mu_k = \mu$
2. $0 \le f_s \le (f_s)_{max} = \mu N = f_k$
3. $\tan\phi = \mu$ (ϕ = angle of friction)

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Chapter 7: WORK & ENERGY

Linear Momentum ($\vec{P}$):

$\vec{P} = m \cdot \vec{V}_{cm}$

$$\frac{d\vec{P}}{dt} = m \cdot \frac{d\vec{V}_{cm}}{dt} \Rightarrow \frac{d\vec{P}}{dt} = \vec{m} \cdot \vec{a}_{cm} = \sum\vec{F}_{ext}$$

$$\boxed{\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt}}$$

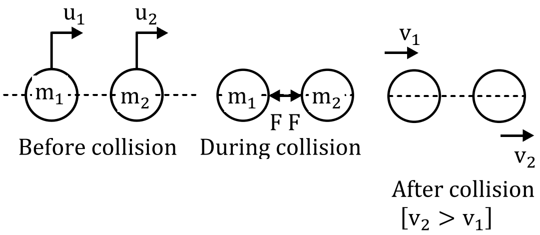
Conservation of Linear Momentum $\vec{P}$:

$\vec{P} =$ Constant

$d\vec{P} = 0$

Only when $\sum\vec{F}_{ext} = 0$

$P_x =$ constant; $P_y =$ Constant



P_x = constant

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

Collision:

1. Perfectly elastic collision $e = 1$ $(m_1 + m_2)$ system

   $(k\varepsilon)_{\text{initial of system}} = (k\varepsilon)_{\text{Final of system}}$

   $$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2} \rightarrow (1)$$

   $$\left[\frac{1}{2}m_1u_1 + \frac{1}{2}m_2u_2 = \frac{1}{2}m_1v_1 + \frac{1}{2}m_2v_2\right] \rightarrow (2)$$

   From (1) & (2)

   $(u_1 - u_2 = v_2 - v_1)$

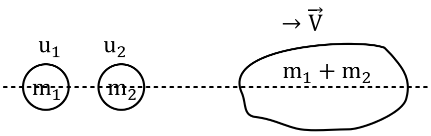
   $u_1 - u_2$ =Velocity of approach

   $v_2 - v_1$ = Velocity of separation

   $$\left(e = \frac{v_2 - v_1}{u_1 - u_2}\right)$$

   e = coefficient of restitution

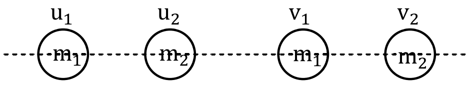
2. Perfectly Inelastic (Plastic) collision (e = 0)



   $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

   $(k\varepsilon)_{loss} = (k\varepsilon)_{initial} - (k\varepsilon)_{final}$

3. Partially Elastic Collision: $(0 < e < 1)$

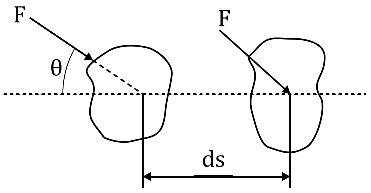


No complete region in size $\Rightarrow$ Same energy loss due to deformation.

$$\boxed{k\varepsilon \neq \text{conserved}}$$

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$

Work done:



$$\boxed{WD = \vec{F}\cdot\vec{ds} = F\,ds\cos\theta}$$

$$(WD)_{spring} = \int Fs\,dx = -\frac{1}{2}kx^2$$

Work Energy Theorem:

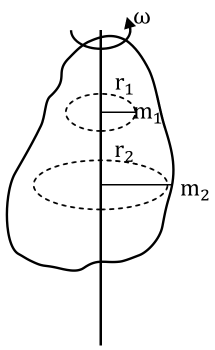
Work done by all the forces acting on a particle = change in kinetic energy

$$\boxed{W_{net} = (k\varepsilon)_{final} - (k\varepsilon)_{initial}}$$

If $(\text{work done})_{\text{non conservative}} = 0$

Then mechanical energy = constant

Rotation of Rigid Bodies:



Mass moment of inertia

$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \cdots$

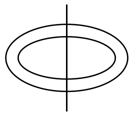
$I = \sum m_ir_i^2$

Let $\sum M_i = m$ = Total mass

$I = mk^2$

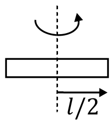
k = radius of gyration

Ring/Hollow Cylinder:



$I = mR^2$

Rod:

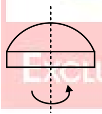


$$I_{cm} = \frac{mL^2}{12}$$

Solid Sphere

$$I = \frac{2}{5}\, mR^2$$

Disc/Hollow Cylinder

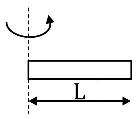


$$I = \frac{MR^2}{2}$$

Hollow sphere

$$I = \frac{2}{3} mR^2$$

Rod about end



$$J = \frac{mL^2}{3}$$

Dynamics of Rigid Body Rotation:

If $\sum \vec{T}_{net} = 0$ then $\vec{\alpha} = 0$

If $\sum \vec{T} \neq 0$ then $\vec{\alpha} \neq 0$

$$\boxed{\sum \vec{T}_{net} = I \cdot \vec{\alpha}}$$

$$\sum \vec{T}_{cm} = I_{cm} \cdot \alpha$$

$$\sum \vec{T}_{Axis} = I_{Axis} \cdot \alpha$$

Angular Momentum ($\vec{L}$) (Moment of linear momentum)

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\boxed{\sum \vec{T} = \frac{dL}{dt}}$$

Conservation of Angular Momentum:

$\vec{L}$ = Constant, $dL = 0$, when $\sum \vec{T} = 0$

Eg: $I_1\omega_1 = I_2\omega_2$

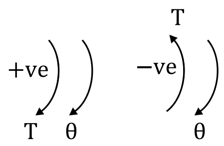
Rotational Kinetic Energy

$$k\varepsilon = \sum \frac{1}{2} m_i\, V_i^2 = \sum \frac{1}{2}\, m_i r_i^2\, w_i^2$$

$$\boxed{k\varepsilon = \frac{1}{2} I \cdot \omega^2}$$

Word done:

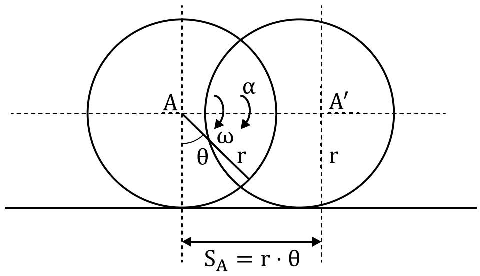
$(wD)_T = T \cdot d\theta$



$$\text{Power (P)} = T \cdot \frac{d\theta}{dt} = T \cdot w$$

$$= \frac{2\pi NT}{60} (N \rightarrow rpm)$$

General Motion = Rotation + Translation



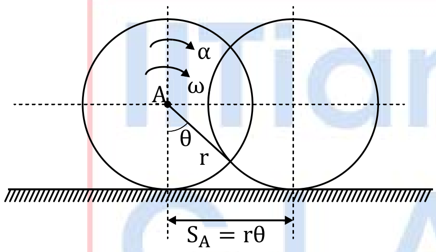
Conditions:

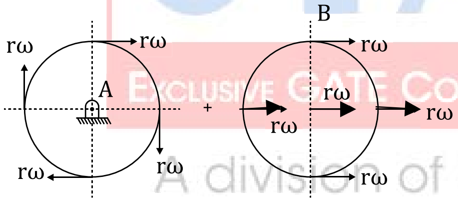
If $S_A = r\theta \rightarrow$ Pure rolling ($V = r\omega \quad a = r\alpha$)

If $S_A > r\theta \rightarrow$ Skidding

If $S_A > r\theta \rightarrow$ Slipping

Velocity Analysis:





(Rotation) (Translation)

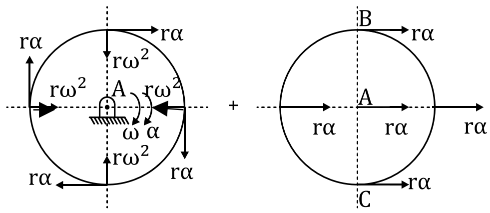
$\vec{V}_T = \vec{V}_{Translation} + \vec{V}_{rotation}$

$\vec{V}_A = rw\hat{i} + 0 = r\omega$

$\vec{V}_B = r\omega i + r\omega\hat{i} = 2r\omega\hat{i}$

$V_c = r\omega\hat{i} - r\omega\hat{i} = 0$ (Pure rolling)

Acceleration Analysis:



Rotation Transatation

$\vec{a}_T = \vec{a}_{Trass} + \vec{a}_{Rot}$

$a_B = r\alpha\hat{i} + r\alpha\hat{i} - r\omega^2\hat{j} = 2r\alpha\hat{i} - r\omega^2\hat{j}$

$a_C = r\alpha\hat{i} - r\alpha\hat{i} + r^2\alpha\hat{j}$

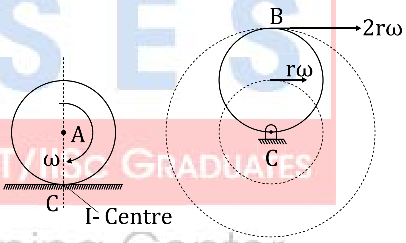
$\boxed{\vec{a}_C \neq 0}$

Note:

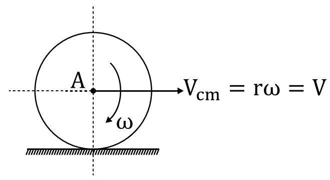
1. In pure rolling static friction will be there on the roller, because contact point has zero velocity.
2. Energy of a roller rolling on rough surface remains conserved because the work done by static friction = 0.

I-Centre (Instantaneous Centre)

Point about which a body is in general motion can be assumed in pure rotation to find velocity only.



Kinetic Energy in Rolling:



Body is rolling due to applied torque (T)

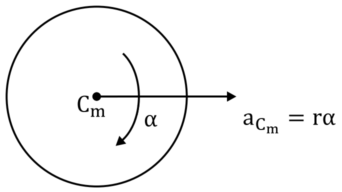
$$(k\varepsilon)_{Rolling} = \frac{1}{2}mV^2 + \frac{1}{2}I_A\omega^2$$

$$= \frac{1}{2}mr^2\omega^2 + \frac{1}{2}I_A\omega^2$$

$$= \frac{\omega^2}{2}[I_A + mr^2]$$

$$(K\varepsilon)_{rolling} = \frac{1}{2}I_C \cdot \omega^2$$

Dynamics:



$\Sigma F_{net} = m a_{cm}$ (NSL)

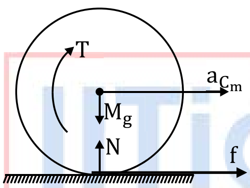
$\Sigma F_{net} = m \cdot r\alpha \rightarrow$ ①

$\Sigma T_{C_m} = I_{C_m} \times \alpha$

$\Sigma T_{Axis} = I_{Axis} \times \alpha$

Friction in Rolling:

Case 1:



Body is rolling due to applied torque (T)

$N = mg \rightarrow$ NFL

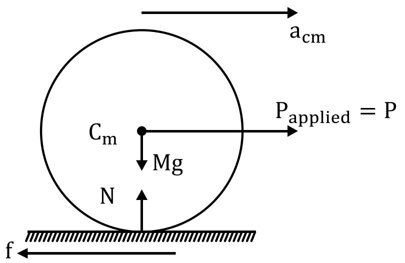
$F = m\, a_{cm} \rightarrow$ NSL

$F = m\, r\, \alpha \quad (T_{app} = T)$

$\Sigma T_{cm} = I_{cm}$

$\left(T_{app} - F(r)\right) = I_{C_m}\alpha$

Case 2:



In this case friction will roll the body

$P_{app} - f(r) = m \cdot a_{cm}$

$a_{cm} = r\alpha$

$\boxed{F(r) = I_{cm}\alpha}$

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