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for

GATE -ME MECHANICAL VIBRATION

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MECHANICAL VIBRATIONS

Chapter 1: Basic of Mechanical Vibrations and Undamped Free Vibration

Free (Natural) Vibration:

Elastic vibrations in which there is no friction and external forces after the initial releases of the body are known as a free or natural vibration.

Damped Vibrations:

When the energy of a vibrating system is gradually dissipated by friction and other resistances. The vibrations are said to be damped. The vibrations gradually cause and the system rests in its equilibrium position.

Forced Vibrations:

When a repeated force continuously acts on a system, the vibrations are said to be forced. The frequency of the vibrations is that of the applied force and is independent of their own natural frequency of vibrations.

Periodic Motion:

A motion which repeats after equal interval time. **Example:** Simple harmonic motion (SHM)

Time Period:

Time to taken for complete one cycle.

Frequency (f) – No. of cycle per unit time

$\omega = 2\pi f(\text{rad/sec})$

Amplitude:

The maximum displacement of a vibrating body from the mean position.

Natural Frequency (ω_n):

- It is a dynamic characteristic of system.
- It is a load independent.
- It is the frequency of free vibration system.

Resonance:

When frequency of system becomes equal to natural frequency of system.

Amplitude of vibration becomes excessive.

Damping

The resistance to the motion of vibrating system.

Basics of Springs

1. **Equation of Stiffness of a Spring:**

   For A helical Spring

   $$k_{\text{equivalent}} = k_{eq} = \frac{d^4 G}{8D^3 n}$$

   d – Wire diameter

   D – Coil diameter

   G – Shear modulus

   n – Number of turns

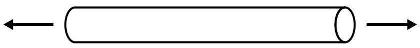
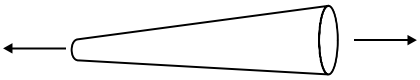
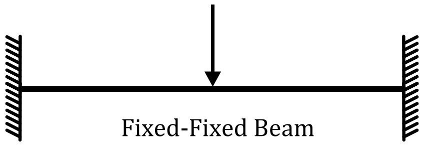
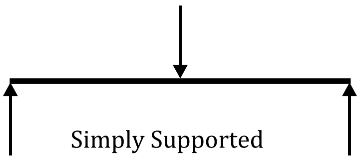
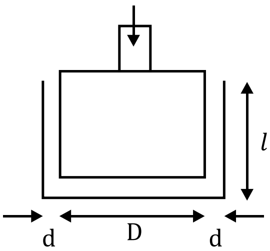
2. **Springs in Series:**

   $$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}$$

   Springs in parallel

   $$k_{eq} = k_1 + k_2 + k_3 + \cdots + k_n$$

3. Equivalent Springs of System

System	Formula of Stiffness
 Rod Under Axial Load	$k = \frac{EA}{l}$
 Tapered Rod	$k = \frac{\pi EDd}{4l}$
 Fixed-Fixed Beam	$k = \frac{192EI}{l^3}$
 Cantilever Beam	$k = \frac{3EI}{l^3}$
 Simply Supported	$k = \frac{48EI}{l^3}$
 Axial motion of Piston in a cylinder (Dashpot damper).	$C_{eq} = \frac{\mu 3\pi D^3 l}{4d^3}\left(1 + \frac{2d}{D}\right)$

4. **Combination of Dampers:**

   a. Parallel $C_{eq} = C_1 + C_2 + \cdots + C_n$

   b. Series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}$

5. **Categories of Vibration:**

   a. $m\ddot{x} + kx = 0$; Free undamped

   b. $m\ddot{x} + c\dot{x} + kx = 0$; Free damped

   c. $m\ddot{x} + kx = F(x)$; Forced undamped

   d. $m\ddot{x} + c\dot{x} + kx =$ $F(x)$; Forced damped

Undamped Free Vibration:

Translational Vibration

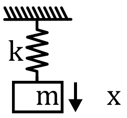
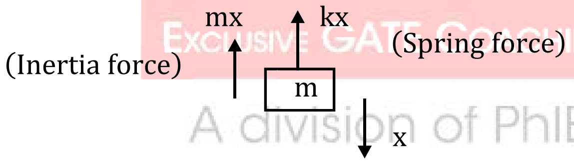


Fig (a) Single degree of freedom

For initial state equilibrium, mg = kΔ

Where Δ = state deflection due to mass



When deflected by x then equation can be written as,

$m\ddot{x} + kx = 0$ .... (1)

This is the equation of simple harmonic and is analogous to,

$\ddot{x} + \omega_n^2 x = 0$

By comparing above equation with equation (1a), we can find

$$\omega_n = \sqrt{\frac{k}{m}}$$

By putting $\frac{k}{m}$ from initial static equilibrium,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\Delta}}$$

Equation of motion for

$m\ddot{x} + kx = 0$

$x = X\sin(\omega_n t + \phi)$ ... ... (2)

Initial Conditions

If the motion is started by displacing the mass through a distance x_0 and giving a velocity V_0, then the solution of equation (2) can be find as,

$t = 0, \quad x = x_0 ; \dot{x} = V_0$

$x = X\sin(\omega_n t + \phi)$

$x_0 = X\sin\phi$

$\dot{x} = X\omega_n \cos(\omega_n t + \phi)$

$V_0 = X\omega_n \cos\phi$

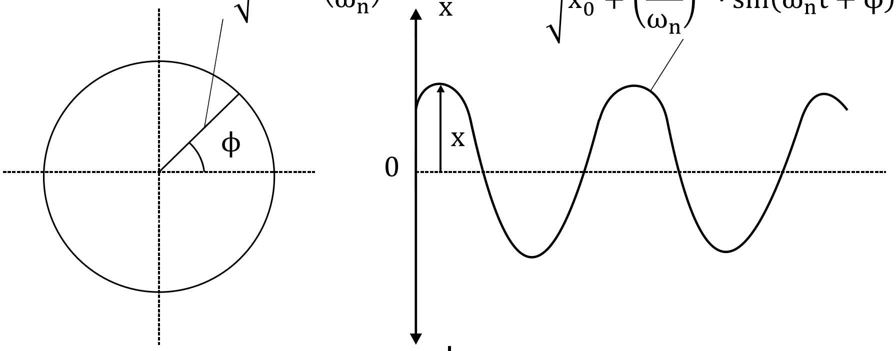
$\rightarrow x_0^2 + \left(\frac{V_0}{\omega_n}\right)^2 = X^2$

$$X = \sqrt{\left[x_0^2 + \left(\frac{V_0}{\omega_n}\right)^2\right]}$$

$$\tan\phi = \frac{x_0}{V_0/\omega_n}$$

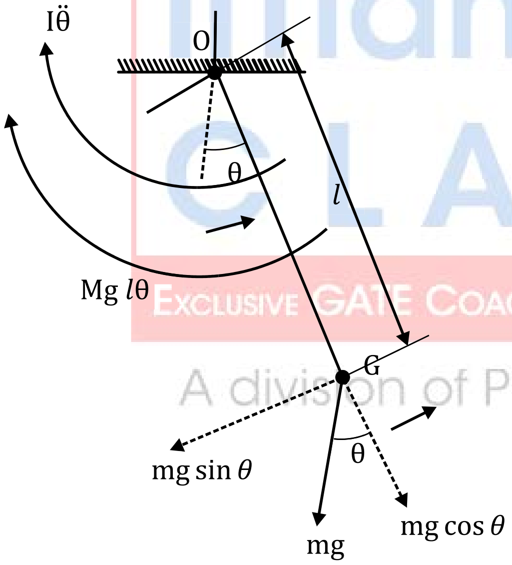
$$\phi = \tan^{-1}\left[\frac{x_0}{V_0/\omega_n}\right]$$

Solution can be plotted graphically



Rotational Vibration

Let Pendulum is displayed by an angle θ of small value.



Here G is the centre of gravity.

Restoring torque = mgsin θ × l = mgl. θ

Inertial torque = $I\ddot{\theta}$

$\therefore I\ddot{\theta} + mgl.\theta = 0$

$$\omega_n = \sqrt{\frac{mgl}{I}}$$

Torsional Vibration

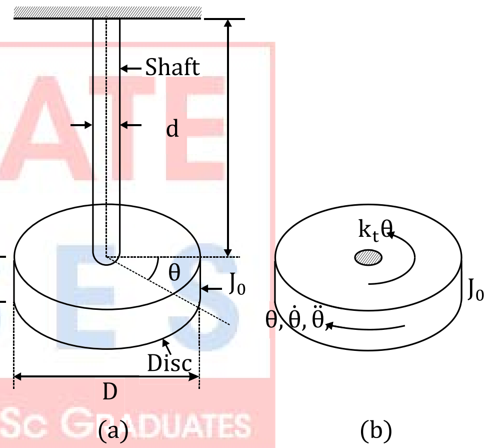


Fig: Torsional Vibration of a Disc

$$J_0\ddot{\theta} + k_t\theta = 0$$

$$\omega_n = \left(\frac{k_t}{J_0}\right)^{1/2}$$

And the period in seconds and frequency of vibration in cycles per second are

$$\tau_n = 2\pi\left(\frac{J_0}{k_t}\right)^{1/2}$$

$$f_n = \frac{1}{2\pi}\left(\frac{k_t}{J_0}\right)^{1/2}$$

Whirling of Shaft:

e = eccentricity of rotor mass from shaft axis.

y = Transverse displacement of shaft

$$y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$

$$\text{Whirling speed} = \omega = \omega_n = \sqrt{\frac{k}{m}}\ \text{rad/sec}$$

Whirling speed/Higher critical speed

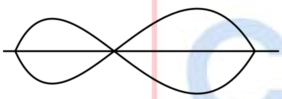
Due to own weight of shaft (W)

ω_1 = First/Basic speed

$$n^4 = \frac{W}{gEI}\,\omega^2$$

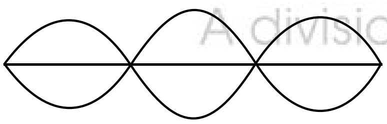
$$\omega_n = n^2\omega_1$$

$$\omega_2 = 4\omega_1$$



2nd fundamental frequency

$$\omega_3 = 9\omega_n$$

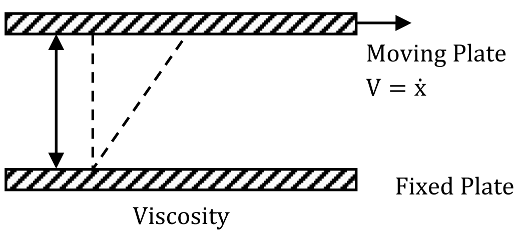


3rd fundamental frequency

Chapter 2: Free Damped Vibration

Types of Damping

- **Viscous Damping**



$$F = \frac{\mu A}{t}\dot{x}$$

Where, A = Area of the plate;

t = Thickness

μ = Coefficient of absolute viscosity of the film

$F = c\,\dot{x}$

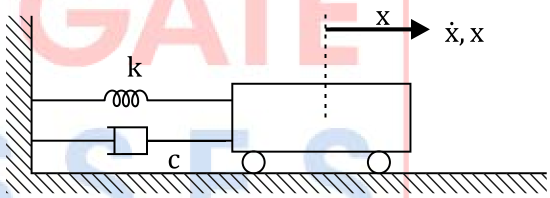
$$\text{Where, } c = \frac{\mu A}{t}$$

Where, c is viscous damping coefficient.

Energy Dissipation in Viscous Damping

$$\Delta E = \pi c \omega A^2$$

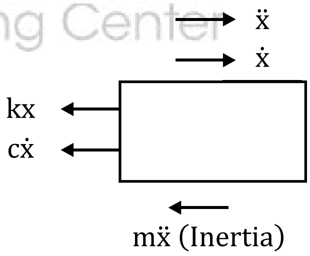
Differential Equation of Damped Free Translational Vibration



The equation of motion,

$$m\ddot{x} + c\dot{x} + kx = 0$$

Free body diagram



For damped $(m\ddot{x} + c\dot{x} + kx = 0)$

Using $x = Ae^{-st}$

$$S_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$= \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

Critical Damping Coefficient:

$C_c = 2m\omega_n = 2\sqrt{km}$

Natural Frequency:

$\boldsymbol{\omega_n = \sqrt{k/m}}$

Damping Factor or Damping Ratio $\zeta = \frac{C}{C_C}$

Now, $\frac{C}{2m} = \frac{C}{C_c} \cdot \frac{C_c}{2m} = \zeta\omega_n$

So,

$S_{1,2} = (-\zeta \pm \sqrt{\zeta^2 - 1})\,\omega_n$

If $C > C_c$ or $\zeta > 1$, system is over damped

$C = C_c$ or $\zeta = 1$, system is critically- damped

$C < C_c$ or $\zeta < 1$, system is under-damped

Over Damped System ($\zeta > 1$)

General solution,

$$x = C_1 e^{(-\zeta + \sqrt{\zeta^2 - 1})\,\omega_n t} + C_2 e^{(-\zeta - \sqrt{\zeta^2 - 1})\,\omega_n t}$$

For Intial condition, $\left.\begin{matrix} x = X_0 \\ \text{And } \dot{x} = 0 \end{matrix}\right\}$

$\rightarrow$ $\boxed{\text{At } t = 0}$

Solving for the condition,

$$x = \frac{X_0}{2\sqrt{\zeta^2 - 1}}\left[\left(-\zeta + \sqrt{\zeta^2 - 1}\right) e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + \left(-\zeta - \sqrt{\zeta^2 - 1}\right) e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}\right]$$

Critical Damped System ($\zeta = 1$)

$S_1 = S_2 = -\omega_n$

Solution, $x = (C_1 + C_2 t)e^{-\omega_n t}$

For initial condition, $\left.\begin{matrix} x = X_0 \\ \text{And } \dot{x} = 0 \end{matrix}\right\}$

$\rightarrow$ $\boxed{\text{At } t = 0}$

$\rightarrow x = X_0(1 + \omega_n t)\, e^{-\omega_n t}$

- Critical damping is smallest damping for which response is non-oscillatory
- With Critical damping system will have steep fall in displacement

Under – Damped System ($\zeta < 1$)

$x = \frac{X_0}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \phi)$ Where, $\phi = \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)$

Amplitude $= \frac{X_0}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t}$

Which is seems to decay exponentially with time. Theoretically, the system will never come to rest, although the amplitude of vibration may become infinitely small.

Period, $T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n(\sqrt{1 - \zeta^2})}$

Damped Frequency:

$\omega_d = \omega_n\sqrt{1 - \zeta^2}$

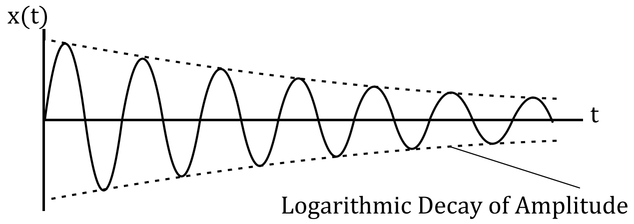
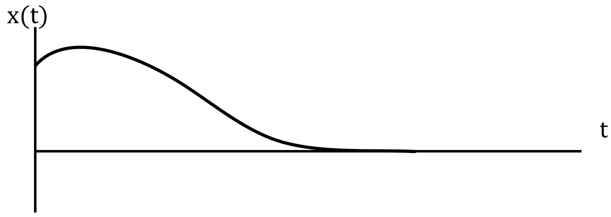
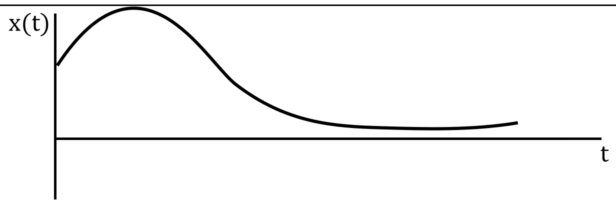
Logarithmic Decrement:

It is the ratio of any two successive amplitudes for an under-damped system vibrating freely is constant and it is a function of damping only

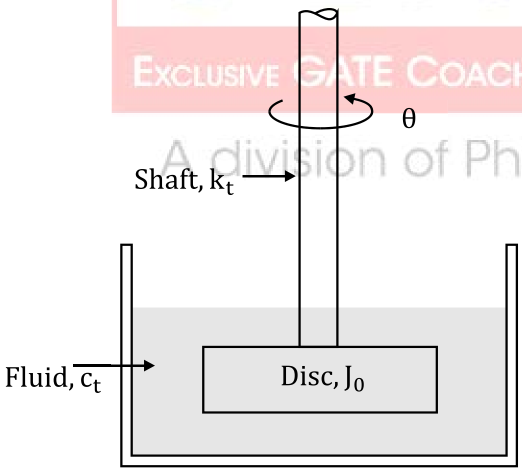
$$\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right) = \frac{2\pi\zeta}{\sqrt{1 - \zeta^2}} \approx 2\pi\zeta(\zeta << 1)$$

Here, number of amplitudes = n+1

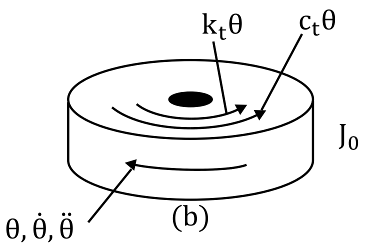
Number of cycles = n - 0 = n

Case	Damping Ratio	Curve
Underdamped	$\zeta < 1$	
Critical	$\zeta = 1$	
Overdamped	$\zeta > 1$	

Differential Equation of Damped Free Torsional Vibration



(a)



(b)

$$J_0\ddot{\theta} + c_t\dot{\theta} + k_t\theta = 0$$

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

$$\omega_n = \sqrt{k_t/J_0}$$

$$\zeta = \frac{c_1}{c_{tc}} = \frac{c_t}{2J_0\omega_n} = \frac{c_t}{2\sqrt{k_t J_0}}$$

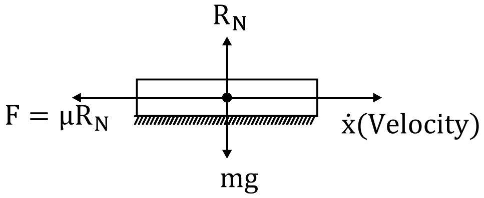
where c_{tc} is the critical torsional damping constant

Note- In case of rotational vibration also equation of motion and other quantities will be like rotational vibration

- **Coulomb Damping**

This type of damping occurs when two machine parts rubs against each other, dry or un- lubricated. The damping resistance in the case is practically constant and is independent of the rubbing velocity.

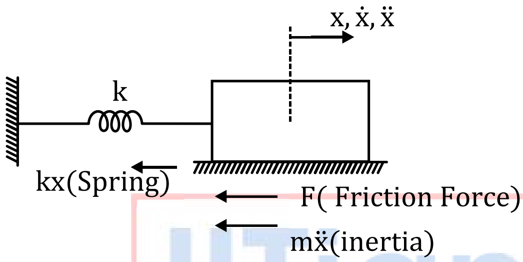
General expression for coulomb damping is:

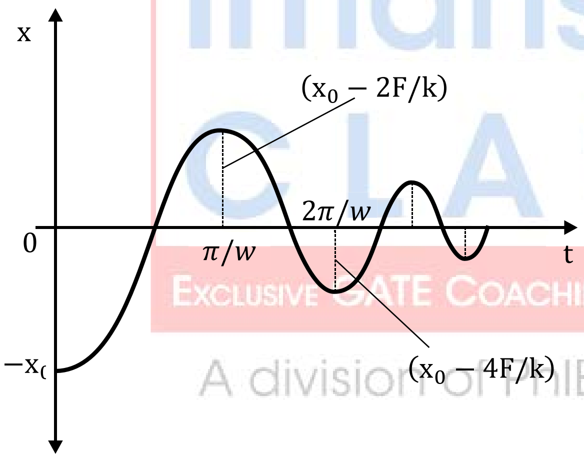


$\boxed{F = \mu R_N}$

Where, μ = coefficient of friction

R_N = Normal reaction





1. The equation of motion is nonlinear with Coulomb damping, whereas it is linear with viscous damping.

2. The natural frequency of the system is unaltered with the addition of Coulomb dumping, whereas it is reduced with the addition of viscous damping.

3. The motion is periodic with Coulomb damping, whereas it can be nonperiodic in a viscously damped (overdamped) system.

4. The system comes to rest after some lime with Coulomb damping, whereas the motion theoretically continues forever (perhaps with an infinitesimally small amplitude) with viscous and hysteresis damping.

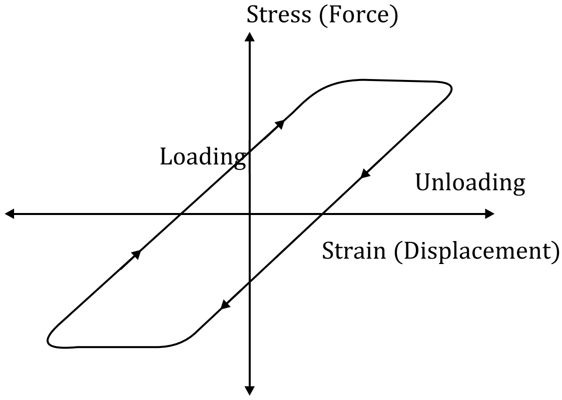
5. The amplitude reduces linearly with Coulomb damping, whereas it reduces exponentially with viscous damping.

6. In each successive cycle, the amplitude of motion is reduced by tire amount 4fiN/k, so the amplitudes at the end of any two consecutive cycles are related:

$$X_n = X_{n-1} - \frac{4\mu N}{k}$$

- **Structural Damping**

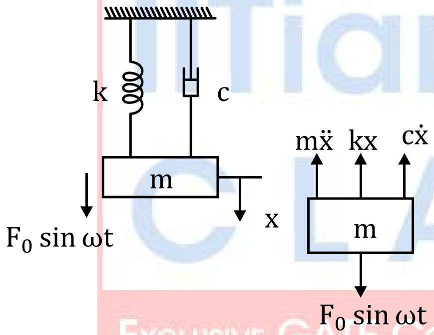
This type of damping is due to the internal friction of the molecules. The stress – strain diagram for vibrating body is not a straight line but forms a hysteresis loop. The area of which represents the energy dissipated due to molecular friction per cycle per unit volume.



The energy loss per cycle,

$E = \pi k \lambda A^2$

Chapter 3: Forced Vibration



$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$

Above differential equation will have two part in solution

The complementary function is obtained from

$m\ddot{x} + c\dot{x} + kx = 0$

$$x_c = A_2 e^{-\zeta \omega_n t} \sin\left[\left(\sqrt{1-\zeta^2}\right)\omega_n t + \phi_2\right]$$

If the particular solution is x_p then

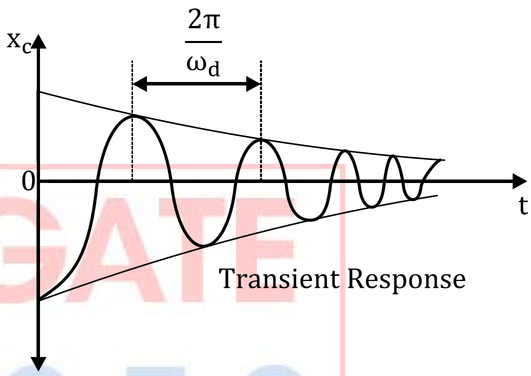
$x_p = X \sin(\omega t - \phi)$

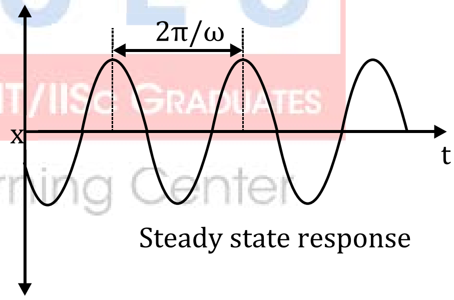
$$\text{Then, } X = \frac{X_{st}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$\phi = \tan^{-1}\left[\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$x = x_c + x_p$$

$$= A_2 e^{-\zeta\omega_n t}\sin\left[\left(\sqrt{1-\zeta^2}\right)\omega_n t + \phi_2\right] + \frac{X_{st}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}} \sin(\omega t - \phi)$$





Magnification Factor

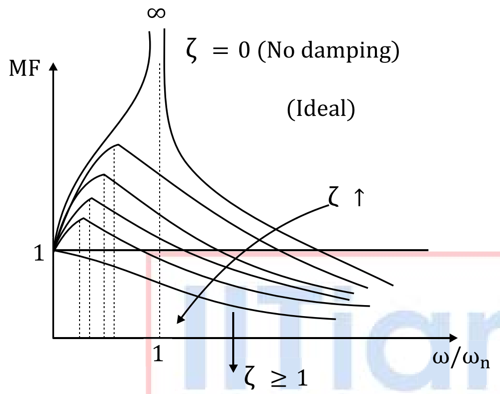
The ratio of the steady state amplitude to the zero frequency (static) deflection i.e., $\frac{X}{X_{st}}$ is defined as the magnification factor and is denoted by M.F.

$$\text{M. F.} = \frac{X}{X_{st}}$$

$$= \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

At resonance, $\frac{\omega}{\omega_n} = 1$

$$M.F. = \frac{X}{X_{st}} = \frac{1}{2\zeta}$$



under damping ↑ ⇒ ζ ↓

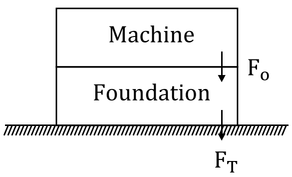
⇒ MF ↑⇒Amplitude ↑⇒Fatigue life ↓↓

Quality Factor

The magnification factor at resonance is known as quality factor and is denoted by R.

$$R = \frac{1}{2\zeta}$$

Vibration Isolation:



$$\epsilon = \frac{F_T}{F_o}$$

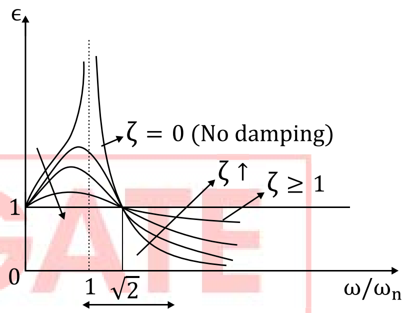
If $0 < \epsilon < 1$ (Machine support)

If $\epsilon > 1$(Machine adverse)

ϵ = Machine Transmissibility

$$\epsilon = \frac{\sqrt{1+\left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

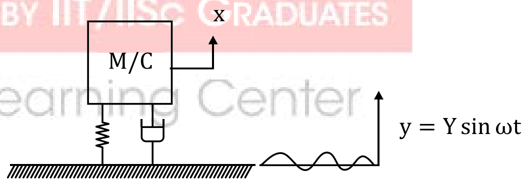
ϵ depends on ζ and ω/ω_n



Damper controlled zone $\left(\frac{\omega}{\omega_n} < \sqrt{2}\right)$

Spring controlled zone $\left(\frac{\omega}{\omega_n} > \sqrt{2}\right)$

Motion Transmissibility:



$$\epsilon = \frac{\text{Aplitude of mass}(X)}{\text{Amplitude of foundation }(Y)}$$

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$\epsilon = \frac{X}{Y} = \pm\frac{\sqrt{k^2 + (c\omega)^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$